PROCEEDINGS OF SCIENCE

# Hadronic atoms

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We apply the non-relativistic effective Lagrangian approach to the description of the $\bar{K}d$ atom, which is presently studied by the SIDDHARTA experiment at LNF-INFN. In particular, we demonstrate that a systematic calculation of the retardation effect in the $\bar{K}d$ scattering length is possible within this framework. It is argued that a relatively small size of the net correction for the double scattering diagrams, despite the large value of the ratio $M_K/m_N$, is related to the cancellations occuring at the leading order [1].

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## 1. Introduction

Hadronic atoms are loosely bound states of hadrons and nuclei, which are created predominately by a static Coulomb force. For the states which are composed only of elementary hadrons or of an elementary hadron and the deuteron, one observes a clear hierarchy of various physical scales: the range of strong interactions (typically, of the order of fm) is much smaller than average separation of the hadronic atom constituents, which is given by the Bohr radius and which amounts up to a few hundreds of fm for various bound states. This hierarchy of scales ensures that the observable characteristics of hadronic atoms – the energy levels and the decay width – can be described in terms of a few parameters characterizing (arbitrarily complicated) strong interactions of the constituent hadrons: the scattering length, the effective radius and so on. The situation with the hadronic atoms involving heavy nuclei is conceptually different. In this case, the hadron, which is bound in the atom, penetrates into the nucleus and feels strong interactions at a shorter distance. For this reason, the exact position of the energy levels depend on the details of strong interactions and can not be described merely in terms of the effective-range expansion parameters. This case is not considered here.

Deser, Goldberger, Baumann and Thirring (DGBT) were first to derive [2] the expression for the (complex) displacement of the ground state energy level of a hadronic atom, which emerges due to strong interactions. The real and imaginary parts of this displacement, which define the energy shift $\Delta E^{\rm str}$ and the line width $\Gamma$ for the ground state, are given by

$$\Delta E^{\rm str} - \frac{i}{2}\Gamma = -2\alpha^3\mu_c^2\,\mathscr{T} + \cdots\,, \tag{1.1}$$

where $\alpha$ stands for the fine structure constant, $\mu_c$ is the reduced mass of the hadronic atom constituents, $\mathscr{T}$ denotes the elastic threshold scattering amplitude of the constituents, and the ellipses stand for the higher-order terms in $\alpha$. At this order, $\mathscr{T}$ can be regarded as a purely hadronic quantity, being proportional to the scattering length in the elastic channel, evaluated in pure QCD. Further, if there are open channels below threshold, the threshold scattering amplitude is complex and the hadronic atom decays through strong interactions. Note also that the spacing between the pure Coulomb levels in the hydrogen-like systems is a quantity of the order $\alpha^2$ and, consequently, the energy level displacement due to strong interactions represents a small perturbation to the Coulomb levels.

The equation (1.1) suggests that measuring the hadronic atom energy shift and width, one may directly extract the (complex) scattering length of the hadronic constituents. This simple observation provides rationale behind the experiments on various hadronic bound systems, which are performed in different high-energy laboratories. In turn, the hadronic scattering lengths are important characteristics of strong interactions and the precise measurement of these quantities allows one to gain deeper insight in the fundamental properties of hadron dynamics at low energy. A detailed review on the physics case for DIRAC, Pionic Hydrogen, SIDDHARTA experiments can be found, e.g., in Refs. [3, 4] (see further references therein).

It turns out that at present experimental precision the accuracy of Eq. (1.1) does not suffice. Isospin-breaking corrections at leading order in $\alpha$ and light quark mass difference $m_d - m_u$ are not negligible. The evaluation of these corrections can be systematically carried out within the

framework of the non-relativistic effective field theory. The resulting expression at the first non-leading order in isospin breaking is given by

$$\Delta E^{\rm str} - \frac{i}{2}\Gamma = -2\alpha^3\mu_c^2\,\mathscr{T}\left(1 - 2\alpha\mu_c(\ln\alpha - 1)\,\mathscr{T}\right) + \cdots, \tag{1.2}$$

where now the quantity $\mathscr{T}$ denotes the threshold amplitude calculated at the next-to-leading order in isospin breaking parameters $\alpha$ and $m_d - m_u$ (see, e.g., Ref. [3] for an exact definition), and the ellipses stand for the next-to-next-to-leading order corrections.

Since 1998, the non-relativistic effective Lagrangian approach has been applied to study various hadronic atoms, and, in particular, to evaluate the leading-order isospin-breaking corrections. Details can be found in the recent papers [3–19]. In particular, Refs. [3, 4] provide a review on the subject. Here, we do not intend to review the subject again. In the following, we rather focus our attention on (perhaps theoretically the most challenging and still unsolved) case of $K^-p$ (kaonic hydrogen) and $K^-d$ (kaonic deuterium) atoms, which are measured by SIDDHARTA collaboration at LNF-INFN [20]. Applying Eq. (1.2) to the measured ground-state spectrum of these atoms[1], one may extract the complex threshold amplitudes $\mathscr{T}_{\bar{K}p}$ and $\mathscr{T}_{\bar{K}d}$. The final aim of the experiment is to determine the exact values of the S-wave $\bar{K}N$ scattering lengths in the isospin symmetry limit $a_0, a_1$, which correspond to the total isospin $I = 0, 1$. Eventually, comparing these scattering lengths with the prediction obtained within the unitarized ChPT, one may test the present theoretical knowledge of the $\bar{K}N$ dynamics at low energy. Below we discuss how the above goal can be achieved.

## 2. Essentials

As discussed above, our final goal is to go one step further from Eq. (1.2), expressing the measured quantities $\mathscr{T}_{\bar{K}p}$ and $\mathscr{T}_{\bar{K}d}$ in terms of $a_0, a_1$ and then to invert these expressions extracting the values of $a_0, a_1$ from the data. Below, we present only a brief summary of our approach. More details can be found in Ref. [1]. In the beginning, the following remarks are in order.

i) The quantities $a_0, a_1$ are strongly absorptive, because inelastic channels are present below threshold. This means that measuring *both* kaonic hydrogen and kaonic deuterium is necessary in order to determine $a_0$ and $a_1$ independently. Note that in case of the pionic hydrogen and pionic deuterium the situation is different: the $\pi N$ scattering lengths are real, so the deuterium data plays an auxiliary role only.

ii) Relation of $\mathscr{T}_{\bar{K}p}$ to $a_0, a_1$ can be easily worked out. On the contrary, relating $\mathscr{T}_{\bar{K}d}$ to $a_0, a_1$ to a sufficient precision represents a serious theoretical challenge. Note that Faddeev studies alone do not suffice for our purpose: what is needed is an *explicit* relation (of a type of the multiple-scattering series), which can be inverted to eventually determine $a_0$ and $a_1$.

iii) In order to study the $\bar{K}d$ scattering near threshold, we propose to apply the non-relativistic effective Lagrangian approach, which has been successfully used to describe hadronic atoms

[1] It turns out that in case of kaonic atoms the higher-order corrections in Eq. (1.2) are not totally negligible (see, e.g., [21]). The bulk of this correction comes, however, from the Coulomb photon exchanges in the loops and can be easily taken into account to all orders.

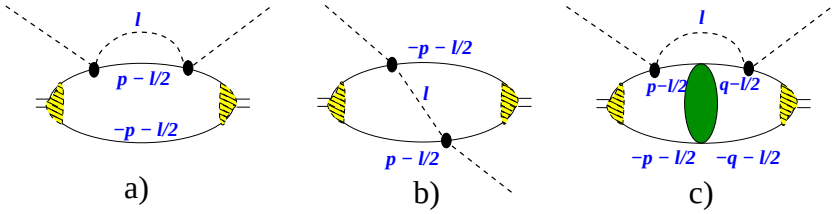


**Figure 1:** Double scattering diagrams contributing to the $\bar{K}d$ scattering amplitude. The dashed and solid lines denote the kaons and nucleons, respectively. The shaded blob in the diagram c) stands for the $NN$ scattering amplitude which emerges after iteration of the $NN$ potential to all orders.

in the past. Our theory is based on the assumption that the range of the $\bar{K}N$ interactions is smaller than that of the $NN$ and $\bar{K}NN$ interactions. Consequently, in our approach $\bar{K}N$ interactions are described by the local vertices, whereas two-nucleon and 3-body forces are assumed to be described by non-local potentials. Eventually, these potentials, which can be considered as an input in this approach, are calculated in ChPT with non-perturbative pions [22]. The advantage of the present approach is that, since the $\bar{K}N$ interactions are local, the usual perturbation theory directly produces multiple-scattering series.

iv) The effect of the 3-body force can be estimated and is found to be small. We believe that it can be safely neglected. In addition, we expect that there is no need to explicitly include hyperonic channels in the non-relativistic approach.

v) Due to the large values of the scattering length the multiple-scattering series in the $\bar{K}d$ scattering are divergent and should be summed up to all orders. This is possible only in the static limit $m_N \to \infty$. However, in reality the ratio $\xi = M_K/m_N \simeq 0.5$ is not small and this may potentially lead to sizable retardation corrections, which represent the largest source of the uncertainty in the presently available theoretical results on $\bar{K}d$ scattering. On the other hand, Faddeev calculations usually find a moderate effect, typically of the order of 20%. Thus the challenge consists in understanding the size of the effect.

We emphasize that the sole method known to us, which enables one to systematically handle the retardation corrections, is the use of the non-relativistic effective field theory. Applying the perturbative uniform expansion [23] to an arbitrary Feynman integral appearing in the theory we obtain the expansion of these integrals in powers of a variable $\sqrt{\xi}$.

## 3. Double scattering

As mentioned already, the multiple-scattering series for the quantity $\mathcal{T}_{\bar{K}d}$ are divergent in the static approximation and should be summed up to all orders. To understand the formalism which will be used to treat retardation corrections we, however, start from the double scattering diagrams shown in Fig. 1. The sum of the three diagrams shown in Fig. 1 can be expanded in powers of $\sqrt{\xi}$

$$\mathcal{M}^{[2]} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c = \mathcal{M}^{\rm stat} + \xi^{1/2}\mathcal{M}_1 + \xi\mathcal{M}_2 + \xi^{3/2}\mathcal{M}_3 + \cdots, \qquad (3.1)$$

with only the diagram b) contributing in the static limit.

A detailed treatment of the diagrams depicted in Fig. 1 is given in Ref. [1]. Below, we shall give a brief summary of the calculation. It should be understood that the expansion in Eq. (3.1) can be obtained without calculating the Feynman integrals explicitly. In each of the diagrams the loop integrations are performed over the kaon (**l**) and nucleon (**p**, **q**) momenta. The latter are always of the order of the average nucleon momenta in the deuteron $\mathbf{p},\mathbf{q}\sim\langle 1/r\rangle_{\mathsf{wf}}$, where the subscript "wf" denotes the averaging in the deuteron ground state. As for the kaon momentum, the following regimes should be distinguished:

1. The low-**l** regime: $\mathbf{l}\sim\sqrt{\xi}\,\mathbf{p}\ll\mathbf{p}$
2. The high-**l** regime: $\mathbf{l}\sim\mathbf{p}$
3. Intermediate regime: $\sqrt{\xi}\,\mathbf{p}\ll\mathbf{l}\ll\mathbf{p}$

Taylor-expanding the integrand in the Feynman integrals for each momentum regime separately, and summing everything up (with appropriate signs, see Refs. [1, 23]), we recover the original Feynman integral expanded in powers of $\sqrt{\xi}$. Note that the low-**l** regime produces only non-integer powers of $\xi$, i.e. $\xi^{1/2},\xi^{3/2},\cdots$, whereas the integer powers of $\xi$ stem from the high-**l** regime. The contribution coming from the intermediate regime cancels the divergences which arise after the Taylor expansion in the first two regimes. If the dimensional regularization is used, the perturbative uniform expansion described here coincides with the threshold expansion introduced in Ref. [24]. The advantage of the former method is, however, that it is not tied to any particular form of the regularization of the Feynman integrals. It is also clear that the method can be applied to study the higher-order diagrams without any modification.

The straightforward calculations yield the following results [1]:

i) The leading-order correction to the static limit, which is proportional to $\xi^{1/2}$, cancels in channels both with isospin 0 and 1 if the contributions from all three diagrams shown in Fig. 1 are taken into account. Although similar findings have been reported earlier [25], the method described here enables one to systematically address such cancellations at higher orders in the parameter $\xi$ as well.

ii) The expansion in powers of $\sqrt{\xi}$ is convergent, albeit rather slowly. The net correction is not very large (less than 20%) even at $\xi\simeq 0.5$. We believe that this fact is largely explained by the cancellation of the leading-order corrections although we also observe some cancellations among higher-order terms.

## 4. Summing up multiple-scattering series

As seen, the retardation correction in double scattering diagrams is relatively small. This brings us to the following conjecture. We assume that the retardation corrections in the $\bar{K}d$ scattering can be treated perturbatively, whereas the series in the static limit should be summed up to all orders. This means that, e.g., in order to obtain a correction at $O(\xi^{1/2})$ in the multiple-scattering series, one has to sandwich the piece of the amplitude $\xi^{1/2}\mathscr{M}_1$ (see (Eq. 3.1)) with infinite number

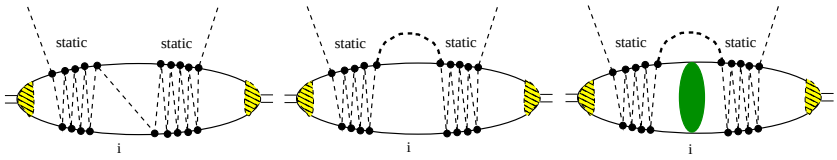


**Figure 2:** One insertion of the retarded piece of the double scattering $\bar{K}NN \to \bar{K}NN$ transition amplitude in the static series for $\bar{K}d$ scattering (cf with Fig. 1).

of the static kaon exchanges from the right and from the left. To obtain the correction at $O(\xi)$, we have to consider one insertion of $\xi\mathscr{M}_2$ or two insertions of $\xi^{1/2}\mathscr{M}_1$ in the static series, and so on (see Fig. 2).

The investigation of the recoil correction in multiple-scattering series is in progress and the results will be reported elsewhere. Note that there is no complete cancellation of the leading-order corrections which are proportional to $\xi^{1/2}$ in the multiple-scattering series. It remains to be seen, whether there are mechanisms of suppression of the retardation correction in this case.

## 5. Conclusions

Using the non-relativistic effective field theory, we investigate the retardation effect in the low-energy $\bar{K}d$ scattering. In particular, we obtain the systematic expansion of the $\bar{K}d$ scattering amplitude in the half-integer powers of $\xi = M_K/m_N$. Such an expansion can be carried out even if the multiple-scattering series diverge and should be summed up to all orders. It is conceivable that a relatively moderate size of the retardation correction even at $\xi \simeq 0.5$ might originate from some kind of cancellations, such as an exact cancellation of the leading-order retardation corrections in the double scattering diagrams.

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