

## Pion-nucleon scattering around the delta resonance

---

**Bingwei Long\***

*European Centre for Theoretical Studies in Nuclear Physics and Related Areas (ECT\*)*

*Strada delle Tabarelle 286*

*I-38100 Villazzano (TN), Italy*

*E-mail: long@ect.it*

We develop a generalized version of heavy-baryon chiral perturbation theory to describe pion-nucleon scattering in a kinematic domain that extends continuously from threshold to the delta-isobar peak. The  $P$ -wave phase shifts are used to illustrate this framework.

*6th International Workshop on Chiral Dynamics*

*July 6-10 2009*

*Bern, Switzerland*

---

\*Speaker.

## 1. Introduction

A prominent feature of pion-nucleon ( $\pi N$ ) scattering is the delta resonance,  $\Delta(1232)$ , a peak in the elastic cross section at the center-of-mass (CM) energy  $m_\Delta \equiv m_N + \delta \simeq 1230$  MeV, where  $\delta \sim 290$  MeV is the nucleon-delta mass splitting [1]. A resonance can be studied by considering the unitarity and analyticity of the  $S$  matrix; however, the accuracy is hard to improve systematically with these general principles alone. Our goal here is to investigate  $\pi N$  scattering from threshold up to the delta resonance in an effective field theory (EFT) (for more details, see Ref. [2]).

Following several seminal papers [3], EFTs have been developed as model-independent approximations to low-energy strong interactions, which can be systematically improved by a series in powers of  $Q/M_{\text{QCD}}$ , where  $Q$  refers generically to small external momenta and  $M_{\text{QCD}} \sim 1$  GeV is the characteristic QCD scale. For reviews, see, for example, Refs. [4, 5]. Chiral perturbation theory (ChPT) specializes in processes involving at most one nucleon [4]. ChPT with only pion and nucleon fields has been extensively applied to near-threshold  $\pi N$  scattering [6], resulting in a perturbative expansion in powers of  $Q/\delta$  and  $m_\pi/\delta$ , which converges slowly as  $m_\pi/\delta \simeq 1/2$ . The convergence can be improved with an explicit delta field. The explicit delta in  $\pi N$  scattering within standard ChPT has been explored [7] and demonstrated in a fully consistent calculation [8].

Nevertheless, the perturbative nature of standard ChPT makes it impossible to describe the delta resonance, a non-perturbative phenomenon. A non-perturbative treatment of the delta within ChPT was considered in Ref. [9]; however, a systematic resummation did not exist until the seminal work of Ref. [10], where it was justified by a power counting based on three separate scales  $m_\pi \ll \delta \ll M_{\text{QCD}}$ , and this idea has been applied to various electromagnetic reactions in the delta region, but for  $\pi N$  scattering few results have been published [11].

We employ a power counting developed for generic narrow resonances [12], in which there are only two scales  $M_{\text{lo}} \sim \delta \sim m_\pi$  and  $M_{\text{hi}} \sim M_{\text{QCD}}$ . Thus the EFT expansion of the  $\pi N$  scattering amplitude pursued here is in powers of  $Q/M_{\text{hi}}$  and  $M_{\text{lo}}/M_{\text{hi}}$ . The kinematic region under consideration spans over both threshold and the resonance.

## 2. Effective Lagrangian

To establish the notation, we review how the effective Lagrangian is constructed (for more details of building chiral Lagrangian, see *e.g.*, Refs. [4, 5, 14, 15]). The effective Lagrangian should inherit the symmetries of QCD: Lorentz invariance, (approximate) two-flavor chiral symmetry ( $SU(2)_L \times SU(2)_R$ ), parity, time-reversal invariance, and baryon-number conservation.

In the kinematic region where the EFT holds, external momenta are much smaller than the nucleon mass,  $Q \ll m_N$ , and thus Lorentz invariance can be fulfilled perturbatively in powers of  $Q/m_N$ . One can start with a relativistic Lagrangian using the Rarita-Schwinger field for the delta, and then reduce from it its nonrelativistic version [10, 16]. This way, however, extra effort needs to be taken in order to control the spurious spin-1/2 sectors of the Rarita-Schwinger field. We employ another approach that starts with heavy-baryon fields  $N$  for the nucleon and  $\Delta$  for the delta, which are, respectively, two- and four-component spinors in spin and isospin spaces. Eventually, the effective Lagrangian only has the baryon degrees of freedom that represent forward propagation.

The crucial ingredient in this approach is to develop an order-by-order Lorentz transformation, by which one can constrain the coefficients of the rotation-invariant operators [15, 17].

Due to the presence of the delta field  $\Delta$ , one needs  $2 \times 4$  matrices  $\vec{S}$  in spin space to make a three-vector  $N\Delta$  bilinear, and  $\Omega_{ij}$  a three-tensor. Similar transition matrices,  $\mathbf{T}$  and  $\Xi_{ab}$ , can be defined in isospace.

The chiral-invariant operators are isoscalars that are made of pion covariant-derivative  $\mathbf{D}_\mu \equiv D^{-1} \partial_\mu \boldsymbol{\pi} / 2f_\pi$  with  $D \equiv 1 + \boldsymbol{\pi}^2 / 4f_\pi^2$ ,  $N$ ,  $\Delta$ , and their covariant derivatives, for example,  $\mathcal{D}_\mu \Delta \equiv \left( \partial_\mu + \mathbf{t}^{(\frac{3}{2})} \cdot \mathbf{E}_\mu \right) \Delta$  with  $\mathbf{E}_\mu \equiv i \boldsymbol{\pi} / f_\pi \times \mathbf{D}_\mu$ .

We use the so-called chiral index  $\nu$  [3] to organize the operators of the effective Lagrangian  $\nu = d + m + n_\delta + f/2 - 2$ , where  $d$ ,  $m$ ,  $n_\delta$ , and  $f$  are the numbers of derivatives, powers of  $m_\pi$ , powers of  $\delta$ , and fermion fields, respectively. In constructing the Lagrangian, we use integration by parts and field redefinitions to remove time derivatives on baryon fields except for the kinetic terms. The Lagrangian terms with the two lowest indices are given by [15]

$$\begin{aligned} \mathcal{L}^{(0)} = & 2f_\pi^2 \mathbf{D}^2 - \frac{1}{2D} m_\pi^2 \boldsymbol{\pi}^2 + N^\dagger i \mathcal{D}_0 N + g_A N^\dagger \boldsymbol{\tau} \boldsymbol{\sigma} N \cdot \vec{\mathbf{D}} \\ & + \Delta^\dagger (i \mathcal{D}_0 - \delta) \Delta + 4g_A^\Delta \Delta^\dagger \mathbf{t}^{(\frac{3}{2})} \vec{S}^{(\frac{3}{2})} \Delta \cdot \vec{\mathbf{D}} + h_A \left( N^\dagger \mathbf{T} \vec{S} \Delta + H.c. \right) \cdot \vec{\mathbf{D}} + \dots \end{aligned} \quad (2.1)$$

and

$$\mathcal{L}^{(1)} = \frac{1}{2m_N} \left( N^\dagger \vec{\mathcal{D}}^2 N + \Delta^\dagger \vec{\mathcal{D}}^2 \Delta \right) - \frac{h_A}{m_N} \left( i N^\dagger \mathbf{T} \vec{S} \cdot \vec{\mathcal{D}} \Delta + H.c. \right) \cdot \mathbf{D}_0 + \dots, \quad (2.2)$$

while the next-higher index yields

$$\begin{aligned} \mathcal{L}^{(2)} = & -\frac{\delta}{2m_N^2} \Delta^\dagger \vec{\mathcal{D}}^2 \Delta + \frac{h_A}{2m_N^2} \left[ \left( N^\dagger \mathbf{T} \vec{S} \vec{\mathcal{D}}^2 \Delta - N^\dagger \mathbf{T} \vec{S} \cdot \vec{\mathcal{D}} \vec{\mathcal{D}} \Delta \right) + H.c. \right] \cdot \vec{\mathbf{D}} \\ & + \frac{h_A}{8m_N^2} \left[ \left( \delta_{lm} N^\dagger \mathbf{T} \vec{S} \cdot \vec{\mathcal{D}} \Delta + 3N^\dagger \mathbf{T} S_l \mathcal{D}_m \Delta + 2\varepsilon_{ijl} N^\dagger \mathbf{T} \Omega_{im} \mathcal{D}_j \Delta \right) + H.c. \right] \cdot \mathcal{D}_l \mathbf{D}_m \\ & + d_1 \left( N^\dagger \mathbf{T} \vec{S} \Delta + H.c. \right) \cdot \vec{\mathcal{D}}^2 \vec{\mathbf{D}} + d_2 \frac{m_\pi^2}{D} \left( 1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2} \right) \left( N^\dagger \mathbf{T} \vec{S} \Delta + H.c. \right) \cdot \vec{\mathbf{D}} + \dots \end{aligned} \quad (2.3)$$

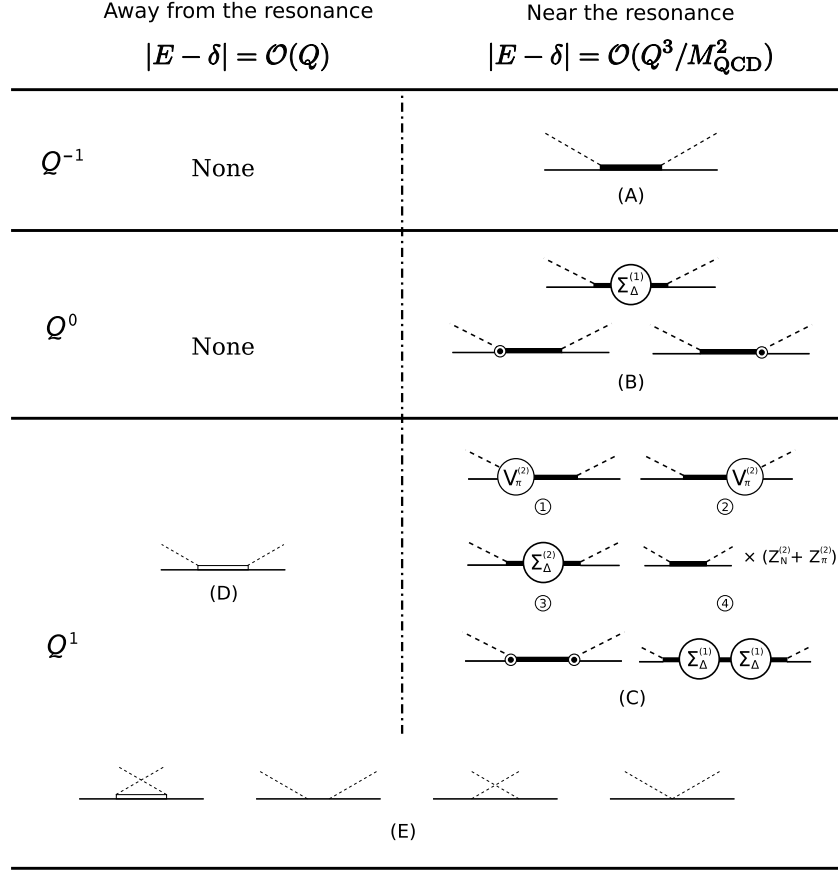
Here,  $g_A$  ( $g_A^\Delta$ ) is the  $\nu = 0$  axial-vector coupling of the nucleon (delta) and  $h_A$  ( $d_{1,2}$ ) is (are) the  $\nu = 0$  ( $\nu = 2$ )  $\pi N \Delta$  coupling(s).

### 3. Power counting

When the CM (heavy-baryon) energies  $E$  are much below the delta peak, the power counting is standard [3, 4, 5] with the simple generalization that  $\delta$  counts as  $Q$ . The contribution of a diagram with  $A$  nucleons (here  $A = 1$ ),  $L$  loops, and  $V_i$  vertices with chiral index  $\nu_i$  is proportional to  $Q^\rho$ , with

$$\rho = 2 - A + 2L + \sum_i V_i \nu_i. \quad (3.1)$$

However, in the small region spanning the delta peak whose size is of the leading-order (LO) delta self-energy,  $|E - \delta| \sim \Sigma_\Delta^{(0)} = \mathcal{O}(Q^3/M_{\text{QCD}}^2)$ , a resummation is needed in one- $\Delta$ -reducible



**Figure 1:** Contributions to  $\pi N$  scattering up to order  $Q^1$ : (A)  $Q^{-1}$  pole diagram; (B)  $Q^0$  pole diagrams; (C)  $Q^1$  pole diagrams; (D)&(E)  $Q^1$  tree diagrams, of which (E) apply to both regions.  $\Sigma_{\Delta}^{(n)}$  is the  $n$ -th order delta self-energy,  $V_{\pi}^{(n)}$  the  $\pi N \Delta$  vertex function, and  $Z_N^{(n)}$  ( $Z_{\Delta}^{(n)}$ ) the nucleon (delta) field renormalization constant.

diagrams because one insertion of  $\Sigma_{\Delta}^{(0)}$  and the bare delta propagator contributes  $\mathcal{O}(1)$ :  $\Sigma_{\Delta}^{(0)}/(E - \delta) = \mathcal{O}(1)$ . The resummation thus amounts to a dressed propagator

$$S_{\Delta}^{(0)}(E) = \left[ E - \delta + \Sigma_{\Delta}^{(0)}(\delta) \right]^{-1}, \quad (3.2)$$

which scales as  $M_{\text{QCD}}^2/Q^3$ . This is an enhancement of two powers over the generic situation. As a consequence, in one- $\Delta$ -irreducible diagrams the standard ChPT power counting (3.1) still applies; dressed delta propagators only need to be included in one- $\Delta$ -reducible diagrams. We thus arrive at a new power counting for one- $\Delta$ -reducible diagrams within a narrow window around the delta peak,

$$\rho = 2 - A - 2n_{\Delta} + 2L + \sum_i V_i v_i, \quad (3.3)$$

where  $n_{\Delta}$  is the number of dressed delta propagators. This is the non-electromagnetic version of  $\rho$  derived in a slightly different power counting in Ref. [10]. Diagrams up to  $Q^1$  are listed in FIG. 1.

It seems that the two different power-counting schemes, which are applicable in two different regions, would lead to an EFT amplitude in the form of a piecewise function in the energy. Even worse, separating these two regions is somewhat arbitrary. A piecewise EFT is actually unnecessary

if we enforce the pole diagrams even in the off-the-pole region, which is equivalent to shifting a subset of higher diagrams into lower orders, *i.e.*, a rearrangement of diagrams. This sort of rearrangement still retains the essence of the original power counting as long as one does not claim a higher accuracy by doing so.

#### 4. $\pi N$ -scattering $T$ matrix

The partial-wave  $T$  matrix is related to the phase shifts, in the channel with total angular momentum  $j$ , orbital angular momentum  $l$ , and isospin  $t$ , by

$$T_{jlt}(E) \equiv -i \{ \exp [2i\theta_{jlt}(E)] - 1 \} . \quad (4.1)$$

In the following we will use a more conventional notation for a specific partial wave:  $l_{2t,2j}$ . For example,  $P_{13}$  refers to the  $l = 1$  ( $P$  wave),  $t = 1/2$ , and  $j = 3/2$ .

Here the exact relation between  $E$  and the CM momentum  $k$ ,  $E = (m_N^2 + k^2)^{1/2} + (m_\pi^2 + m_N^2)^{1/2} - m_N$ , is assumed, meaning that certain trivial, kinematic  $k/m_N$  terms are resummed — what we refer to as semi-resummation. A strict heavy-baryon expansion can be readily obtained afterwards.

At LO ( $Q^{-1}$ ) there is only a pole diagram, FIG. 1(A), which contributes only to the  $P_{33}$  wave,

$$T_{P_{33}}^{\text{LO}} = -\frac{\gamma^{(0)}(\delta)}{E - \delta + i\gamma^{(0)}(\delta)/2} \left[ 1 + \mathcal{O} \left( \frac{Q}{M_{\text{QCD}}} \right) \right], \quad (4.2)$$

where

$$\gamma^{(0)}(\delta) = \frac{h_A^2}{24\pi f_\pi^2} (\delta^2 - m_\pi^2)^{\frac{3}{2}} \left[ 1 + \delta/m_N + (\delta^2 - m_\pi^2)/(2m_N)^2 \right]^{\frac{3}{2}} \frac{1 + \delta/m_N + (\delta^2 - m_\pi^2)/2m_N^2}{(1 + \delta/m_N)^5}. \quad (4.3)$$

The NLO ( $Q^0$ ) amplitude has the same form as LO,

$$T_{P_{33}}^{\text{NLO}} = -\frac{\gamma^{(0)}(\delta) + \gamma^{(1)}(\delta)}{E - \delta + i[\gamma^{(0)}(\delta) + \gamma^{(1)}(\delta)]/2} \left[ 1 + \mathcal{O} \left( \frac{Q^2}{M_{\text{QCD}}^2} \right) \right]. \quad (4.4)$$

However,  $\gamma^{(1)}(\delta)$  vanishes in the CM frame when we do not expand kinematic relations in powers of  $\delta/m_N$ .

Summing up the pole (FIG. 1(C)) and tree (FIG. 1(E)) diagrams, one first finds the NNLO amplitude in the  $P_{33}$  channel,

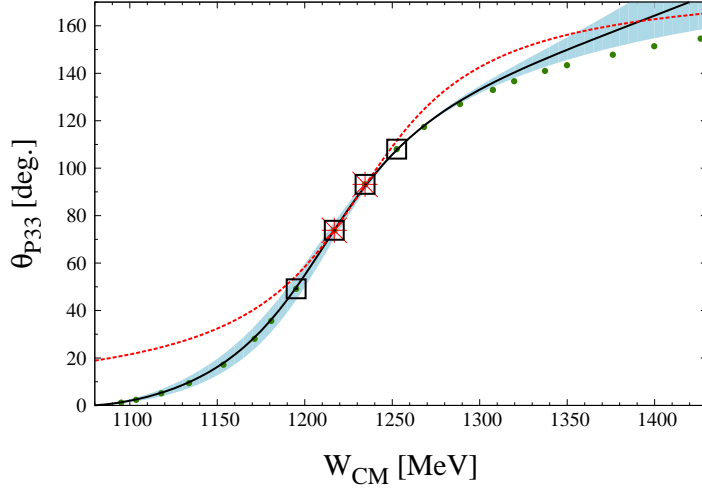
$$T_{P_{33}}^{\text{NNLO}} = -\frac{\Gamma(E)}{E - \delta + i\Gamma(E)/2} [1 + iT_B(E)] + T_B(E) + \mathcal{O} \left( T_{P_{33}}^{\text{LO}} \frac{Q^3}{M_{\text{QCD}}^3} \right), \quad (4.5)$$

where

$$T_B(E) = \frac{k^3}{6\pi f_\pi^2} \left( \frac{g_A^2}{E} + \frac{1}{36} \frac{h_A^2}{E + \delta} \right) \quad \text{and} \quad \Gamma(E) = \frac{(m_N^2 + k^2)^{1/2}}{E + m_N} \frac{[h_A(1 + \varkappa)]^2}{24\pi f_\pi^2} k^3 \quad (4.6)$$

with

$$\varkappa \equiv \frac{k_\delta^2}{(4\pi f_\pi)^2} \left[ \frac{(4\pi f_\pi)^2}{h_A} \left( -d_1 + d_2 \frac{m_\pi^2}{k_\delta^2} \right) + \text{Re}\mathcal{G}(m_\pi/k_\delta) \right], \quad (4.7)$$



**Figure 2:**  $P_{33}$  phase shifts (in degrees) as a function of  $W_{\text{CM}}$  (in MeV), the CM energy including the nucleon mass. The EFT strict heavy-baryon expansion at LO (NNLO) is represented by the red dashed (black solid) line. The NLO curve coincides with LO. The light-blue band outlines the estimated systematic error of the NNLO curve. The green dots are the results of the GW phase-shift analysis [13]. Points marked by a red star (black square) are inputs for LO (NNLO).

where

$$\mathcal{G}(x) = \frac{2}{3} (1+x^2)^{-\frac{1}{2}} \left\{ -\pi \left( g_A^2 - \frac{81}{16} g_A^{\Delta^2} \right) x^3 + 2\pi i \left( g_A^2 + \frac{1}{72} h_A^2 \right) + \left[ g_A^2 - \frac{1}{72} h_A^2 (13 + 15x^2) + \frac{81}{16} g_A^{\Delta^2} \right] \ln \left( \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1} \right) \right\} \quad (4.8)$$

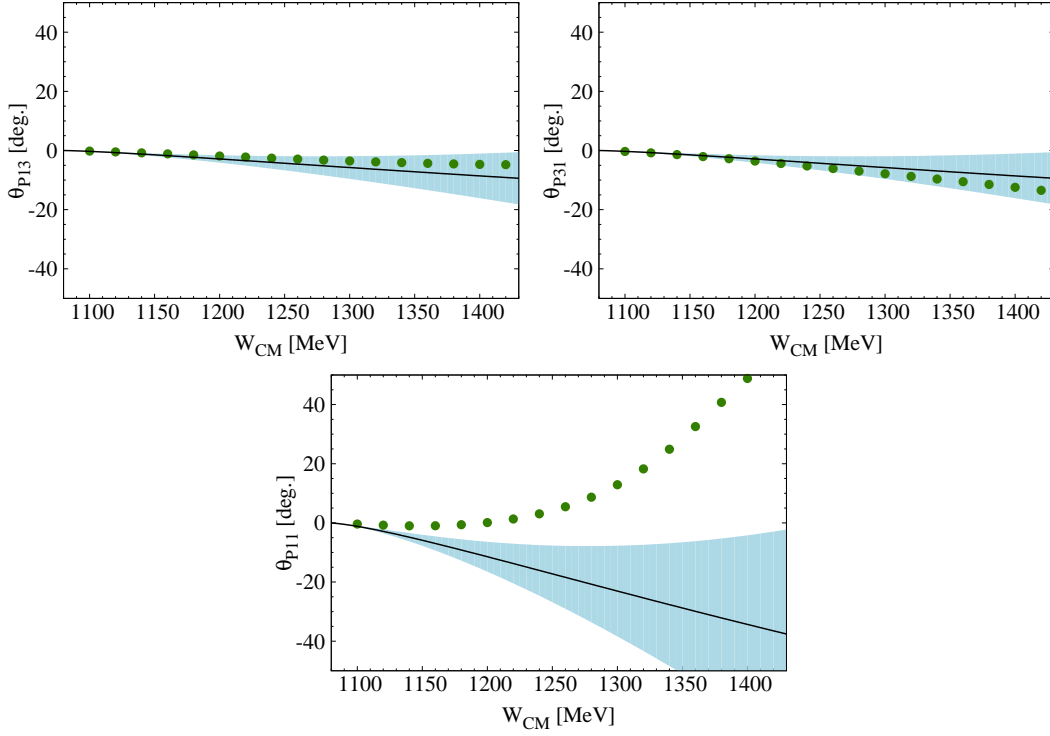
and  $k_\delta$  satisfies  $\delta = (m_N^2 + k_\delta^2)^{1/2} + (m_\pi^2 + k_\delta^2)^{1/2} - m_N$ . Other channels are easy to calculate from the one- $\Delta$ -irreducible tree diagrams in FIG. 1(E). For the remaining  $P$ -wave channels,

$$T_{P_{13}}^{\text{NNLO}} = T_{P_{31}}^{\text{NNLO}} = \frac{1}{4} T_{P_{11}}^{\text{NNLO}} = -\frac{k^3}{12\pi f_\pi^2} \left( \frac{g_A^2}{E} - \frac{2}{9} \frac{h_A^2}{E + \delta} \right) \left[ 1 + \mathcal{O} \left( \frac{Q}{M_{\text{QCD}}} \right) \right]. \quad (4.9)$$

## 5. $P$ -wave phase shifts

A number of low-energy constants (LECs) can be determined from other processes, such as pion decay and neutron decay. We adopt the following values:  $m_\pi = 139$  MeV,  $m_N = 939$  MeV,  $g_A = 1.26$ , and  $f_\pi = 92.4$  MeV. Our strategy of fitting is to determine the free parameters,  $\delta$ ,  $h_A$ , and  $\varkappa$  from the  $P_{33}$  phase shifts around the delta peak and then predict the phase shifts at lower energies in all  $P$  waves. Shown in FIGs. 2 and 3 are the EFT curves (strict heavy-baryon expansion used) fitted to the partial-wave analysis (PSA) by the George Washington (GW) group [13]. The PSA points used to determine the free parameters are explicitly marked. Based on the power counting we use, systematic errors of the EFT curves can be estimated, shown in FIGs. 2 and 3 as light-colored bands.

The LECs extracted from the  $P_{33}$  fit are given in TABLE 1. One can estimate the errors in the NNLO values as the variation in each LEC within which the NNLO  $P_{33}$  curve in FIG. 2 roughly



**Figure 3:** Predicted phase shifts (in degrees) in the  $P_{13}$ ,  $P_{31}$ , and  $P_{11}$  channels as functions of  $W_{\text{CM}}$  (in MeV), the CM energy including the nucleon mass. LO and NLO vanish in these channels; NNLO EFT results in the strict heavy-baryon expansion are given by the black solid lines. The light-blue bands outline the estimated systematic errors of the NNLO curves. The green dots are the results of the GW phase-shift analysis [13].

**Table 1:** Low-energy constants extracted at LO, NLO, and NNLO from the fits using the strict heavy-baryon expansion.

$\delta$ (MeV)			$h_A$			$\varkappa$
LO	NLO	NNLO	LO	NLO	NNLO	NNLO
293	293	321	1.98	4.21	2.85	0.046

stays within the error band. This way we find  $\delta/\text{MeV}$ ,  $h_A$ , and  $\varkappa$  to be within  $\sim \pm 4$ ,  $\pm 0.30$ , and  $\pm 0.030$ , respectively, of the NNLO values in TABLE 1.

## 6. Summary

We have extended standard ChPT to deal with the non-perturbative delta resonance in an EFT framework. The delta is treated as a nonrelativistic particle from the beginning, rather than being represented by the Rarita-Schwinger field.

Like other EFTs that deal with non-perturbative phenomena, ours captures the non-perturbative structure in LO. Subsequently, the power counting leads to a systematic, perturbative improvement beyond LO. We applied this power counting to low-energy  $\pi N$  scattering, where we built the am-

plitudes up to NNLO. We fitted our  $P$ -wave amplitudes to the phase shifts given by Ref. [13]. With just three free parameters, we obtained a good fit in the  $P_{33}$  channel.

The EFT approach presented here also provides the basis for a model-independent, unified description, from threshold to past the delta resonance without discontinuity, of reactions involving other probes and targets, including nuclei.

## References

- [1] C. Amsler *et al.* (Particle Data Group), *Phys. Lett.* **B667** (2008) 1.
- [2] Bingwei Long and U. van Kolck, arXiv:0907.4569 [hep-ph].
- [3] S. Weinberg, *Physica* **96A** (1979) 327; J. Gasser and H. Leutwyler, *Ann. Phys.* **158** (1984) 142; *Nucl. Phys.* **B250** (1985) 465.
- [4] V. Bernard, N. Kaiser, and U.-G. Meißner, *Int. J. Mod. Phys.* **E4** (1995) 193; V. Bernard, *Prog. Part. Nucl. Phys.* **60** (2008) 82.
- [5] U. van Kolck, *Prog. Part. Nucl. Phys.* **43** (1999) 337; E. Epelbaum, *Prog. Part. Nucl. Phys.* **57** (2006) 654.
- [6] S. Weinberg, *Phys. Rev. Lett.* **17** (1966) 616; Y. Tomozawa, *Nuovo Cim.* **46A** (1966) 707; M. Mojžiš, *Eur. Phys. J.* **C2** (1998) 181; N. Fettes and U.-G. Meißner, *Nucl. Phys.* **A676** (2000) 311; *Nucl. Phys.* **A693** (2001) 693; T. Becher and H. Leutwyler, *JHEP* **0106** (2001) 017.
- [7] A. Datta and S. Pakvasa, *Phys. Rev.* **D56** (1997) 4322.
- [8] N. Fettes and U.-G. Meißner, *Nucl. Phys.* **A679** (2001) 629.
- [9] P.J. Ellis and H.-B. Tang, *Phys. Rev.* **C57** (1998) 3356; K. Torikoshi and P.J. Ellis, *Phys. Rev.* **C67** (2003) 015208.
- [10] V. Pascalutsa and D.R. Phillips, *Phys. Rev.* **C67** (2003) 055202; V. Pascalutsa, *Prog. Part. Nucl. Phys.* **61** (2008) 27.
- [11] V. Pascalutsa, M. Vanderhaeghen, and S.N. Yang, *Phys. Rept.* **437** (2007) 125.
- [12] P.F. Bedaque, H.W. Hammer, and U. van Kolck, *Phys. Lett.* **B569** (2003) 159.
- [13] R.A. Arndt, W.J. Briscoe, I.I. Strakovsky, and R.L. Workman, *Phys. Rev.* **C74** (2006) 045205; R.A. Arndt, W.J. Briscoe, I.I. Strakovsky, R.L. Workman, and M.M. Pavan, *Phys. Rev.* **C69** (2004) 035213; R.A. Arndt *et al.*, The SAID program, <http://gwdac.phys.gwu.edu/>.
- [14] S. Weinberg, *Quantum Theory of Fields*, Vols. 1 and 2, Cambridge University Press, New York (1996).
- [15] U. van Kolck, Ph.D dissertation, U. of Texas (1993); C. Ordóñez, L. Ray, and U. van Kolck, *Phys. Rev. Lett.* **72** (1994) 1982; *Phys. Rev.* **C53** (1996) 2086; T.D. Cohen, J.L. Friar, G.A. Miller, and U. van Kolck, *Phys. Rev.* **C53** (1996) 2661.
- [16] T.R. Hemmert, B.R. Holstein, and J. Kambor, *Phys. Lett.* **B395** (1997) 89; *J. Phys.* **G24** (1998) 1831.
- [17] Bingwei Long and V. Lensky, in preparation.