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Theory of the Hadronic Light-by-Light Contribution to Muon g-2

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I report on the theory, recent calculations and present status of the hadronic light-by-light contribution to muon $g-2$. In particular, I report on work done together with Eduardo de Rafael and Arkady Vainshtein where we get $a^{\mathrm{HLbL}} = (10.5 \pm 2.6) \times 10^{-10}$ as our present result for this quantity.

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1. Introduction

There are six possible momenta configurations contributing to the hadronic light-by-light to muon g-2, one of them is depicted in Fig. 1 and described by the vertex function

$$\Gamma^\mu(p_2,p_1) = -e^6 \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{\Pi^{\mu\nu\rho\sigma}(q,k_1,k_2,k_3)}{k_1^2 k_2^2 k_3^2} \times \gamma_\nu (\not{p}_2 + \not{k}_2 - m)^{-1} \gamma_\rho (\not{p}_1 - \not{k}_1 - m)^{-1} \gamma_\sigma \tag{1.1}$$

where $q \to 0$ is the momentum of the photon that couples to the external magnetic source, $q = p_2 - p_1 = -k_1 - k_2 - k_3$ and m is the muon mass.

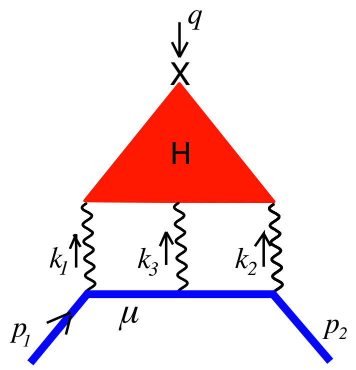

Figure 1: Hadronic light-by-light scattering contribution.

The dominant contribution to the hadronic four-point function

$$\Pi^{\rho\nu\alpha\beta}(q,k_1,k_3,k_2) = i^3 \int d^4x \int d^4y \int d^4z \, e^{i(-k_1\cdot x + k_3\cdot y + k_2\cdot z)} \langle 0|T\left[V^\mu(0)V^\nu(x)V^\rho(y)V^\sigma(z)\right]|0\rangle \tag{1.2}$$

comes from the three light-quark ($q = u,d,s$) components in the electromagnetic current $V^\mu(x) = \left[\overline{q}\widehat{Q}\gamma^\mu q\right](x)$ where $\widehat{Q} \equiv \mathrm{diag}(2,-1,-1)/3$ denotes the light-quark electric charge matrix. For $g-2$ we are interested in the limit $q \to 0$ where current conservation implies

$$\Gamma^\mu(p_2,p_1) = -\frac{a^{\mathrm{HLbL}}}{4m}[\gamma^\mu,\gamma^\nu]\, q_\nu \,. \tag{1.3}$$

Therefore, the muon anomaly can then be extracted as

$$a^{\mathrm{HLbL}} = \frac{e^6}{48m} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{1}{k_1^2k_2^2k_3^2} \left[\frac{\partial}{\partial q^\mu}\Pi^{\lambda\nu\rho\sigma}(q,k_1,k_3,k_2)\right]_{q=0} \times \mathrm{tr}\left\{(\not{p}+m)[\gamma_\mu,\gamma_\lambda](\not{p}+m)\gamma_\nu(\not{p}+\not{k}_2-m)^{-1}\gamma_\rho(\not{p}-\not{k}_1-m)^{-1}\gamma_\sigma\right\} . \tag{1.4}$$

Here I report on the results of [1] and [2]. Previous work on the hadronic light-by-light contribution to muon $g-2$ can be found in [3–12] and recent reviews are in [13–16].

The hadronic four-point function $\Pi^{\mu\nu\rho\sigma}(q,k_1,k_3,k_2)$ is an extremely difficult object involving many scales and no full first principle calculation of it has been reported yet –even in the simpler large numbers of colors N_c of QCD limit. Notice that we need that hadronic four-point function with momenta k_1, k_2 and k_3 varying from 0 to ∞ and $q \to 0$. Unfortunately, unlike the hadronic vacuum polarization, there is neither a direct connection of $a^{\rm HLbL}$ to a measurable quantity. Two lattice groups have started exploratory calculations [17, 18] but the final uncertainty that these calculations can reach is not clear yet.

Attending to a combined large number of colors of QCD N_c and chiral perturbation theory (CHPT) counting, one can distinguish four types of contributions [19]. Notice that we use the CHPT counting only for organization of the contributions and refers to the lowest order term contributing in each case. In fact, Ref. [1] shows that there are chiral enhancement factors that demand more than Nambu-Goldstone bosons in the CHPT expansion in the light-by-light contribution to the muon anomaly. See more comments on this afterwards.

The four different types of contributions mentioned above are:

- Nambu-Goldstone boson exchanges contribution are $\mathcal{O}(N_c)$ and start contributing at $\mathcal{O}(p^6)$ in CHPT.

- One-meson irreducible vertex contribution and non-Goldstone boson exchanges contribute also at $\mathcal{O}(N_c)$ but start contributing at $\mathcal{O}(p^8)$ in CHPT.

- One-loop of Goldstone bosons contribution are $\mathcal{O}(1/N_c)$ and start at $\mathcal{O}(p^4)$ in CHPT.

- One-loop of non-Goldstone boson contributions which are $\mathcal{O}(1/N_c)$ but start contributing at $\mathcal{O}(p^8)$ in CHPT.

Based on the counting above there are two full calculations [3, 4, 6] and [5, 7]. There is also a detailed study of the π^0 exchange contribution [8] putting emphasis in obtaining analytical expressions for this part. Recently, two new calculations of the pion exchange using also the organization above have been made. In Ref. [10], the pion pole term exchange is evaluated within an effective chiral model, NχQM. These authors also study the box diagram one-meson irreducible vertex contribution. The results are numerically very similar to the ones found in the literature as can be seen in Table 1. In Ref. [11], the author uses a large N_c model $\pi^0\gamma^*\gamma^*$ form factor with the pion also off-shell. This has to be considered as a first step and more work has to be done in order to have the full light-by-light within this approach. In particular, it would be very interesting to calculate the contribution of one-meson irreducible vertex contribution within this model.

There is also model independent short-distance QCD information on the relevant form factor. Using operator product expansion (OPE) in QCD, the authors of [12] pointed out a short-distance constraint of the reduced full four-point Green function (form factor)

$$\langle 0|T\left[V^\nu(k_1)V^\rho(k_3)V^\sigma(-(k_1+k_2+q))\right]|\gamma(q)\rangle \tag{1.5}$$

when $q \to 0$ and in the special momenta configuration $-ks_1^2 \simeq -k_3^2 >> -(k_1+k_3)^2$ Euclidean and large. In that kinematical region,

$$T\left[V^\nu(k_1)V^\rho(k_3)\right] \sim \frac{1}{\hat{k}^2}\,\varepsilon^{\nu\rho\alpha\beta}\hat{k}_\alpha\left[\overline{q}\hat{Q}^2\gamma_\beta\gamma_5 q\right] \tag{1.6}$$

with $\hat{k} = (k_1-k_3)/2 \simeq k_1 \simeq -k_3$. See also [20]. This short distance constraint was not explicitly imposed in calculations previous to [12].

2. Leading in $1/N_c$ Results

Using effective field theory techniques, the authors of [9] shown that the leading large N_c contribution to $a^{\rm HLbL}$ contains an enhanced $\log^2(M_\rho/m_\pi)$ term at low energy. Where the rho mass M_ρ acts as an ultraviolet scale and the pion mass m_π provides the infrared scale. The leading logarithm term is generated by Nambu-Goldstone boson exchange contributions and is fixed by the Wess–Zumino–Witten (WZW) vertex $\pi^0\gamma\gamma$.

$$a^{\rm HLbL}(\pi^0) = \left(\frac{\alpha}{\pi}\right)^3 N_c\frac{m^2N_c}{48\pi^2 f_\pi^2}\left[\ln^2\frac{M_\rho}{m_\pi} + \mathscr{O}\left(\ln\frac{M_\rho}{m_\pi}\right) + \mathscr{O}(1)\right] \tag{2.1}$$

In the chiral limit, where quark masses are neglected, and at large N_c, the coefficient of this double logarithm is model independent and has been calculated and shown to be positive in [9]. All the calculations we discuss here agree with these leading behaviour and its coefficient including the sign. A global sign mistake in the π^0 exchange in the results presented in [3–5] was found by [8, 9] and confirmed by [6, 7] and by others [21, 22]. The subleading ultraviolet scale μ-dependent terms [9], namely, $\log(\mu/m_\pi)$ and a non-logarithmic term $\kappa(\mu)$, are model dependent and calculations of them are implicit in the results presented in [3–5, 7, 12]. In particular, $\kappa(\mu)$ contains the large N_c contributions from one-meson irreducible vertex and non–Nambu-Goldstone boson exchanges. In the next section we review the recent model calculations of the full leading in the $1/N_c$ expansion contributions.

2.1 Model Calculations

The pseudo-scalar exchange is the dominant numerical contribution and was saturated in [3–8, 10, 11] by Nambu-Goldstone boson exchange. This contribution is depicted in Fig. 2 with $M = \pi^0, \eta, \eta'$. The relevant four-point function was obtained in terms of the off-shell $\pi^0\gamma^*(k_1)\gamma^*(k_3)$ form factor $\mathscr{F}(k_1^2,k_3^2)$ and the off-shell $\pi^0\gamma^*(k_2)\gamma(q=0)$ form factor $\mathscr{F}(k_2^2,0)$ modulating each one of the two WZW $\pi^0\gamma\gamma$ vertex.

In all cases discussed here, several short-distance QCD constraints were imposed on these form-factors. In particular, they all have the correct QCD short-distance behaviour

$$\mathscr{F}(Q^2,Q^2) \to \frac{A}{Q^2} \quad \text{and} \quad \mathscr{F}(Q^2,0) \to \frac{B}{Q^2} \tag{2.2}$$

when Q^2 is Euclidean and large and are in agreement with $\pi^0\gamma^*\gamma$ low-energy data [1]. They differ

[1] See however the new measurement of the $\gamma\gamma^* \to \pi_0$ transition form factor by BaBar [23] at momentum transfer energies between 4 GeV^2 and 40 GeV^2

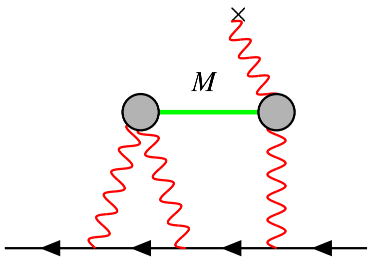

Figure 2: A generic meson exchange contribution to the hadronic light-by-light part of the muon $g-2$.

References	$10^{10} \times a$	
	π^0 only	π^0, η and η'
[3, 4, 6]	5.7	8.3 ± 0.6
[5, 7]	5.6	8.5 ± 1.3
[8] with $h_2 = 0$	5.8	8.3 ± 1.2
[8] with $h_2 = -10$ GeV^2	6.3	
[10]	$6.3 \sim 6.7$	
[11]	7.2	9.9 ± 1.6
[12]	7.65	11.4 ± 1.0

Table 1: Results for the π^0, η and η' exchange contributions.

slightly in shape due to the different model assumptions (VMD, ENJL, Large N_c, NχQM) but they produce small numerical differences always compatible within quoted uncertainty $\sim (1.3-1.6) \times 10^{-10}$ –see Table 1.

Within the models used in [3–8, 10, 11], to get the full contribution at leading in $1/N_c$ one needs to add the one-meson irreducible vertex contribution and the non-Goldstone boson exchanges. In particular, below some hadronic scale Λ, the one-meson irreducible vertex contribution was identified in [5, 7] with the ENJL quark box contribution with four dressed photon legs. While to mimic the contribution of short-distance QCD quarks above Λ, a loop of bare massive heavy quark with mass Λ and QCD vertices was used. The results are in Table 2. There, one can see a very nice stability region when Λ is in the interval [0.7, 4.0] GeV. Similar results for a constituent quark-box contribution below Λ were obtained in [3, 4], though these authors didn't discuss any short-distance–long-distance matching.

In [5, 7], non-Goldstone boson exchanges were saturated by the hadrons appearing in the model, i.e. the lowest scalar and pseudo-vector hadrons. There, both states were used in nonet-

Λ [GeV]	0.7	1.0	2.0	4.0
$10^{10} \times a^{\rm HLbL}$	2.2	2.0	1.9	2.0

Table 2: Sum of the short- and long-distance quark loop contributions [5] as a function of the matching scale Λ.

References	$10^{10} \times a^{\rm HLbL}$
[3, 4, 6]	0.17 ± 0.10
[5, 7]	0.25 ± 0.10

Table 3: Results for the axial-vector exchange contributions from [3, 4, 6] and [5, 7].

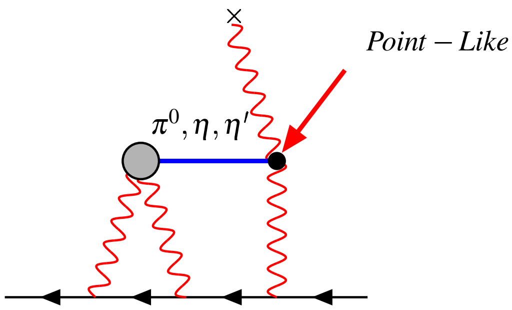

Figure 3: Goldstone boson exchange in the model in [12] contributing to the hadronic light-by-light.

symmetry –this symmetry is exact in the large N_c limit of QCD.

Within the ENJL model, the one-meson irreducible vertex contribution is related trough Ward identities to the scalar exchange which we discuss below and *both* have to be included within this model [5, 7]. The result of the scalar exchange obtained in [5] is

$$a^{\rm HLbL}({\rm Scalar}) = -(0.7 \pm 0.2) \times 10^{-10} \, . \tag{2.3}$$

The scalar exchange was not included in [3, 4, 6, 8]. The result of the axial-vector exchanges in [3, 4, 6] and [5, 7] can be found in Table 3.

Melnikov and Vainshtein used a model that saturates the hadronic four-point function in (1.2) at leading order in the $1/N_c$ expansion by the exchange of the Nambu-Goldstone π^0, η, η' and the lowest axial-vector f_1 states. In that model, the new OPE constraint of the reduced four-point function found in [12] mentioned above, forces the $\pi^0 \gamma^*(q) \gamma(p_3 = 0)$ vertex to be point-like rather than including a $\mathscr{F}(q^2, 0)$ form factor. There are also OPE constraints for other momenta regions [24] which are not satisfied by the model in [12] though the authors argued that this mismatch

Full Hadronic Light-by-Light	$10^{10} \times a_\mu$
[3, 4, 6]	8.9± 1.7
[5, 7]	8.9 ± 3.2
[12]	13.6 ± 2.5

Table 4: Results for the full hadronic light-by-light contribution to $a^{\rm HLbL}$.

makes only a small numerical difference of the order of 0.05×10^{-10}. In fact, within the large N_c framework, it has been shown [25] that in general for other than two-point functions, to satisfy fully the QCD short-distance properties requires the inclusion of an infinite number of narrow states.

3. Next-to-leading in $1/N_c$ Results

For the next-to-leading in $1/N_c$ contributions to the $a^{\rm HLbL}$ there is no model independent result at present and is possibly the most difficult component. Charged pion and kaon loops saturated this contribution in [3 – 7]. To dress the photon interacting with pions, a particular Hidden Gauge Symmetry (HGS) model was used in [3, 4, 6] while a full VMD was used in [5, 7]. The results obtained in these two models are $-(0.45 \pm 0.85) \times 10^{-10}$ in [3] and $-(1.9 \pm 0.5) \times 10^{-10}$ in [5] while using a point-like vertex one gets -4.6×10^{-10}.

Both models (HGS and VMD) satisfy the known constraints though start differing at $\mathcal{O}(p^6)$ in CHPT. Some studies of the cut-off dependence of the pion loop using the full VMD model was done in [5] and showed that their final number comes from fairly low energies where the model dependence should be smaller.

The authors of [12] analyzed the model used in [3, 4] and showed that there is a large cancellation between the first three terms of an expansion in powers of $(m_\pi/M_\rho)^2$ and with large higher order corrections when expanded in CHPT orders but the same applies to the π^0 exchange as can be seen from Table 6 in the first reference in [2] by comparing the WZW column with the others. The authors of [12] took $(0 \pm 1) \times 10^{-10}$ as a guess estimate of the total NLO in $1/N_c$ contribution. This seems too simply and certainly with underestimated uncertainty.

4. Comparing Different Calculations

The comparison of individual contributions in [3 – 8, 10 – 12] has to be done with care because they come from different model assumptions to construct the full relevant four-point function. In fact, the authors of [10] have shown that their constituent quark-box provides the correct asymptotics and in particular the new OPE found in [12]. It has more sense to compare results for $a^{\rm HLbL}$ either at leading order or at next-to-leading order in the $1/N_c$ expansion.

The results for the final hadronic light-by-light contribution to $a^{\rm HLbL}$ quoted in [3 – 7, 12] are in Table 4. The apparent agreement between [3, 4, 6] and [5, 7] hides non-negligible differences which numerically almost compensate between the quark-loop and charged pion and [12] are in Table 4. Notice also that [3, 4, 6] didn't include the scalar exchange.

Comparing the results of [5, 7] and [12], as discussed above, we have found several differences of order 1.5×10^{-10} which are not related to the new short-distance constraint used in [12]. The different axial-vector mass mixing accounts for -1.5×10^{-10}, the absence of the scalar exchange in [12] accounts for -0.7×10^{-10} and the use of a vanishing NLO in $1/N_c$ contribution in [12] accounts for -1.9×10^{-10}. These model dependent differences add up to -4.1×10^{-10} out of the final -5.3×10^{-10} difference between the results in [5, 7] and the ones in [12] –see Table 4. Clearly, the new OPE constraint used in [12] accounts only for a small part of the large numerical final difference.

5. Conclusions and Prospects

To give a result at present for the hadronic light–by–light contribution to the muon anomalous magnetic moment, the authors of [1] concluded, from the above considerations, that it is fair to proceed as follows:

Contribution to $a^{\rm HLbL}$ from π^0, η and η' exchanges

Because of the effect of the OPE constraint discussed above, we suggested [1] to take as central value the result of Ref. [12] with, however, the largest error quoted in Refs. [5, 7]:

$$a^{\rm HLbL}(\pi,\eta,\eta') = (11.4\pm1.3)\times10^{-10}\,. \tag{5.1}$$

Recall that this central value is quite close to the one in the ENJL model which includes the short–distance quark-loop contribution.

Contribution to $a^{\rm HLbL}$ from pseudo-vector exchanges

The analysis made in Ref. [12] suggests that the errors in the first and second entries of Table 3 are likely to be underestimates. Raising their ±0.10 errors to ±1 puts the three numbers in agreement within one sigma. We suggested [1] then as the best estimate for this contribution at present

$$a^{\rm HLbL}({\rm pseudo-vectors}) = (1.5\pm1)\times10^{-10}\,. \tag{5.2}$$

Contribution to $a^{\rm HLbL}$ from scalar exchanges

The ENJL–model should give a good estimate for these contributions. We kept [1], therefore, the result of Ref. [5, 7] with, however, a larger error which covers the effect of other unaccounted meson exchanges,

$$a^{\rm HLbL}({\rm scalars}) = -(0.7\pm0.7)\times10^{-10}\,. \tag{5.3}$$

Contribution to $a^{\rm HLbL}$ from dressed charged pion and kaon loop

Because of the instability of the results for the charged pion loop and unaccounted loops of other mesons, we suggested [1] using the central value of the ENJL result but wit a larger error:

$$a^{\rm HLbL}(\pi-{\rm dressed\ loop}) = -(1.9\pm1.9)\times10^{-10}\,. \tag{5.4}$$

From these considerations, adding the errors in quadrature, as well as the small charm contribution 0.23×10^{-10}, we get

$$a^{\rm HLbL} = (10.5\pm2.6)\times10^{-10}\,, \tag{5.5}$$

as our final estimate.

The proposed new muon $g-2$ experiments at Fermilab [28] with 1.6×10^{-10} accuracy goal and at J-PARC [29] with even higher accuracy goal between 1.2×10^{-10} and 0.6×10^{-10} call for a considerable improvement in the present calculations of $a^{\rm HLbL}$. The use of further theoretical and experimental constraints could result in reaching such accuracy soon enough. In particular, imposing as many as possible short-distance QCD constraints [3 – 8, 11] has result in a better understanding of the numerically dominant π^0 exchange. At present, none of the light-by-light hadronic parametrization satisfy fully all short distance QCD constraints. In particular, this requires the inclusion of infinite number of narrow states for other than two-point functions and two-point functions with soft insertions [25]. A numerical dominance of certain momenta configuration can help to minimize the effects of short distance QCD constraints not satisfied, as in the model in [12].

More experimental information on the decays $\pi^0 \to \gamma\gamma^*$, $\pi^0 \to \gamma^*\gamma^*$ and $\pi^0 \to e^+e^-$ (with radiative corrections included [22, 26, 27]) in the low- and intermediate-energy regions (below a few GeVs) can also help to confirm some of the neutral pion exchange results. A better understanding of other smaller contributions but with comparable uncertainties needs both more theoretical work and experimental information. This refers in particular to pseudo-vector exchanges. Experimental data on radiative decays and two-photon production of these and other C-even resonances can be useful in that respect.

New approaches to the pion dressed loop contribution, together with experimental information on the vertex $\pi^+\pi^-\gamma^*\gamma^*$ in the intermediate energy region (0.5 − 1.5) GeV would also be very welcome. Measurements of two-photon processes like $e^+e^- \to e^+e^-\pi^+\pi^-$ can be useful to give information on that vertex and again could reduce the model dependence. The two-gamma physics program low energy facilities like the experiment KLOE-2 at DAΦNE will be very useful and well suited in the processes mentioned above which information can help to decrease the present model dependence of $a^{\rm HLbL}$.

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