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Hadronic light-by-light scattering in the muon $g-2$: a new short-distance constraint on pion exchange

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We summarize our recent new evaluation of the pion-exchange contribution to hadronic light-by-light scattering in the muon $g-2$. We first derive a new short-distance constraint on the off-shell pion-photon-photon form factor at the external vertex in a_μ which relates the form factor to the quark condensate magnetic susceptibility in QCD. We then evaluate the pion-exchange contribution in the framework of large-N_C QCD using an off-shell form factor which fulfills all short-distance constraints and obtain the new estimate $a_\mu^{\mathrm{LbyL};\pi^0} = (72 \pm 12) \times 10^{-11}$. Updating our earlier results for the contributions from the exchanges of the η and η' using simple vector-meson dominance form factors, we get $a_\mu^{\mathrm{LbyL;PS}} = (99 \pm 16) \times 10^{-11}$ for the sum of all light pseudoscalars. Combined with available evaluations for the other contributions to hadronic light-by-light scattering this leads to the estimate $a_\mu^{\mathrm{LbyL;had}} = (116 \pm 40) \times 10^{-11}$. The corresponding contributions to the anomalous magnetic moment of the electron are also given.

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1. Introduction

The muon $g-2$ has served over many decades as an important test of the Standard Model (SM). It is also sensitive to contributions from New Physics slightly above the electroweak scale. In fact, for several years now a discrepancy of more than three standard deviations has existed between the SM prediction and the experimental value, see the recent reviews Refs. [1, 2, 3] on the muon $g-2$. The main error in the theoretical SM prediction comes from hadronic contributions, i.e. hadronic vacuum polarization and hadronic light-by-light (had. LbyL) scattering. Whereas the hadronic vacuum polarization contribution can be related to the cross section $e^+e^- \to$ hadrons, no direct experimental information is available for had. LbyL scattering. One therefore has to rely on hadronic models to describe the strongly interacting, nonperturbative dynamics at the relevant scales from the muon mass up to about 2 GeV. This leads to large uncertainties, see Refs. [4, 5, 3] for recent reviews on had. LbyL scattering, largely based on the original works [6, 7, 8, 9].

Essentially, these models describe the interactions of hadrons with photons, usually with the help of some form factors. One can reduce this model dependence and the corresponding uncertainties by relating the form factors at low energies to results from chiral perturbation theory (ChPT) [10] and at high energies (short distances) to the operator product expansion (OPE) [11]. In this way, one connects the form factors to the underlying theory of QCD. In particular, this has been done in Refs. [6, 7, 12, 8, 9] for the numerically dominant contribution from the exchange of light pseudoscalars π^0, η, η'.

The pseudoscalar-exchange contributions to had. LbyL scattering are given by the diagrams shown in Fig. 1.

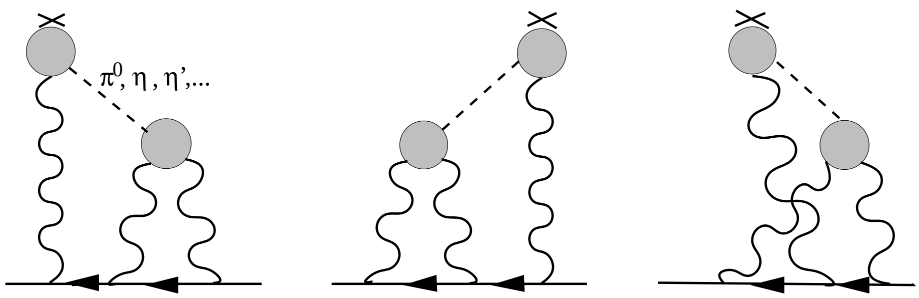

Figure 1: The pseudoscalar-exchange contributions to had. LbyL scattering. The shaded blobs represent the form factor $\mathscr{F}_{\mathrm{PS}^*\gamma^*\gamma^*}$ where $\mathrm{PS} = \pi^0, \eta, \eta', \pi^{0\prime}, \ldots$.

It was pointed out recently in Ref. [2], that one should use fully *off-shell* form factors for the evaluation of the LbyL scattering contribution. This seems to have been overlooked in the recent literature, in particular, in Refs. [12, 8, 9, 4, 5]. The on-shell form factors as used in Refs. [8, 12] actually violate four-momentum conservation at the external vertex, as observed already in Ref. [9].

The exchange of the lightest state π^0 yields the largest contribution and therefore warrants special attention. In these proceedings, based on the results obtained in Ref. [13], we present a new QCD short-distance constraint on the off-shell pion-photon-photon form factor $\mathscr{F}_{\pi^{0*}\gamma^*\gamma^*}$ at the external vertex by relating it to the quark condensate magnetic susceptibility of QCD. We then evaluate this contribution in the framework of large-N_C QCD [14], using a form factor which fulfills this new and other relevant short-distance constraints.

2. On-shell versus off-shell form factors

For the pion, the key object which enters the diagrams in Fig. 1 is the *off-shell* form factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)$ which can be defined via the QCD Green's function $\langle VVP \rangle$ [6, 7, 13]

$$\int d^4x\, d^4y\, e^{i(q_1\cdot x + q_2\cdot y)} \,\langle 0|T\{j_\mu(x) j_\nu(y) P^3(0)\}|0\rangle$$
$$= \varepsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta \,\frac{i\langle\overline{\psi}\psi\rangle}{F_\pi}\, \frac{i}{(q_1+q_2)^2 - m_\pi^2}\, \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) \;+\; \ldots, \tag{2.1}$$

up to small mixing effects with the states η and η' and neglecting exchanges of heavier states like $\pi^{0\prime}, \pi^{0\prime\prime}, \ldots$. Here $j_\mu(x)$ is the light quark part of the electromagnetic current and $P^3 = \overline{\psi} i\gamma_5 \frac{\lambda^3}{2}\psi$.

The corresponding contribution to the muon $g-2$ may be worked out with the result [8]

$$a_\mu^{\mathrm{LbyL};\pi^0} = -e^6 \int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 [(p+q_1)^2 - m_\mu^2][(p-q_2)^2 - m_\mu^2]}$$
$$\times \left[\frac{\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q_2^2, q_1^2, (q_1+q_2)^2)\; \mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q_2^2, q_2^2, 0)}{q_2^2 - m_\pi^2}\; T_1(q_1, q_2; p) \right.$$
$$\left. + \frac{\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)\; \mathcal{F}_{\pi^{0*}\gamma^*\gamma}((q_1+q_2)^2, (q_1+q_2)^2, 0)}{(q_1+q_2)^2 - m_\pi^2}\; T_2(q_1, q_2; p) \right], \tag{2.2}$$

where the external photon has now zero four-momentum. See Ref. [8] for the expressions for T_i.

Instead of the representation in Eq. (2.2), Refs. [12, 8] considered *on-shell* form factors which would yield the so called *pion-pole* contribution, e.g. for the term involving T_2, one would write [2]

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_2^2) \;\times\; \mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, (q_1+q_2)^2, 0). \tag{2.3}$$

Although pole dominance might be expected to give a reasonable approximation, it is not correct as it was used in those references, as stressed in Refs. [9, 2]. The point is that the form factor sitting at the external photon vertex in the pole approximation $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, (q_1+q_2)^2, 0)$ for $(q_1+q_2)^2 \neq m_\pi^2$ violates four-momentum conservation, since the momentum of the external (soft) photon vanishes. The latter requires $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}((q_1+q_2)^2, (q_1+q_2)^2, 0)$. Ref. [9] then proposed to use instead

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_2^2) \;\times\; \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, m_\pi^2, 0)\,. \tag{2.4}$$

Note that putting the pion on-shell at the external vertex automatically leads to a constant form factor, given by the Wess-Zumino-Witten (WZW) term [15]. However, this prescription does not yield the *pion-exchange* contribution with off-shell form factors, which we calculate with Eq. (2.2).

Strictly speaking, the identification of the pion-exchange contribution is only possible, if the pion is on-shell. If one is off the mass shell of the exchanged particle, it is not possible to separate different contributions to the $g-2$, unless one uses some particular model where elementary pions can propagate. In this sense, only the pion-pole contribution with on-shell form factors can be defined, at least in principle, in a model-independent way. On the other hand, the pion-pole contribution is only a part of the full result, since in general, e.g. using some resonance Lagrangian, the form factors will enter the calculation with off-shell momenta. In this respect, we view our evaluation as being a part of a full calculation of had. LbyL scattering using a resonance Lagrangian whose coefficients are tuned in such a way as to systematically reproduce the relevant QCD short-distance constraints, along the lines of the resonance chiral theory developed in Ref. [16].

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3. A new short-distance constraint on the off-shell pion-photon-photon form factor

The form factor $\mathscr{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)$ defined in Eq. (2.1) is determined by nonperturbative physics of QCD and cannot (yet) be calculated from first principles. Therefore, various hadronic models have been used in the literature. At low energies, the form factor is normalized by the decay amplitude, $\mathscr{A}(\pi^0 \to \gamma\gamma) \equiv e^2 \mathscr{F}_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0)$. To a good approximation, all hadronic models thus have to satisfy the constraint $\mathscr{F}_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0) = -N_C/(12\pi^2 F_\pi)$.[1]

For an on-shell pion, there is also experimental data available for one on-shell and one off-shell photon, from the process $e^+e^- \to e^+e^-\pi^0$. Several experiments [19] thereby fairly well confirm the Brodsky-Lepage [20] behavior for large Euclidean momentum $\lim_{Q^2\to\infty} \mathscr{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0) \sim -2F_\pi/Q^2$ and any model should reproduce this behavior, maybe with a different prefactor.[2]

Apart from these experimental constraints, any consistent hadronic model for the off-shell form factor $\mathscr{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)$ should match at large momentum with short-distance constraints from QCD that can be calculated using the OPE. In Ref. [22] the short-distance properties for the three-point function $\langle VVP \rangle$ in Eq. (2.1) in the chiral limit and assuming octet symmetry have been worked out in detail. Two limits are of interest. In the first case, the two momenta become simultaneously large, which describes the situation where the space-time arguments of all three operators tend towards the same point at the same rate. The second situation corresponds to the case where the relative distance between only two of the three operators in $\langle VVP \rangle$ becomes small. When the space-time arguments of the two vector currents in $\langle VVP \rangle$ approach each other, the leading term in the OPE leads to the Green's function $\langle AP \rangle$. The explicit results for both these cases can be found in Refs. [22, 13].

The new short-distance constraint on the off-shell form factor at the external vertex in had. LbyL scattering arises when the space-time argument of one of the vector currents in $\langle VVP \rangle$ approaches the argument of the pseudoscalar density. This leads to the two-point function $\langle VT \rangle$ of the vector current and the antisymmetric tensor density

$$\delta^{ab}(\Pi_{\rm VT})_{\mu\rho\sigma}(p) = \int d^4x e^{ip\cdot x} \langle 0|T\{V_\mu^a(x)(\overline{\psi}\sigma_{\rho\sigma}\frac{\lambda^b}{2}\psi)(0)\}|0\rangle, \qquad \sigma_{\rho\sigma} = \frac{i}{2}[\gamma_\rho, \gamma_\sigma]. \tag{3.1}$$

Conservation of the vector current and invariance under parity then give $(\Pi_{\rm VT})_{\mu\rho\sigma}(p) = (p_\rho\eta_{\mu\sigma} - p_\sigma\eta_{\mu\rho})\Pi_{\rm VT}(p^2)$. In this way one obtains the relation (up to corrections of order α_s) [22, 13]

$$\lim_{\lambda\to\infty} \mathscr{F}_{\pi^{0*}\gamma^*\gamma^*}((\lambda q_1+q_2)^2, (\lambda q_1)^2, q_2^2) = -\frac{2}{3}\frac{F_0}{\langle\overline{\psi}\psi\rangle_0}\Pi_{\rm VT}(q_2^2) + \mathscr{O}\left(\frac{1}{\lambda}\right). \tag{3.2}$$

In particular, at the external vertex in LbyL scattering in Eq. (2.2), the limit $q_2 \to 0$ is relevant.

As pointed out in Ref. [23], the value of $\Pi_{\rm VT}(p^2)$ at zero momentum is related to the quark condensate magnetic susceptibility χ in QCD in the presence of a constant external electromagnetic field, introduced in Ref. [24]: $\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F = e\, e_q\, \chi\, \langle\overline{\psi}\psi\rangle_0 F_{\mu\nu}$, with $e_u = 2/3$ and $e_d = -1/3$.

[1] We note that in our work [13] and in Refs. [6, 7, 8, 9] simply $F_\pi = 92.4$ MeV is used, without any error attached. Maybe this could be an additional source of uncertainty in $a_\mu^{\rm LbyL;\pi^0}$, in particular in view of the new value $\Gamma(\pi^0 \to \gamma\gamma) = (7.82 \pm 0.23)$ eV presented in Ref. [17]; see also the discussion in Ref. [18] and references therein.

[2] Note, however, that a recent measurement of the form factor by the BABAR collaboration [21] at momentum transfers Q^2 between 4 GeV^2 and 40 GeV^2 does not show such a falloff. We will come back to this issue in Section 4.

With our definition of $\Pi_{\rm VT}$ in Eq. (3.1) one obtains the relation $\Pi_{\rm VT}(0) = -(\langle\overline{\psi}\psi\rangle_0/2)\chi$ (see also Ref. [25]) and the new short-distance constraint at the external vertex can be written as [13]

$$\lim_{\lambda\to\infty} \mathscr{F}_{\pi^{0*}\gamma^*\gamma}((\lambda q_1)^2,(\lambda q_1)^2,0) = \frac{F_0}{3}\,\chi + \mathscr{O}\left(\frac{1}{\lambda}\right). \tag{3.3}$$

Note that there is no falloff in this limit, unless χ vanishes.

Unfortunately there is no agreement in the literature what the actual value of χ should be. In comparing different results one has to keep in mind that χ actually depends on the renormalization scale μ. In Ref. [24] the estimate $\chi(\mu = 0.5~\text{GeV}) = -(8.16^{+2.95}_{-1.91})~\text{GeV}^{-2}$ was given in a QCD sum rule evaluation of nucleon magnetic moments. A similar value $\chi = -N_C/(4\pi^2 F_\pi^2) = -8.9~\text{GeV}^{-2}$ was obtained in Ref. [26], probably again for a low scale $\mu \sim 0.5$ GeV as argued in Ref. [26].

On the other hand, saturating the leading short-distance behavior of the two-point function $\Pi_{\rm VT}$ [27] with one multiplet of lowest-lying vector mesons (LMD) [28, 23, 22] leads to the estimate $\chi^{\rm LMD} = -2/M_V^2 = -3.3~\text{GeV}^{-2}$ [28]. Again, it is not obvious at which scale this relation holds, it might be at $\mu = M_V$. This LMD estimate was soon afterwards improved by taking into account higher resonance states (ρ', ρ'') in the framework of QCD sum rules, with the results $\chi(0.5~\text{GeV}) = -(5.7\pm0.6)~\text{GeV}^{-2}$ [23] and $\chi(1~\text{GeV}) = -(4.4\pm0.4)~\text{GeV}^{-2}$ [29]. A more recent analysis [30] yields, however, a smaller absolute value $\chi(1~\text{GeV}) = -(3.15\pm0.30)~\text{GeV}^{-2}$, close to the original LMD estimate.[3] For a quantitative comparison of all these estimates for χ we would have to run them to a common scale, for instance, 1 GeV or 2 GeV, which can obviously not be done within perturbation theory starting from such low scales as $\mu = 0.5$ GeV.

4. New evaluation of the pseudoscalar-exchange contribution in large-N_C QCD

In the spirit of the minimal hadronic Ansatz [33] for Green's functions in large-N_C QCD, an *off-shell* form factor $\mathscr{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)$ has been constructed in Ref. [22]. It contains the two lightest multiplets of vector resonances, the ρ and the ρ' (LMD+V), and fulfills all the OPE constraints discussed earlier:

$$\mathscr{F}^{\rm LMD+V}_{\pi^{0*}\gamma^*\gamma^*}(q_3^2,q_1^2,q_2^2) = \frac{F_\pi}{3}\,\frac{q_1^2 q_2^2 (q_1^2+q_2^2+q_3^2) + P_H^V(q_1^2,q_2^2,q_3^2)}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}, \tag{4.1}$$

$$\begin{aligned} P_H^V(q_1^2,q_2^2,q_3^2) = {} & h_1 (q_1^2+q_2^2)^2 + h_2 q_1^2 q_2^2 + h_3 (q_1^2+q_2^2) q_3^2 + h_4 q_3^4 \\ & + h_5 (q_1^2+q_2^2) + h_6 q_3^2 + h_7, \qquad q_3^2 = (q_1+q_2)^2. \end{aligned} \tag{4.2}$$

Below we reevaluate the pion-exchange contribution using off-shell LMD+V form factors at both vertices. The constants h_i in the Ansatz for $\mathscr{F}^{\rm LMD+V}_{\pi^{0*}\gamma^*\gamma^*}$ in Eq. (4.1) are determined as follows. The normalization with the pion decay amplitude $\pi^0 \to \gamma\gamma$ yields $h_7 = -N_C M_{V_1}^4 M_{V_2}^4/(4\pi^2 F_\pi^2) - h_6 m_\pi^2 - h_4 m_\pi^4 = -14.83~\text{GeV}^6 - h_6 m_\pi^2 - h_4 m_\pi^4$, where we used $M_{V_1} = M_\rho = 775.49$ MeV and $M_{V_2} = M_{\rho'} = 1.465$ GeV. The Brodsky-Lepage behavior can be reproduced by choosing $h_1 = 0~\text{GeV}^2$.

[3] After the publication of our paper Ref. [13], two new estimates for χ appeared, both based on the analysis of the zero-modes of the Dirac operator. Ref. [31] presents an analytical approach which yields $\chi(1~\text{GeV}) = -3.52~\text{GeV}^{-2}$ with an estimated error of $30-50\%$. A quenched lattice calculation [32] for $N_C = 2$ gives a very small absolute value $\chi = -1.547(6)~\text{GeV}^{-2}$. No scale dependence is given, the lattice spacing corresponds to 2 GeV.

In Ref. [22] a fit to the CLEO data [19] for the on-shell form factor $\mathscr{F}^{\rm LMD+V}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ was performed, with the result $h_5 = (6.93 \pm 0.26)$ GeV$^4 - h_3 m_\pi^2$. The constant h_2 can be obtained from higher-twist corrections in the OPE with the result $h_2 = -10.63$ GeV^2 [9].

Within the LMD+V framework, the vector-tensor two-point function reads [22, 13]

$$\Pi^{\rm LMD+V}_{\rm VT}(p^2) = -\langle\overline{\psi}\psi\rangle_0 \frac{p^2 + c_{\rm VT}}{(p^2 - M_{V_1}^2)(p^2 - M_{V_2}^2)}, \qquad c_{\rm VT} = \frac{M_{V_1}^2 M_{V_2}^2 \chi}{2}. \tag{4.3}$$

As shown in Ref. [22], the OPE constraint from Eq. (3.2) for $\mathscr{F}^{\rm LMD+V}_{\pi^{0*}\gamma^*\gamma^*}$ leads to the relation

$$h_1 + h_3 + h_4 = 2 c_{\rm VT}. \tag{4.4}$$

The LMD estimate $\chi^{\rm LMD} = -2/M_V^2 = -3.3$ GeV^{-2} is close to $\chi(\mu = 1 \text{ GeV}) = -(3.15 \pm 0.30)$ GeV^{-2} obtained in Ref. [30] using QCD sum rules with several vector resonances ρ, ρ', and ρ''. Assuming that the LMD/LMD+V framework is self-consistent, we will therefore take $\chi = (-3.3 \pm 1.1)$ GeV^{-2} in our numerical evaluation, with a typical large-N_C uncertainty of about 30%. We will vary h_3 in the range ± 10 GeV^2 and determine h_4 from Eq. (4.4) and vice versa.

The coefficient h_6 in the LMD+V Ansatz is undetermined as well. It enters at order p^6 in the low-energy expansion of $\langle VVP\rangle$ in one combination of low-energy constants from the chiral Lagrangian of odd intrinsic parity, $A^{\rm LMD+V}_{V,(p+q)^2} = -F_\pi^2 h_6/(8 M_{V_1}^4 M_{V_2}^4)$ [22]. The LMD ansatz with only one multiplet of vector resonances yields $A^{\rm LMD}_{V,(p+q)^2} = -F_\pi^2/(8M_V^4) = -0.26\ (10^{-4}/F_\pi^2)$ [22]. If the LMD/LMD+V framework is self-consistent, the change in these estimates, while going from LMD to LMD+V, should not be too big. Since the size of this low-energy constant seems to be small compared to another combination of low-energy constants which enters at order p^6, we allow for a 100% uncertainty of $A^{\rm LMD}_{V,(p+q)^2}$ and get the range $h_6 = (5 \pm 5)$ GeV^4, see Ref. [13] for details.

The results for $a_\mu^{{\rm LbyL};\pi^0}$ for some selected values of h_3, h_4 and h_6, varied in the ranges discussed above, for $\chi = -3.3$ GeV^{-2}, $h_1 = 0$ GeV^2, $h_2 = -10.63$ GeV^2 and $h_5 = 6.93$ GeV$^4 - h_3 m_\pi^2$ are collected in Table 1, see Refs. [13, 3] for details on the numerics.

	$h_6 = 0$ GeV^4	$h_6 = 5$ GeV^4	$h_6 = 10$ GeV^4
$h_3 = -10$ GeV^2	68.4	74.1	80.2
$h_3 = 0$ GeV^2	66.4	71.9	77.8
$h_3 = 10$ GeV^2	64.4	69.7	75.4
$h_4 = -10$ GeV^2	65.3	70.7	76.4
$h_4 = 0$ GeV^2	67.3	72.8	78.8
$h_4 = 10$ GeV^2	69.2	75.0	81.2

Table 1: Results for $a_\mu^{{\rm LbyL};\pi^0} \times 10^{11}$ obtained with the off-shell LMD+V form factor for $\chi = -3.3$ GeV^{-2} and the given values for h_3, h_4 and h_6. When varying h_3 (upper half of the table), the parameter h_4 is fixed by the constraint in Eq. (4.4). In the lower half the procedure is reversed.

Varying χ by ± 1.1 GeV^{-2} changes the result for $a_\mu^{{\rm LbyL};\pi^0}$ by $\pm 2.1 \times 10^{-11}$ at most. The uncertainty in h_6 affects the result by up to $\pm 6.4 \times 10^{-11}$. The variation of $a_\mu^{{\rm LbyL};\pi^0}$ with h_3 [with h_4 determined from the constraint in Eq. (4.4) or vice versa] is much smaller, at most $\pm 2.5 \times 10^{-11}$. In the absence of more information on the values of the constants h_3, h_4 and h_6, we take the average of the results obtained with $h_6 = 5$ GeV^4 for $h_3 = 0$ GeV^2 and for $h_4 = 0$ GeV^2 as our central value: $a_\mu^{\mathrm{LbyL};\pi^0} = 72.3 \times 10^{-11}$. Adding all uncertainties from the variations of χ, h_3 (or h_4), h_5 and h_6 linearly to cover the full range of values obtained with our scan of parameters, we get [13, 3]

$$a_\mu^{\mathrm{LbyL};\pi^0} = (72 \pm 12) \times 10^{-11}. \tag{4.5}$$

This value replaces the result obtained in Ref. [8] with on-shell LMD+V form factors at both vertices. We think the 16% error should fairly well describe the inherent model uncertainty using the *off-shell* LMD+V form factor. In order to facilitate updates of our result in case some of the parameters h_i in the LMD+V Ansatz in Eq. (4.1) will be known more precisely, we have given in the Appendix of Ref. [13] a parametrization of $a_\mu^{\mathrm{LbyL};\pi^0}$ for arbitrary coefficients h_i.[4]

As far as the contribution to a_μ from the exchanges of the other light pseudoscalars η and η' is concerned, it is not so straightforward to apply the above analysis within the LMD+V framework to these resonances. In particular, the short-distance analysis in Ref. [22] was performed in the chiral limit and assumed octet symmetry. We therefore resort to a simplified approach which was also adopted in other works [6, 7, 8, 9] and take a simple VMD form factor normalized to the experimental decay width $\Gamma(\mathrm{PS} \to \gamma\gamma)$. In this way we obtain the results $a_\mu^{\mathrm{LbyL};\eta} = 14.5 \times 10^{-11}$ and $a_\mu^{\mathrm{LbyL};\eta'} = 12.5 \times 10^{-11}$, which update the values given in Ref. [8]. Adding up the contributions from all the light pseudoscalar exchanges, we obtain the estimate [13, 3]

$$a_\mu^{\mathrm{LbyL;PS}} = (99 \pm 16) \times 10^{-11}, \tag{4.6}$$

where we have assumed a 16% error, as inferred above for the pion-exchange contribution.[5]

5. Discussion and conclusions

We would like to stress that although our result for the pion-exchange contribution is not too far from the value $a_\mu^{\mathrm{LbyL};\pi^0-\mathrm{pole}} = (76.5 \pm 6.7) \times 10^{-11}$ given in Ref. [9], this is *pure coincidence*. We have used off-shell LMD+V form factors at both vertices, whereas the authors of Ref. [9] evaluated the *pion-pole* contribution using the on-shell LMD+V form factor $\mathscr{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) \equiv \mathscr{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_2^2)$ at the internal vertex and a constant WZW form factor at the external vertex, see for instance Eq. (18) in Ref. [9]. Since only the pion-pole contribution is considered in Ref. [9], their short-distance constraint cannot be applied to our approach either. However, our ansatz for the pion-exchange contribution agrees qualitatively with the short-distance behavior of the quark-loop derived in Ref. [9], see the discussion in Refs. [13, 3].

[4] A fit of the on-shell LMD+V form factor to the recent BABAR data [21] yields $h_1 = (-0.17 \pm 0.02)$ GeV^2 and $h_5 = (6.51 \pm 0.20)$ GeV$^4 - h_3 m_\pi^2$ with $\chi^2/\mathrm{dof} = 15.0/15 = 1.0$. In this way we would get the new average value $a_\mu^{\mathrm{LbyL};\pi^0} = 71.8 \times 10^{-11}$, i.e. the result is essentially unchanged from Eq. (4.5).

[5] Applying the same procedure to the electron, we get $a_e^{\mathrm{LbyL};\pi^0} = (2.98 \pm 0.34) \times 10^{-14}$ [13]. This number supersedes the value given in Ref. [8]. Note that the naive rescaling $a_e^{\mathrm{LbyL};\pi^0}(\mathrm{rescaled}) = (m_e/m_\mu)^2\, a_\mu^{\mathrm{LbyL};\pi^0} = 1.7 \times 10^{-14}$ yields a value which is almost a factor of 2 too small. Our estimates for the other pseudoscalars contributions using VMD form factors at both vertices are $a_e^{\mathrm{LbyL};\eta} = 0.49 \times 10^{-14}$ and $a_e^{\mathrm{LbyL};\eta'} = 0.39 \times 10^{-14}$. Therefore we get $a_e^{\mathrm{LbyL;PS}} = (3.9 \pm 0.5) \times 10^{-14}$, where the relative error of about 12% is again taken over from the pion-exchange contribution. Assuming that the pseudoscalar contribution yields the bulk of the result of the total had. LbyL scattering correction, we obtain $a_e^{\mathrm{LbyL;had}} = (3.9 \pm 1.3) \times 10^{-14}$, with a conservative error of about 30%, see Ref. [3]. This value was later confirmed in the published version of Ref. [5] where a leading logs estimate yielded $a_e^{\mathrm{LbyL;had}} = (3.5 \pm 1.0) \times 10^{-14}$.

Our results for the pion and the sum of all pseudoscalars are about 20% larger than the values in Refs. [6, 7] which used other hadronic models. An evaluation of the pion-exchange contribution using an off-shell form factor based on a nonlocal chiral quark model yielded $a_\mu^{\mathrm{LbyL};\pi^0} = (65 \pm 2) \times 10^{-11}$ [34]. In that model, off-shell effects of the pion always lead to a strong damping in the form factor and the result is therefore smaller than the pion-pole contribution obtained in Ref. [9]. In our model, there are some corners of the parameter space where the result is larger than the pion-pole contribution, for instance, we get a maximal value of $a_\mu^{\mathrm{LbyL};\pi^0} = 83.3 \times 10^{-11}$ in the scanned region. Very recently, a value of $a_\mu^{\mathrm{LbyL;PS}} = 107 \times 10^{-11}$ with an estimated error of at most 30% was obtained in Ref. [35] within an AdS/QCD approach.

Combining our result for the pseudoscalars with the evaluation of the axial-vector contribution in Ref. [9] and the results from Ref. [6] for the other contributions, we obtain the estimate [13, 3]

$$a_\mu^{\mathrm{LbyL;had}} = (116 \pm 40) \times 10^{-11} \tag{5.1}$$

for the total had. LbyL scattering contribution to the anomalous magnetic moment of the muon. To be conservative, we have added all the errors linearly, as has become customary in recent years. In the very recent review [5] the central values of some of the individual contributions to had. LbyL scattering are adjusted and some errors are enlarged to cover the results obtained by various groups which used different models. The errors are finally added in quadrature to yield the estimate $a_\mu^{\mathrm{LbyL;had}} = (105 \pm 26) \times 10^{-11}$. Note that the dressed light quark loops are not included as a separate contribution in Ref. [5]. They are assumed to be already covered by using the short-distance constraint from Ref. [9] on the pseudoscalar-pole contribution. Certainly, more work on the had. LbyL scattering contribution is needed to fully control all the uncertainties.

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