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Isospin-breaking corrections to the pion–nucleon scattering lengths

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We analyze isospin breaking through quark mass differences and virtual photons in the pion–nucleon scattering lengths in all physical channels in the framework of covariant baryon chiral perturbation theory. The so-called triangle relation is found to be violated by about 1.5 %. We encounter a substantial isospin-breaking correction to neutral-pion–nucleon scattering beyond Weinberg's prediction due to a cusp effect. Finally, the application to hadronic atoms is briefly discussed.

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1. Introduction

Isospin violation in the Standard Model is driven by strong and electromagnetic interactions, that is by the differences in the light quark masses and charges, respectively. Already in [1] Weinberg stressed that the pion–nucleon scattering lengths offer a particularly good testing ground for strong isospin violation, predicting e.g. a large effect in the difference of the neutral-pion–nucleon scattering lengths $a_{\pi^0 p} - a_{\pi^0 n}$. Isospin violation in πN scattering was addressed in the framework of heavy-baryon chiral perturbation theory (ChPT) in a series of papers about a decade ago [2 – 6]. Recently, new interest arose in high-precision calculations of the pion–nucleon scattering lengths. First, corrections for isospin violation are an essential ingredient to extract the isoscalar and isovector πN scattering lengths a^+ and a^- from pionic hydrogen (πH) and deuterium (πD) measurements to high precision. The isospin-breaking corrections to $a_{\pi^- p\to\pi^- p}$ needed for the ground state of πH were calculated in [7] at third chiral order. A consistent description requires the knowledge of $a_{\pi^- p\to\pi^0 n}$ (width of πH) and $a_{\pi^- n\to\pi^- n}$ (ground state of πD) at the same accuracy. In the analysis of πD isospin violation is particularly important, since the πd scattering length at leading order is proportional to the small isoscalar scattering length a^+ and therefore chirally suppressed [8]. Second, threshold pion photoproduction offers the unique possibility of measuring the so far undetermined $\pi^0 p$ scattering length and gives access to the charge exchange scattering length $a_{\pi^+ n\to\pi^0 p}$ [9, 10]. Such measurements are becoming feasible at HIγS and at MAMI. In view of these developments, we have extended the work of [7] to *all* charge channels in pion–nucleon scattering in [11].

2. Formalism

The calculation of the scattering lengths is performed at $\mathscr{O}(p^3)$ in manifestly covariant baryon ChPT [12] and at first order in the isospin-breaking parameter $\delta = \mathscr{O}(e^2, m_\mathrm{d} - m_\mathrm{u})$, using the effective Lagrangian for nucleons, pions, and virtual photons, as constructed in [7]. We denote the masses of proton, neutron, charged and neutral pion by m_p, m_n, M_π, and M_{π^0}, respectively, and define the isospin limit by the charged particles. The mass differences are expressed by $\Delta_\pi = M_\pi^2 - M_{\pi^0}^2$ and $\Delta_\mathrm{N} = m_\mathrm{n} - m_\mathrm{p}$.

The one-loop topologies are depicted in Fig. 1. As soon as we take into account virtual photons, we have to specify how to deal with the threshold divergences: first of all, we subtract all one-photon-reducible diagrams, since they diverge $\sim 1/t$ (s and t are the usual Mandelstam variables), and denote the resulting amplitude by $\tilde{T}$. The additional divergences due to photon loops may be regularized in the form

$$e^{iQ\alpha\theta_\mathrm{C}(|\mathbf{p}|)}\tilde{T}\Big|_{|\mathbf{p}|\to 0} = \frac{\beta_1}{|\mathbf{p}|} + \beta_2 \log\frac{|\mathbf{p}|}{\mu_\mathrm{c}} + T_\mathrm{thr} + \mathscr{O}(|\mathbf{p}|), \tag{2.1}$$

where $\mathbf{p}$ denotes the center-of-mass momentum, $\alpha = e^2/4\pi$ the fine structure constant, $\theta_\mathrm{C}(|\mathbf{p}|)$ the infrared divergent Coulomb phase given by

$$\theta_\mathrm{C}(|\mathbf{p}|) = -\frac{\mu_\mathrm{c}}{|\mathbf{p}|}\log\frac{m_\gamma}{2|\mathbf{p}|}, \tag{2.2}$$

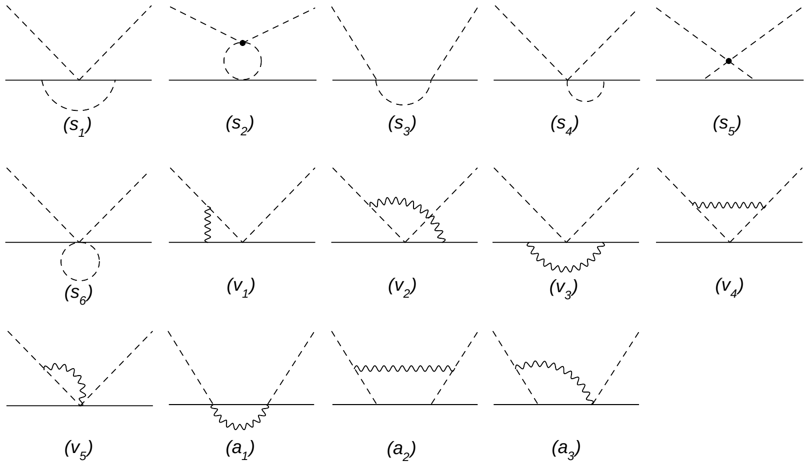

Figure 1: One-loop topologies for πN scattering at threshold. Solid, dashed, and wiggly lines, denote nucleons, pions, and photons, respectively. Crossed diagrams and diagrams contributing via wave function renormalization only are not shown.

$\mu_c = m_p M_\pi/(m_p + M_\pi)$ the reduced mass of the incoming particles, and Q accounts for the charges of the particles involved. The term $\propto 1/|\mathbf{p}|$ referred to as Coulomb pole is solely generated by (v_1) at this order, while β_2 only enters at two-loop level. The scattering lengths are finally given by

$$a = \frac{T_{\text{thr}}}{8\pi\sqrt{s}}. \tag{2.3}$$

It turns out that for the charged-pion elastic channels the triangle graph (s_5) yields a very large contribution. In addition, (s_3) can generate cusp effects, which are proportional to $\sqrt{\delta}$ and thus enhanced by $\sqrt{\delta}$. For the analytical results and more details of the calculation we refer to [11].

3. Numerical results

In the isospin limit, the πN scattering lengths are solely determined by a^+ and a^-. Subtracting these contributions, we obtain the following isospin-breaking shifts (in units of $10^{-3}M_\pi^{-1}$):

isospin limit	channel	shift	channel	shift
$a^+ + a^-$	$\pi^- p \to \pi^- p$	$-3.4^{+4.3}_{-6.5} + 5.0i$	$\pi^+ n \to \pi^+ n$	$-4.3^{+4.3}_{-6.5} + 6.0i$
$a^+ - a^-$	$\pi^+ p \to \pi^+ p$	$-5.3^{+4.3}_{-6.5}$	$\pi^- n \to \pi^- n$	$-6.2^{+4.3}_{-6.5}$
$-\sqrt{2}a^-$	$\pi^- p \to \pi^0 n$	0.4 ± 0.9	$\pi^+ n \to \pi^0 p$	2.3 ± 0.9
a^+	$\pi^0 p \to \pi^0 p$	-5.2 ± 0.2	$\pi^0 n \to \pi^0 n$	-1.8 ± 0.2

The precise values of the low-energy constants as well as a detailed estimate of the theoretical uncertainties can be found in [11]. Our result for the triangle relation, which vanishes in the isospin limit and is thus a convenient way to quantify isospin violation in terms of measurable quantities, reads

$$R = 2\frac{a_{\pi^+ p\to\pi^+ p} - a_{\pi^- p\to\pi^- p} - \sqrt{2}\,a_{\pi^- p\to\pi^0 n}}{a_{\pi^+ p\to\pi^+ p} - a_{\pi^- p\to\pi^- p} + \sqrt{2}\,a_{\pi^- p\to\pi^0 n}} = (1.5 \pm 1.1)\,\%. \tag{3.1}$$

The difference between the elastic neutral-pion–nucleon scattering lengths is found to be

$$a_{\pi^0 p} - a_{\pi^0 n} = \frac{m_{\mathrm{p}}}{4\pi(m_{\mathrm{p}}+M_\pi)}\left\{\frac{4c_5 B(m_{\mathrm{d}}-m_{\mathrm{u}})}{F_\pi^2} - \frac{M_\pi^2}{8\pi F_\pi^4}\left(\sqrt{\Delta_\pi+2M_\pi\Delta_{\mathrm{N}}} - \sqrt{\Delta_\pi-2M_\pi\Delta_{\mathrm{N}}}\right)\right\}$$
$$= \big((-2.3\pm0.4)-1.1\big)\cdot 10^{-3}M_\pi^{-1} = (-3.4\pm0.4)\cdot 10^{-3}M_\pi^{-1}. \tag{3.2}$$

The first term was already given by Weinberg in [1], while the second one is due to a cusp effect and contributes roughly one third to the final result.

4. Application to hadronic atoms

a^+ and a^- can be related to the strong shift of the ground state of πH and πD and to the width of πH via Deser-type formulae [13 – 16], to which isospin-breaking corrections are an essential ingredient. The impact of our isospin-breaking corrections on the extraction of a^- and

$$\tilde{a}^+ = a^+ + \frac{m_{\mathrm{p}}}{4\pi(m_{\mathrm{p}}+M_\pi)}\left\{\frac{4\Delta_\pi}{F_\pi^2}c_1 - 2e^2 f_1\right\} \tag{4.1}$$

is displayed in Fig. 2:[1] the constraint due the width of πH barely changes, while the the bands for the level shift of πH and πD significantly move upwards when going from $\mathcal{O}(p^2)$ to $\mathcal{O}(p^3)$, corresponding to the small corrections to the charge exchange reaction and a large shift in the charged-pion elastic channels due to the triangle graph alluded to above.

However, Fig. 2 does not provide a complete picture at $\mathcal{O}(p^3)$: the few-body contributions to $a_{\pi d}$ [20 – 22] are still based on the assumption of isospin symmetry. It remains to be seen whether the nice consistency between the three bands persists once isospin violation in the few-body part is included.

5. Summary and outlook

We have systematically analyzed isospin violation in the πN scattering lengths in all channels, including an estimate of the theoretical uncertainties. We find that isospin breaking is quite small in $\pi^- p \to \pi^0 n$, at the order of one percent at most, whereas the charged-pion elastic channels display more sizeable effects on the few-percent level. In particular, the triangle relation is violated by about 1.5% consistent with earlier findings in heavy-baryon ChPT and inconsistent with the 5–7% deviation extracted from the data at lowest pion momenta in [23, 24]. In addition, we find a substantial isospin-breaking correction to the neutral-pion–proton scattering length. Finally, we have shown that in a full $\mathcal{O}(p^3)$ calculation the value of the isoscalar scattering length will increase as compared to previous analyses of hadronic atoms [8].

An extension of this analysis beyond threshold for $\pi^\pm p \to \pi^\pm p$ and $\pi^- p \to \pi^0 n$ can be found in [25], while a full calculation of isospin violation in the few-body contributions to the πd scattering length will be addressed in [26].

[1] Experimental input: level shift of πH: $\varepsilon_{1s} = (-7.120\pm0.017)\,\mathrm{eV}$ [17], width of πH: $\Gamma_{1s} = (0.823\pm0.019)\,\mathrm{eV}$ [18], and $\pi^- d$ scattering length: $a_{\pi d} = (-0.0261\pm0.0005)\,M_\pi^{-1}$[19].

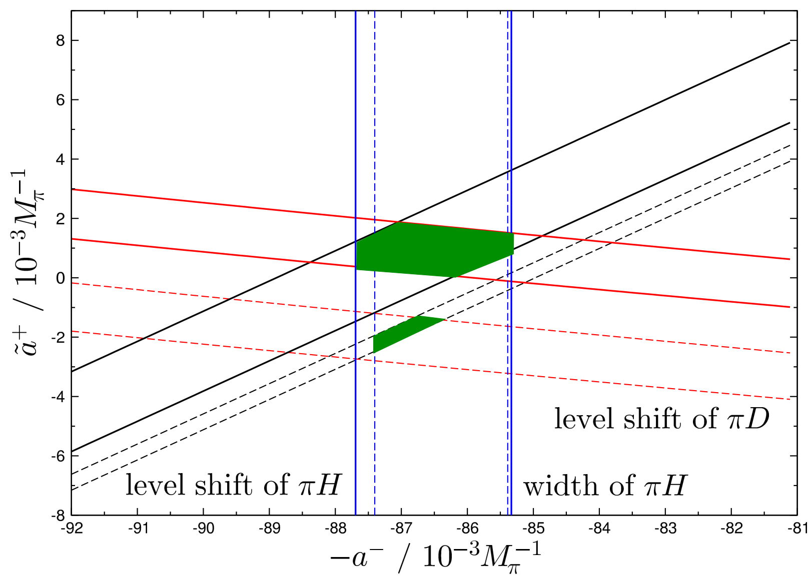

Figure 2: Constraints on a^- and $\tilde{a}^+$ provided by the level shift of πH and πD and the width of πH. Solid (dashed) bands refer to isospin-breaking corrections at $\mathscr{O}(p^3)$ ($\mathscr{O}(p^2)$), respectively.

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