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pion pion scattering lengths measurement at NA48-CERN

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Recent experimental and theoretical works allowed the measurement of the pion pion scattering lengths with an experimental accuracy comparable with the precision on the same quantities achieved by Chiral perturbation theory predictions. The excellent agreement between prediction and a measurement of the pion pion scattering lengths by NA48 collaboration is an important validation of the Chiral Dynamics approach.

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1. Introduction

It is well known that at low energy ($E < 1$ GeV), the strong sector of the Standard Model (QCD) has a serious limitation due to its non-perturbative regime. For instance it means that the effect due to quarks confinement in the case of meson decay cannot be computed by usual series expansion technique. A solution for this inconvenient is the replacement of the fundamental QCD Lagrangian with an effective one. Chiral perturbation theory (CHPT) is an effective field theory introduced nearly 30 years ago [1] and soon thereafter developed [2], [3] into a powerful tool able to overcome the mentioned QCD limitation. At the cost of a certain number of free parameters determined from experimental data, CHPT can quantitatively account for effects due to mesons structure and form factors. Quantitative predictions on low energy processes involved in Standard Model tests are in general dependent on the corrections provided by CHPT and as a consequence the interpretation of possible traces of new physics at the confinement regime requires CHPT. Measurements testing the coherence of CHPT and its assumptions are of fundamental importance.

2. Pion pion scattering

Let's consider a pair of pions strongly interacting and let r be the finite size of the interaction range and k the momentum of the pion in the centre of mass frame. At low energy ($kr << 1$), the S-wave dominates the total cross section. The isospin state of a pion pair with zero angular momentum could only be $I = 0$ or $I = 2$ because of Bose-Einstein statistics. The scattering produces a phase shift in the wave function so that the scattering matrix can be parametrised in term of two phases

$$S|\pi\pi >= e^{2i\delta}|\pi\pi >$$

$$\delta_{0,2} = a_{0,2}k + O(k^3)$$

where the subscripts $0,2$ refer to the isospin state. If natural units are used a_0 and a_2 are dimensioned as length and therefore are called scattering lengths. The first prediction for a_0 and a_2 is due to S. Weinberg (1966) [4]

$$a_0 m_\pi = \frac{7m_\pi^2}{32\pi F_\pi^2} = 0.16 \quad , \quad a_2 m_\pi = -\frac{m_\pi^2}{16\pi F_\pi^2} = -0.045$$

where m_π and F_π are the mass and decay constant of the charged pion. Pion pion scattering at low energy is a fundamental process for CHPT, particularly sensitive to the mechanism of spontaneous chiral symmetry breaking. In this context, a more recent prediction, based on numerical solutions of the Roy equation and CHPT to two loops [5], gives the values: $(a_0 - a_2)m_\pi = 0.265 \pm 0.004$ and $a_2 m_\pi = -0.0444 \pm 0.0010$. These values confirm the two-loop result of Ref. [6], although with considerably smaller uncertainties.Analiticity and chiral symmetry provides an extra-constraint to a_0 and a_2 given by the formula [7]

$$a_2 = (-0.0444 \pm 0.0008) + 0.236(a_0 - 0.22) - 0.61(a_0 - 0.22)^2 - 9.9(a_0 - 0.22)^3.$$

In the following we refer to this constraint as the the CHPT constraint. The accuracy in the predictions is at the level of few percent and it is a challenge for experimental physics to measure the pion scattering lengths at the same level of precision. The main sources of data used for this purpose are:

- Ke4 decays: from the final state interaction of the two pions one can extract the phase shift difference $\delta_0 - \delta_1$ where δ_0, δ_1 are the phase shifts for the $I = 0$ S-wave and $I = 1$ P-wave, respectively. The study of the phase shift as a function of the pion-pion invariant mass can be used to extract the pion pion scattering lengths. Details of the method and recent experimental results are discussed in the contribution of Bloch-Deveaux in these Proceedings [8].
- Decay of pionium: the decay rate of pionium is proportional to $(a_0 - a_2)^2$ and the combination $|a_0 - a_2|$ can be extracted from a measurement of pionium lifetime [9].
- Cusp structure in $K \to 3\pi$: this is the most recent topic and will be discussed in the following.

3. First observation of the cusp: a question of calibration

The NA48 collaboration scientific program came to end in 2006 after 10 years of activities mainly focused on K-meson physics summarised in the following items

- 1996-2002: Study of direct CP violation in $K_{L,S} \to 2\pi$ decay and determination of the CP violating parameter $\mathscr{R}e(\varepsilon'/\varepsilon)$ [10].
- 2003-2006: Study of CP violation in $K^{\pm} \to 3\pi$ decay by looking at possible difference between positive and negative Kaon in the Dalitz plot distribution [11] and measurement of the $\pi\pi$ phase shift in $K^{\pm} \to e^{\pm}\nu\pi\pi$ (Ke4) decay [12].

The experiment sensitivity to CP violation effects is at the level of 10^{-4} when good statistics of K-meson decays and sufficient control of systematics induced by the experimental apparatus are provided. NA48 experiment detects final state photons by using a calorimeter based on liquid Krypton. The device has an excellent energy resolution measured to be

$$\frac{\sigma(E)}{E} = \frac{0.09}{E_{[GeV]}} \oplus \frac{0.032}{\sqrt{E_{[GeV]}}} \oplus 0.0042$$

in the range $3 < E < 100$ GeV. In order to limit systematics a frequent monitoring of energy scale, calibration and resolution parameters of the device is required. A check of the apparatus performances is done by measuring masses of mesons and by comparing results with the accepted values. The method essentially makes use of $\pi^0, \eta \to 2\gamma$ decay samples cross-checked with $\eta \to 3\pi^0$ events. As a by product, the collaboration published a very precise measurement of the η meson mass [13] later confirmed by a more recent measurement at Frascati collider [14]. In year 2003, while NA48 was collecting a large sample of $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ decays, it was suggested to consider pionium atom as a valid candidate for a third calibration point set in the middle between π^0 and η masses, the pionium mass being twice the charged pion mass m_+. Pionium ($A_{2\pi}$) is the electromagnetically bound state formed by two pions with opposite charges and it is known to decay mostly into $2\pi^0$. An excess of $O(1000)$ events due to pionium formation was expected on top of

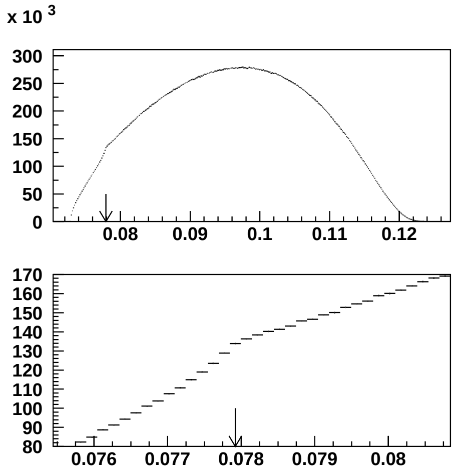

Figure 1: m^2_{00} distribution. Entire range (above) and zoom around the value $m_{00} = 2m_+$ indicated by the arrow (below). The statistical error bars are also shown in these plots.

the natural $\pi^0\pi^0$ invariant mass (m_{00}) distribution. The estimation is based on the prediction given in reference [15]

$$\frac{\Gamma(K^+ \to \pi^+ A_{2\pi})}{\Gamma(K^+ \to \pi^+\pi^+\pi^-)} \simeq 10^{-5}$$

After the collection of 60×10^6 $K^\pm \to \pi^\pm\pi^0\pi^0$ events, the m^2_{00} distribution appeared as in figure 1 showing an unexpected cusp structure located at the pionium mass value.

4. First interpretation of the cusp

The first answer to the question "What is that cusp ? " came from N. Cabibbo [16]. His explanation tells that the weak decay $K^\pm \to \pi^\pm\pi^0\pi^0$ has a dominant contribution due to the three pions direct emission with amplitude

$$\mathscr{M}_0 = 1 + gu/2 + hu^2/2 + kv^2/2$$

where u, v are the usual Dalitz plot variables and g, h, k are the linear and quadratic slopes; a second order contribution is due to a three charged pions direct emission followed by charge exchange reaction $\pi^+\pi^- \to \pi^0\pi^0$. The amplitude for this second order effect is given by

$$\mathscr{M}_1 = i\frac{2}{3}m_+(a_0 - a_2)(1+\varepsilon/3)A_{++-}\sqrt{1-\left(\frac{2m_+}{m_{00}}\right)^2}$$

where A_{++-} is the tree level matrix element for $K^+ \to \pi^+\pi^+\pi^-$ decay and a_0 and a_2 are the pion scattering lengths. The combination $a_0 - a_2$ enters as a coupling constant in the charge exchange process.

The cusp finds its origin when $\mathscr{M}_1$ changes from real to imaginary as m_{00} crosses the threshold value $2m_+$ with the consequence that $\mathscr{M}_1$ interferes destructively with $\mathscr{M}_0$ below the threshold while it adds quadratically above it. This singular behaviour of cross section near threshold is a general well known phenomenon studied in the past by Wigner [17] and for the first time applied to K-meson in reference [18].

Isospin breaking effects are accounted for by the factor

$$\varepsilon = \frac{m_+^2 - m_0^2}{m_+^2}$$

which does not appear in the original Cabibbo's paper but it has been included following reference [19].

One can easily realize that a fit to experimental m_{00} distribution can be used to extract the value of $a_0 - a_2$. By analyzing a fraction of the collected statistics, the NA48 collaboration published a preliminary result [21] based on a two loops extension of the Cabibbo's model [20].

5. The Bern-Bonn approach

A more complete approach for pion pion rescattering effect in $K \to 3\pi$ has been proposed in reference [22]. This approach is based on a non-relativistic field theory framework and uses two expansion parameters: a, the generic $\pi\pi$ scattering length at threshold and a formal parameter ε such that in the K-meson rest frame the pion momentum is of order ε and its kinetic energy of order ε^2. The present formulation computes the $K \to 3\pi$ amplitudes including terms up to $O(\varepsilon^4, a\varepsilon^5, a^2\varepsilon^2)$ and it can be used to fit simultaneously the invariant mass distribution of same charge $\pi\pi$ pair (i.e. $\pi^0\pi^0$ from $K^\pm \to \pi^\pm\pi^0\pi^0$ and $\pi^\pm\pi^\pm$ from $K^\pm \to \pi^\pm\pi^+\pi^-$) indicated as m_{00} and $m_{\pm\pm}$, respectively. Radiative corrections to both $K^\pm \to \pi^\pm\pi^0\pi^0$ and $K^\pm \to \pi^\pm\pi^+\pi^-$ have been recently studied by extending the Bern-Bonn approach [22] to include real and virtual photons [23]. The radiative corrections provided can be used over the entire spectrum outside the threshold where they diverge.

6. Extraction of a_0 and a_2

The spectrum shown in figure 1 has been fitted using both Cabibbo's and Bern-Bonn approach over the m_{00}^2 interval from 0.074094 to 0.104244GeV^2. In the case of Bern-Bonn also the $m_{\pm\pm}^2$ distribution has been fitted in the interval from 0.080694 to 0.119844 GeV^2. Intervals have been chosen in order to minimize detection efficiency effects due to the experimental apparatus. The fitting function contains 9 parameters

- $a_0 - a_2$, a_2: pion pion scattering lengths.
- f_{atom}: excess of event due to pionium formation at the threshold.
- g_0, h_0, k_0: linear and quadratic slopes for $K^\pm \to \pi^\pm\pi^0\pi^0$.
- g, h, k: linear and quadratic slopes for $K^\pm \to \pi^\pm\pi^+\pi^-$.

All 9 parameters can be determined from χ^2 minimization by using the Bern-Bonn approach, while in the case of the Cabibbo's model the last 3 parameters are fixed at the value published in the PDG.

Several fits have been performed by leaving all parameters free or by fixing some of them. In some fits a group of seven consecutive bins centred at the threshold (i.e. an interval of ± 0.94 MeV) have been excluded from the fitting to skip the pionium formation region. In other fits the group of seven bins were included and parameter f_{atom} set free. Fits results are given in tables 1 and 2. CI and BB means Cabibbo-Isidori model [20] and Bern-Bonn approach [22], respectively; subscript A means amount of pionium atom kept fixed and 7 bins around threshold excluded; superscript χ means CHPT constraint on. Statistical errors are shown in brackets. Note that the number of degrees of freedom between CI and BB differs because BB has been used to fit simultaneously m_{00} and $m_{\pm\pm}$ distributions while CI fits only m_{00} one. Somehow arbitrarily the radiative corrections

Fit	χ^2/NDF	$(a_0 - a_2)m_\pi$	$a_2 m_\pi$	f_{atom}
CI	206.3/195	0.2727(46)	-0.0392(80)	0.0533(91)
CI_A	201.6/189	0.2689(50)	-0.0344(86)	0.0533
CI^χ	210.6/196	0.2749(21)	-0.0413	0.0441(76)
CI^χ_A	207.6/190	0.2741(21)	-0.0415	0.0441
BB	462.9/452	0.2815(43)	-0.0693(136)	0.0530(95)
BB_A	458.5/446	0.2775(48)	-0.1593(142)	0.0542
BB^χ	467.3/453	0.2737(26)	-0.0417	0.0647(76)
BB^χ_A	459.8/447	0.2722(27)	-0.0421	0.0647

Table 1: Fits results without radiative correction. Parameter values without errors have been kept fixed or calculated using the CHPT constraint.

Fit	χ^2/NDF	$(a_0 - a_2)m_\pi$	$a_2 m_\pi$	f_{atom}
CI	205.6/195	0.2483(45)	-0.0092(91)	0.0625(92)
CI_A	202.9/189	0.2461(49)	-0.0061(98)	0.0625
CI^χ	222.1/196	0.2646(21)	-0.0443	0.0420(77)
CI^χ_A	219.7/190	0.2645(22)	-0.0444	0.0420
BB	477.4/452	0.2571(48)	-0.0241(129)	0.0631(97)
BB_A	474.4/446	0.2544(51)	-0.0194(132)	0.0631
BB^χ	479.8/453	0.2633(24)	-0.0447	0.0538(77)
BB^χ_A	478.1/447	0.2627(25)	-0.0449	0.0538

Table 2: Fits results with radiative correction. Parameter values without errors have been kept fixed or calculated using the CHPT constraint.

[23], developped in the context of BB approach, have been applied also to CI model [20]. This make sense for Coulomb and bremsstrahlung corrections which are universal but it is not correct for direct photon emission. As partial justification, it should be noted that the structure dependent radiative effects are small compared to the others. Radiative corrections produce a shift in the results comparable to the statistical error, the effect is shown in figure 2

The Bern-Bonn approach combined with radiative correction provides presently the most complete description of rescattering effect so we have decide to quote as our final result the numbers

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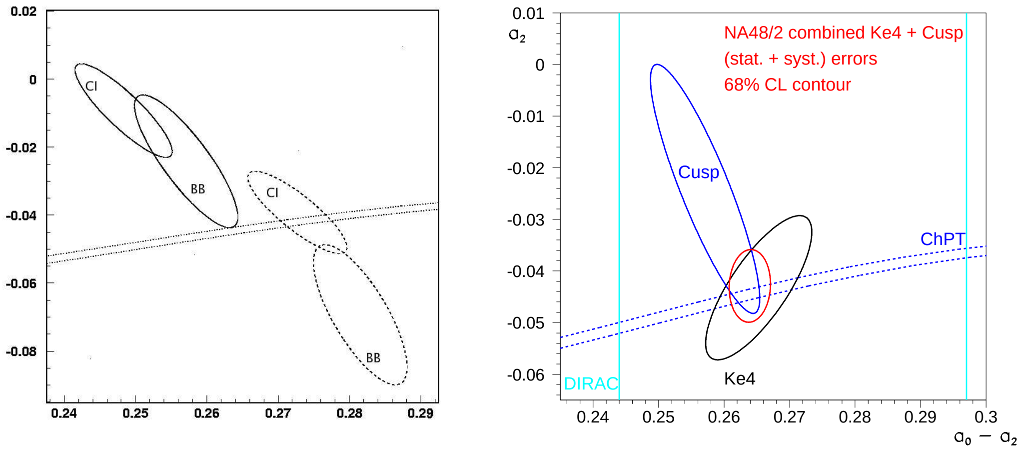

Figure 2: At left : 68% confidence level ellipses taking into account the statistical uncertainties only. Dashed lines ellipses: fits CI and BB without radiative corrections. Solid line ellipses: fits CI and BB with radiative corrections. The theoretical band allowed by CHPT constraint is shown by the dotted curve. At right: 68% confidence level ellipses corresponding to the cusp and to the Ke4 measurement. The band from DIRAC experiment is also shown. The small ellipse is the combination of cusp and Ke4 results

reported in the BB row of table 2 :

$$(a_0 - a_2)m_+ = 0.2571 \pm 0.0048(stat) \pm 0.0025(syst) \pm 0.014(ext)$$

$$a_2 m_+ = -0.024 \pm 0.013(stat) \pm 0.009(syst) \pm 0.002(ext)$$

If CHPT constraint is used (see BB^{χ} of table 2) we obtain

$$(a_0 - a_2)m_+ = 0.2633 \pm 0.0024(stat) \pm 0.0014(syst) \pm 0.0019(ext)$$

The external error quoted originates from the uncertainty on the ratio

$$\frac{\Gamma(K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-})}{\Gamma(K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0})} = 3.175 \pm 0.050$$

The main systematics affecting the measurement is due to the discrepancy between the measured distribution of the K-meson momentum (peaked at 60 GeV with 3 GeV FWHM) in the laboratory frame and the one predicted by the Montecarlo simulation. The second source is due to non linearity in the energy reconstruction of photons below 6 GeV. Figure 2 at right shows a comparison between the result obtained by NA48 from cusp and from Ke4 sample [8] as well as the $(a_0 - a_2)$ measurement from pionium lifetime by DIRAC collaboration [9].

7. Cusp in $K_L \to 3\pi^0$

Recently a cusplike structure has been seen by KTeV collaboration in a 70 M events sample of fully reconstructed $K_L \to 3\pi^0$ [24]. The cusp originates from the same mechanism discussed in section 4. Here we have the interference between the tree level amplitude $\mathscr{M}_0 = 1 + h(u^2 + v^2/3)/2$ with the small second order amplitude $\mathscr{M}_1$,due to $K_L \to \pi^+\pi^-\pi^0$ followed by charge exchange rescattering,

$$\mathscr{M}_1 \propto (a_0 - a_2)A_{+-0}\sqrt{1-\left(\frac{2m_+}{m_{00}}\right)^2}$$

where A_{+-0} is the amplitude relative to the process $K_L \to \pi^+\pi^-\pi^0$. The interference effect in K_L is expected to be smaller than in $K^\pm$ as can be argued by defining the parameter $R = \frac{\mathscr{M}_1}{\mathscr{M}_0}$ and considering the ratio

$$\frac{R(K^+)}{R(K_L)} = \frac{2A_{++-}}{A_{+00}} / \frac{A_{+-0}}{A_{000}}.$$

The factor 2 is due to combinatorics accounting for the two possible pion pairs available for a charge exchange rescattering in the (++-) case. The explicit calculation at the threshold from measured partial widths and slope parameters gives

$$\frac{R(K^+)}{R(K_L)} \simeq 13.$$

The cusp visibility is therefore ~ 13 times higher in K^+ than in K_L. In the paper published by KTeV collaboration [24] the extraction of $(a_0 - a_2)$ is based on the Cabibbo's two loops calculation [20] and the value found is

$$(a_0 - a_2)m_+ = 0.215 \pm 0.014(stat) \pm 0.025(syst) \pm 0.006(ext).$$

The central value is somehow off the mainstream of the other measurements and the systematics is dominated by the choice of the fitting region. Once data have been corrected for detection efficiency, a residual discrepancy between data and Cabibbo's model is seen below the cusp in the interval $0.270 < m_{00} < 0.274$ GeV$/c^2$ and this interval has been excluded from the fitting.

In year 2000 experiment NA48 collected a sample of $\sim 100M$ $K_L \to 3\pi^0$ decays which is now being analysed. Due to its reduced visibility, the cusp can only be seen from data to phase space ratio plotted in figure 3. The analysis is still at a preliminary stage but so far fits tend to give $(a_0 - a_2)$ values smaller than the predicted one confirming the KTeV measurement even if no anomaly below 0.274 GeV has been seen. Fits have been performed by leaving $(a_0 - a_2)$ and the quadratic slope h as free parameters to be determined from χ^2 minimization. A very large statistical errors correlation $\sim 90\%$ between the two parameters is found in agreement with a similar correlation found by KTeV. It has been observed that when the quadratic slope h is frozen to zero the fitted value of $a_0 - a_2$ tends to be consistent with the predicted one. No significant difference has been seen by adopting Cabibbo's model or Bern-Bonn approach.

8. Conclusions and perspectives

Three independent experimental techniques, based on Ke4, pionium lifetime and cusp in charged K-meson, have been adopted to measure the pion pion scattering lengths obtaining results in remarkable agreement with each other and with the prediction of CHPT. Despite the reduced sensitivity to $a_0 - a_2$, the cusp observed in the neutral K-meson by KTeV and NA48 experiments could

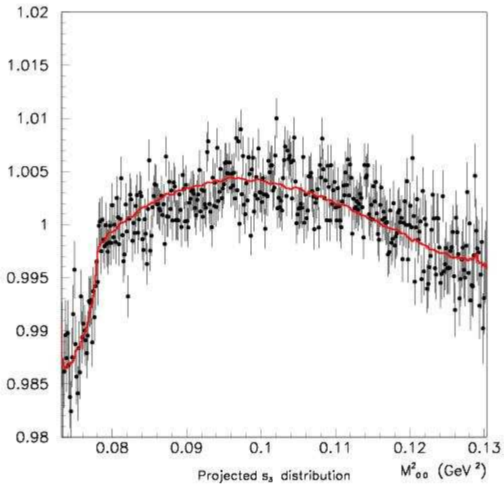

Figure 3: NA48 Data to phase space ratio projection of $K_L \to 3\pi^0$ Dalitz plot

in principle give a new independent measurement but first it should be clarified if the anomaly below the cusp seen by KTeV is instrumental or genuine and if the strong correlation between $a_0 - a_2$ and the quadratic slope is or not a serious limitation. In order to consolidate the pion pion scattering lengths extraction technique, it would be desirable to apply the method in other processes outside the K-meson system. A theoretical estimate for cusp effect in $\eta \to 3\pi^0$ has been recently published in [25] and a corresponding experimental search for a cusplike structure based on a sample of 3×10^6 $\eta \to 3\pi^0$ events has been reported in reference [26]. A better statistics would be needed to draw any final conclusion, however a deviation from the prediction below the $\pi\pi$ threshold can be seen in the data. A more promising channel in term of cusp visibility seems to be the decay $\eta' \to \eta\pi\pi$ [27].

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