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Effective Field Theory for Nuclear Forces

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We address conceptual aspects of renormalization in the context of effective field theories for the two-nucleon system. We emphasize the importance of low-energy theorems for the subthreshold coefficients and consider an exactly solvable model to demonstrate their validity in the effective field theory approach with a finite cutoff Λ provided it is chosen of the order of the hard scale in the problem. Removing the cutoff by taking the limit $\Lambda \to \infty$ yields a finite result for the scattering amplitude but violates the low-energy theorems and is, therefore, not compatible with the effective field theory framework.

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1. Introduction

Almost two decades ago Weinberg proposed a way to extend baryon chiral perturbation theory to few-nucleon systems [1] in which chiral perturbation theory is applied to the effective potential, defined as the sum of all possible N-nucleon irreducible diagrams, rather than to the scattering amplitude. The amplitude is then generated by solving the corresponding dynamical equation such as the Lippmann-Schwinger (LS) equation in the two-nucleon sector. For recent reviews and references the reader is referred to Refs. [3, 4, 5].

While phenomenologically successful, the consistency of Weinberg's approach was questioned by several authors. The resulting nucleon-nucleon (NN) potential is non-renormalizable in the traditional sense, i.e. iterations of the LS equation generate divergent terms with structures which are not included in the original potential. Consequently, renormalization of the Neumann series resulting from iterating the LS equation requires inclusion of contributions of infinitely many higher-order short-range operators in the potential (counterterms). The freedom in the choice of the finite parts of counterterms is compensated by the running of the corresponding renormalized coupling constants. Notice that the above mentioned complications can be avoided if pion-exchange contributions to the potential are treated perturbatively [6, 7]. The resulting perturbative expansion for the scattering amplitude was found not to converge for nucleon momenta of the order of the pion mass at least in certain spin-triplet channels [8], see however Ref. [9], yielding strong evidence that pion-exchange contributions have to be treated non-perturbatively [10, 11]. This is in line with phenomenological successes of Weinberg's approach which treats pion exchange contributions nonperturbatively. In particular, the most advanced analyses of the NN system at next-to-next-to-next-to-leading order in the Weinberg's power counting scheme demonstrate the ability to accurately describe NN scattering data up to center-of-mass momenta at least of the order $\sim 2M_\pi$ [12, 13]. It is important to emphasize that these studies are carried out within the cutoff EFT along the lines of Lepage [14, 15] who argued that the cutoff parameter Λ in such calculations should be taken of the order of the relevant hard large scale such as e.g. the mass of the ρ meson, see also Refs. [16, 17, 13, 18, 19]. The fairly narrow range of cutoffs $\Lambda = 450\ldots600$ MeV used in Refs. [12, 13] was criticized in [20] where low NN partial waves were considered based on the one-pion exchange potential and contact interactions employing a much larger range of cutoffs with $\Lambda < 4$ GeV. Furthermore, several groups are exploring the possibility of manifestly nonperturbative renormalization of the LS equation by taking the limit $\Lambda \to \infty$, see e. g. [21, 22, 23, 24, 25, 26]. In addition, some authors advocate various kinds of mixed procedure by treating certain contributions to the potential and/or high partial waves in perturbation theory, see [20, 25]. Finally, using renormalization-group methods to set up power counting rules for NN interaction is explored in [27].

The purpose of this manuscript is to clarify some conceptual issues related to renormalization and the role of the cutoff in the context of EFT for the two-nucleon system. We first discuss the meaning of low-energy theorems (LETs) for subthreshold parameters using general arguments based on the analytic structure of the scattering amplitude. We argue that LETs provide an important and nontrivial test of long-range physics and thus must be respected in EFT with explicit pions, see also [11]. We then consider effective field theory (EFT) for an exactly solvable model for two nucleons interacting via the long- ($r_l \sim m_l^{-1}$) and short-range ($r_s \sim m_s^{-1} \ll m_l^{-1}$) forces which can

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be regarded as a toy model for chiral EFT [28]. We employ the Weinberg-like (or, more precisely, Lepage-like [14]) formulation with a finite cutoff Λ and demonstrate the validity of the LETs as long as it is chosen of the order $\Lambda \sim m_s$. Taking the limit $\Lambda \to \infty$ is shown to yield a finite result for the amplitude but leads to breakdown of LETs. This procedure is, therefore, not compatible with the EFT framework. We argue that Λ should not be taken (considerably) larger than the short-range scale m_s in the problem.

2. Low-energy theorems and the modified effective range expansion

Consider two non-relativistic nucleons interacting via the local potential V. The ordinary effective range function

$$F_l(k^2) \equiv k^{2l+1} \cot \delta_l(k) \tag{2.1}$$

with k, l and $\delta_l(k)$ denoting the CMS scattering momentum, orbital angular momentum and phase shift, respectively, is well known to be a real meromorphic function of k^2 near the origin for local non-singular potentials of a finite range [29, 30]. It can, therefore, be Taylor-expanded leading to the well-known effective range expansion (ERE)

$$F_l(k^2) = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + \ldots\,, \tag{2.2}$$

with a, r and v_i being the scattering length, effective range and the so-called shape parameters. Generally, the radius of convergence of the ERE is bounded from above by the lowest left-hand singularity associated with the potential. For example, for Yukawa-type potentials corresponding to exchange of a meson of mass M, the maximal radius of convergence of the ERE is given by $k^2 < M^2/4$. For nucleon-nucleon interaction, the ERE is, therefore, expected to converge for energies up to $E_{\rm lab} \sim M_\pi^2/(2m) = 10.5$ MeV, where m denotes the nucleon mass. Notice that pionless EFT in the two-nucleon sector in the absence of external sources is equivalent to ERE since both approaches provide an expansion of the amplitude in powers of k/M_π, have the same validity range and incorporate the same physical principles.

The framework of ERE can be generalized to the case in which the potential is given by a sum of a long-range ($r_l \sim m_l^{-1}$) and short-range ($r_s \sim m_s^{-1} \ll m_l^{-1}$) potentials V_L and V_S, respectively. Following van Haeringen and Kok [31], one can define the modified effective range function F_l^M via

$$F_l^M(k^2) \equiv M_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|} \cot[\delta_l(k) - \delta_l^L(k)]\,, \tag{2.3}$$

where $\delta_l^L(k)$ and $f_l^L(k)$ refer to the phase shift and Jost function associated with the potential V_L and the quantity $M_l^L(k)$ can be computed from the Jost solution $f_l^L(k,r)$ associated with V_L, see [31] for more details and precise definitions. The function $F_l^M(k^2)$ reduces, per construction, to $F_l(k^2)$ for $V_L = 0$ and is a real meromorphic function in a much larger region given by $1/r_s$ as compared to $F_l(k^2)$ since the lowest left-hand singularity due to V_L is removed from $F_l^M(k^2)$.[1] It is, therefore, natural to assume that the coefficients in the modified effective range expansion (MERE), i.e. the Taylor expansion of $F_l^M(k^2)$ near the origin, are driven by the hard scale m_s

[1]The existence of $M_l^L(k)$ implies certain constraints on the small-r behavior of $V_L(r)$.

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(except for the modified scattering length), see [32] for a related discussion. The meaning of the LETs becomes evident if one uses Eq. (2.3) to express the ordinary effective range function $F_l(k^2)$ in terms of the modified one, $F_l^M(k^2)$, and the quantities which are calculable solely from the long-range interaction V_L. The MERE for $F_l^M(k^2)$ then yields an expansion of the subthreshold parameters entering Eq. (2.2) in powers of m_L/m_S. In particular, using the first few terms in the MERE as input allows to make predictions for *all* coefficients in the ERE. We further emphasize that the appearance of the correlations between the subthreshold parameters in the above-mentioned sense is the only signature of the long-range interaction at low energy.

3. Toy model

We now consider two nucleons in the spin-singlet S-wave interacting via the two-range separable potential

$$V(p,p') = v_l F_l(p) F_l(p') + v_s F_s(p) F_s(p'), \quad F_l(p) \equiv \frac{\sqrt{p^2+m_s^2}}{p^2+m_l^2}, \quad F_s(p) \equiv \frac{1}{\sqrt{p^2+m_s^2}}, \tag{3.1}$$

where the masses m_l and m_s fulfill the condition $m_l \ll m_s$. Further, the dimensionless quantities v_l and v_s denote the strengths of the long- and short-range interactions, respectively. The choice of the explicit form of $F_{l,s}(p)$ is entirely motivated by the simplicity of calculations [28]. The coefficients in the ERE generally scale with the mass corresponding to the long-range interaction which gives rise to the first left-hand cut in the T-matrix. Notice that the scattering length can be tuned to any value by adjusting the strength of the interaction. The coefficients in the ERE can be expanded in powers of m_l/m_s leading to the "chiral" expansion:

$$\begin{aligned} a &= \frac{1}{m_l}\left(\alpha_a^{(0)} + \alpha_a^{(1)}\frac{m_l}{m_s} + \alpha_a^{(2)}\frac{m_l^2}{m_s^2} + \ldots\right), \\ r &= \frac{1}{m_l}\left(\alpha_r^{(0)} + \alpha_r^{(1)}\frac{m_l}{m_s} + \alpha_r^{(2)}\frac{m_l^2}{m_s^2} + \ldots\right), \\ v_i &= \frac{1}{m_l^{2i-1}}\left(\alpha_{v_i}^{(0)} + \alpha_{v_i}^{(1)}\frac{m_l}{m_s} + \alpha_{v_i}^{(2)}\frac{m_l^2}{m_s^2} + \ldots\right), \end{aligned} \tag{3.2}$$

where $\alpha_a^{(m)}$, $\alpha_r^{(m)}$ and $\alpha_{v_i}^{(m)}$ are dimensionless constants whose values are determined by the specific form of the interaction potential. We fine tune the strengths of the long- and short-range interactions in such a way that they generate scattering lengths of a natural size. More precisely, we require that the scattering length takes the value $a = \alpha_l/m_l$ ($a = \alpha_s/m_s$) with a dimensionless constant $|\alpha_l| \sim 1$ ($|\alpha_s| \sim 1$) when the short-range (long-range) interaction is switched off. This leads to

$$v_l = -\frac{8\pi m_l^3 \alpha_l}{m\left(\alpha_l m_s^2 + m_l^2 \alpha_l - 2m_s^2\right)}, \quad v_s = -\frac{4\pi m_s \alpha_s}{m\left(\alpha_s - 1\right)}. \tag{3.3}$$

One then finds the following expressions for the first three terms in the "chiral" expansion of the scattering length

$$\alpha_a^{(0)} = \alpha_l, \qquad \alpha_a^{(1)} = (\alpha_l - 1)^2 \alpha_s, \qquad \alpha_a^{(2)} = (\alpha_l - 1)^2 \alpha_l \alpha_s^2, \tag{3.4}$$

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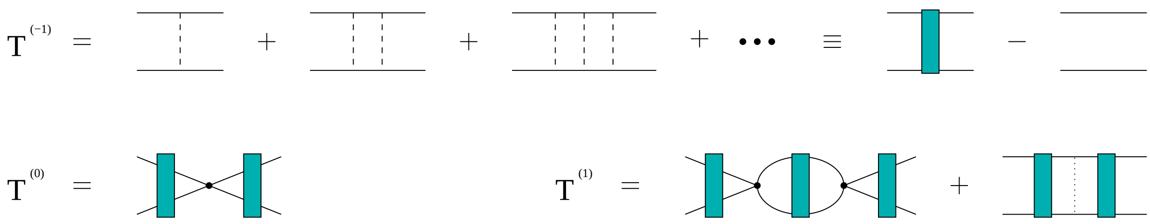

Figure 1: Leading, next-to-leading and next-to-next-to-leading order contributions to the scattering amplitude in the KSW-like approach. The solid lines denote nucleons while the dashed ones represent an insertion of the lowest-order (i.e. $\mathscr{O}(q^{-1})$) long-range interaction. Solid dots (dotted lines) denote an insertion of the lowest-order contact interaction $\propto C_0$ (subleading order-$\mathscr{O}(q)$ contribution to the long-range interaction).

and effective range

$$
\alpha_r^{(0)} = \frac{3\alpha_l - 4}{\alpha_l}, \qquad \alpha_r^{(1)} = \frac{2\,(\alpha_l - 1)\,(3\alpha_l - 4)\,\alpha_s}{\alpha_l^2},
$$
$$
\alpha_r^{(2)} = \frac{(\alpha_l - 1)\,(3\alpha_l - 4)\,(5\alpha_l - 3)\,\alpha_s^2 + (2 - \alpha_l)\,\alpha_l^2}{\alpha_l^3}. \tag{3.5}
$$

Notice that in the model considered the leading terms in the m_l/m_s-expansion of the ERE coefficients are completely fixed by the long-range interaction. The scenario realized corresponds to a strong (at momenta $k \sim m_l$) long-range interaction which needs to be treated non-perturbatively and a weak short-range interaction which can be taken into account perturbatively. This particular hierarchy is not important for our purposes.

At momenta of the order $k \lesssim m_l$, the details of the short-range interaction cannot be resolved. An EFT description emerges by keeping the long-range interaction and replacing the short-range one by a series of contact terms $V_{\rm short}(p, p') = C_0 + C_2(p^2 + p'^2) + \ldots$. Renormalization prescription plays an important role in organizing the EFT expansion. We first consider the most convenient and elegant KSW-like formulation based on the subtractive renormalization which respects dimensional power counting at the level of diagrams. The soft and hard scales in the problem are given by $q = \{k, \mu, m_l\}$ and $\lambda = \{m_s, m\}$, respectively. Here $\mu \sim m_l$ denotes the subtraction point. The contributions to the amplitude up to next-to-next-to-leading order (NNLO) in the q/λ-expansion are visualized in Fig. 1 and can be easily verified using naive dimensional analysis. Notice that the natural size of the short-range effects in our model suggests the scaling of the short-range interactions in agreement with the naive dimensional analysis, i.e. $C_{2n} \sim q^0$. At NNLO, the linearly divergent integral occurs which is treated in the following way

$$
I_1^{\rm reg} \equiv 4\pi m \int_0^\Lambda \frac{l^2 dl}{(2\pi)^3} \frac{1}{k^2 - l^2 + i\varepsilon} = -\frac{m\Lambda}{2\pi^2} - i\frac{mk}{4\pi} + \mathscr{O}(\Lambda^{-1}) \;\; \rightarrow \;\; I_1^{\rm subtr} = -\frac{m\mu}{2\pi^2} - i\frac{mk}{4\pi}. \tag{3.6}
$$

The effective range function at NNLO is given by the perturbative expansion

$$
k\cot\delta = -\frac{4\pi}{m}\frac{1}{T^{(-1)}}\left[1 - \frac{T^{(0)}}{T^{(-1)}} + \left(\frac{T^{(0)}}{T^{(-1)}}\right)^2 - \frac{T^{(1)}}{T^{(-1)}}\right] + ik, \tag{3.7}
$$

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where explicit expressions for the amplitudes $T^{(-1)}$, $T^{(0)}$ and $T^{(1)}$ are given in [28]. The μ-dependence of the renormalized low-energy constant (LEC) $C_0(\mu)$ is determined by the renormalization group equation

$$\frac{d}{d\mu}\left[T^{(-1)}+T^{(0)}+T^{(1)}\right]=0\,. \tag{3.8}$$

One needs two observables to fix the integration constants in the above equation for $C_0(\mu)$, see Ref. [28] for more details which we choose to be $\alpha_a^{(1)}$ and $\alpha_a^{(2)}$.[2] This leads to

$$C_0(\mu)=\frac{4\pi\alpha_s}{mm_s}+\frac{8\mu\alpha_s^2}{mm_s^2}+\mathscr{O}(q^2)\,, \tag{3.9}$$

and the following prediction for the effective range

$$\begin{aligned} r \, = \, \frac{1}{m_l}\Bigg[&\frac{3\alpha_l-4}{\alpha_l}+\frac{2\,(\alpha_l-1)\,(3\alpha_l-4)\,\alpha_s}{\alpha_l^2 m_s}m_l+\frac{(\alpha_l-1)\,(3\alpha_l-4)\,(5\alpha_l-3)\,\alpha_s^2+(2-\alpha_l)\,\alpha_l^2}{\alpha_l^3 m_s^2}m_l^2 \\ &-\frac{4\mu m_l\,(\alpha_l-1)\,(3\alpha_l-4)\,\alpha_s^3\,(\pi m_l\,(3-5\alpha_l)+4\mu\alpha_l)}{\pi^2\alpha_l^3 m_s^3}+\mathscr{O}\left(q^4\right)\Bigg]\,. \end{aligned} \tag{3.10}$$

As expected, the first three terms in the "chiral" expansion of r and shape parameters v_i, see Ref. [28], are correctly reproduced at NNLO being protected by the LETs introduced in the previous section. The knowledge of $\alpha_{x_j}^{(i)}$ for one particular x_j is sufficient to predict $\alpha_{x_k}^{(i)}$ for all $k\neq j$.

An EFT formulation like the one described above which respects the manifest power counting at every stage of the calculation is not available in the realistic case of nucleon-nucleon interaction. Here, one lacks a regularization prescription for *all* divergent integrals resulting from iterations of the potential in the LS equation which would keep regularization artefacts small without, at the same time, introducing a new hard scale in the problem. In the context of pionful EFT for few-nucleon systems, the divergent integrals are usually dealt with by introducing an UV cutoff Λ, which has to be taken of the order $\Lambda\sim m_s$ or higher in order to keep regularization artefacts small. Clearly, cutoff-regularized diagrams do not obey dimensional power counting anymore. *Renormalization* is carried out in this Weinberg-like framework by adjusting the bare LECs C_i to low-energy observables at a given value of Λ which then allows to eliminate the bare LECs in all other quantities of physical interest.[3]

To be specific, consider the effective potential at next-to-leading order in the Weinberg-like approach as depicted in Fig. 2

$$V_{\rm eff}(p,\,p')=v_l\,F_l(p)\,F_l(p')+C_0\,. \tag{3.11}$$

In addition to the divergent integral $I_1^{\rm reg}$ in Eq. (3.6), iteration of the above potential in the LS equation leads to another divergent integral

$$\begin{aligned} I_2^{\rm reg} \, \equiv \, 4\pi m\int_0^\Lambda\frac{l^2\,dl}{(2\pi)^3}\frac{\sqrt{l^2+m_s^2}}{[k^2-l^2+i\varepsilon][l^2+m_l^2]} &= \frac{m}{2\pi^2}\Bigg[k\frac{\sqrt{k^2+m_s^2}}{k^2+m_l^2}\ln\left(\frac{k+\sqrt{k^2+m_s^2}}{m_s}\right) \\ &-\frac{m_l m_s s}{2(k^2+m_l^2)}+\ln\left(\frac{m_s}{2\Lambda}\right)-\frac{i\pi k\sqrt{k^2+m_s^2}}{2\left(k^2+m_l^2\right)}\Bigg)+\mathscr{O}(\Lambda^{-1})\,, \end{aligned} \tag{3.12}$$

[2] In the considered model, the leading terms in the "chiral" expansion of the subthreshold parameters are driven by the long-range interaction alone and are, of course, correctly reproduced at order $\mathscr{O}(q^{(-1)})$ which is parameter free.

[3] Notice that the resulting nonlinear equations for $\{C_i\}$ do not necessarily possess real solutions for all values of Λ.

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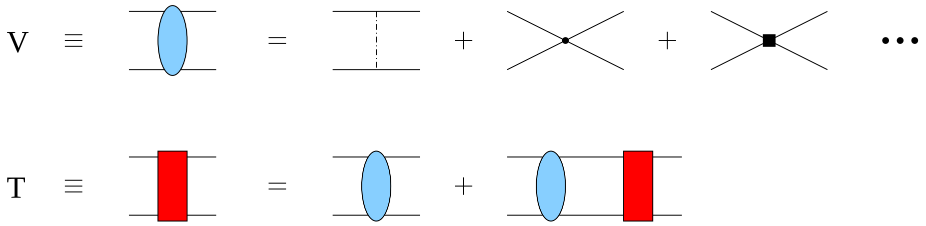

Figure 2: Effective potential and scattering amplitude in the Weinberg-like approach. The dashed-dotted line refers to the full long-range interaction. Solid dot and filled rectangle refer to the leading and subleading contact interactions, respectively. For remaining notation see Fig. 1.

where $s \equiv \left(2\sqrt{m_s^2 - m_l^2}/m_s\right) \operatorname{arccot}\left(m_l/\sqrt{m_s^2 - m_l^2}\right)$. Neglecting, for the sake of simplicity, the finite cutoff artefacts represented by the $\mathcal{O}(\Lambda^{-1})$-terms in Eqs. (3.6) and (3.12) and performing straightforward calculations, one obtains for the scattering length:

$$a_\Lambda = \frac{\pi m_s \left\{C_0 m \left[2\alpha_l \left(m_s (\Lambda - s m_l) + 2m_l^2 \ln(m_s/2\Lambda)\right) + \pi m_l m_s\right] + 4\pi^2 \alpha_l m_s\right\}}{m_l \left\{2\pi m_s^2 \left(C_0 m \Lambda + 2\pi^2\right) - C_0 m m_l \alpha_l \left[s m_s - 2m_l \ln(m_s/2\Lambda)\right]^2\right\}} . \qquad (3.13)$$

Renormalization is carried out by matching the above expression to the value of the scattering length in the model (to be regarded as a data),

$$a_{\rm underlying} = \frac{m_l \,(2\alpha_l - 1)\, \alpha_s - \alpha_l m_s}{m_l \,(m_l \alpha_l \alpha_s - m_s)} , \qquad (3.14)$$

and expressing $C_0(\Lambda)$ in terms of $a_{\rm underlying}$. A straightforward calculation yields the following *renormalized* expression for the effective range:

$$r_\Lambda = \frac{1}{m_l} \left[\frac{3\alpha_l - 4}{\alpha_l} + \frac{2\,(\alpha_l - 1)\,(3\alpha_l - 4)\,\alpha_s}{\alpha_l^2 m_s} m_l + \left(\frac{4\,(\alpha_l - 2)\,\alpha_s}{\pi \alpha_l m_s^2} \left(\ln \frac{m_s}{2\Lambda} + 1\right) \right.\right.$$
$$\left.\left. + \frac{(\alpha_l - 1)\,(3\alpha_l - 4)\,(5\alpha_l - 3)\,\alpha_s^2 + (2 - \alpha_l)\,\alpha_l^2}{\alpha_l^3 m_s^2} \right) m_l^2 + \mathcal{O}\left(m_l^3\right) \right] . \qquad (3.15)$$

In agreement with the LETs discussed above, one observes that the subleading terms in the "chiral" expansion of r (and v_i, see [28]) are correctly reproduced once C_0 is appropriately tuned. The sub-subleading and higher-order terms in the "chiral" expansion of r and v_i are not reproduced correctly being not protected by the LETs at the considered order. Moreover, since the included LEC is insufficient to absorb all divergencies arising from iterations of the LS equation, nothing prevents the appearance of positive powers or logarithms of the cutoff Λ in the expressions for $\alpha_r^{(\geq 2)}$. The results in Eq. (3.15) show that this is indeed the case. The dependence on Λ occurs, however, only in contributions beyond the accuracy of calculation and, obviously, does not affect the predictive power of the EFT as long as the cutoff is chosen to be of the order of the characteristic hard scale in the problem, $\Lambda \sim m_s$. Taking values $\Lambda \gg m_s$ artificially enhances certain higher-order contributions in the "chiral" expansion of the ERE coefficients spoiling the predictive power of the theory.

The appearance of positive powers of Λ and/or logarithmic terms in the predicted "chiral" expansion of the subthreshold parameters, see Eq. (3.15), may give the wrong impression that no

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finite limit exists for r_Λ and $(v_i)_\Lambda$ as $\Lambda \to \infty$. In fact, taking the limit $\Lambda \to \infty$ does not commute with the Taylor expansion of the ERE coefficients in powers of m_l. It is easy to see, that all coefficients in the ERE as well as the on-shell T-matrix stay finite as $\Lambda \to \infty$. In particular, one obtains the following infinite-cutoff prediction for the effective range:

$$r_\infty = \frac{1}{m_l}\left[\frac{3\alpha_l - 4}{\alpha_l} + \frac{4\,(\alpha_l - 1)^2\alpha_s}{\alpha_l^2 m_s} m_l + \frac{\alpha_l^3\left(8\alpha_s^2 - 1\right) + \alpha_l^2\left(2 - 20\alpha_s^2\right) + 16\alpha_l\alpha_s^2 - 4\alpha_s^2}{\alpha_l^3 m_s^2} m_l^2 + \ldots\right], \tag{3.16}$$

where ellipses refer to $\mathscr{O}\left(m_l^3\right)$-terms. One observes that the results after removing the cutoff fail to reproduce the low-energy theorem by yielding wrong values for $\alpha_r^{(1)}$, which also holds true for $\alpha_{v_i}^{(1)}$ [28] (notice that, per construction, the scattering length is still correctly reproduced).

4. Discussion and conclusions

The breakdown of LETs in the Weinberg-like approach in the $\Lambda \to \infty$ limit can be traced back to spurious Λ-dependent contributions still appearing in expressions for observables after renormalization is carried out, see e.g. Eq. (3.15), which are irrelevant (at the order of calculations) in the regime $\Lambda \sim m_s$ but become numerically dominant if $\Lambda \gg m_s$. Due to non-renormalizability of the effective potential as discussed in the introduction, such spurious terms do, in general, involve logarithms and positive powers of Λ which, as Λ gets increased beyond the hard scale m_s, become, at some point, comparable in size with lower-order terms in the "chiral" expansion. For example, the appearance of terms linear in Λ would suggest the breakdown of LETs as the cutoff approaches the scale $\Lambda \sim m_s^2/m_l$. The unavoidable appearance of ever higher power-law divergences when going to higher orders in the EFT expansion implies that the cutoff should not be increased beyond the pertinent hard scale in Weinberg-like or Lepage-like approach to NN scattering leading to $\Lambda \sim m_s$ as the optimal choice.[4] It is furthermore instructive to compare the predictions for the effective range in Eqs. (3.10) and (3.15) corresponding to two different renormalization schemes. One observes that taking $\Lambda \gg m_s$ in Eq. (3.15) has an effect which is qualitatively similar to choosing $\mu \gg m_l$ in Eq. (3.10) and corresponds to an improper choice of renormalization conditions in the EFT framework.

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