PROCEEDINGS OF SCIENCE

Investigations on the Property of $f_0(600)$ and $f_0(980)$ Resonances in $\gamma\gamma \to \pi\pi$ Process

X. G. Wang, O. Zhang, L. C. Jin, H. Q. Zheng,*
Department of Physics, Peking University, Beijing 100871, P. R. China

Z. Y. Zhou
Department of Physics, Southeast University, Nanjing 211189, P. R. China

Using dispersion relation technique and experimental data, a coupled channel analysis on $\gamma\gamma \to \pi\pi$ process is made. Di-photon coupling of $f_0(600)$ and $f_0(980)$ resonances are extracted and their dynamical properties are discussed. Especially we study the physical meaning of the coupling constant $g^2_{\sigma\pi\pi}$, which maintains a negative real part as determined through dispersive analyses.

6th International Workshop on Chiral Dynamics, CD09
July 6-10, 2009
Bern, Switzerland

*Speaker.

1. A dispersive analysis on $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ processes

In recent few years there have been renewed interests on the study of the $\gamma\gamma \to \pi\pi$ process, partly due to the new experimental data provided by Belle Collaboration. [1] The investigation on such a process enables us to extract the di-photon coupling of resonances appearing in this reaction, which, as emphasized by Pennington, [2] affords a unique opportunity in exploring the underlying structure of these states. Along with previous work found in the literature, [3, 4, 5] we performed a dispersive analysis on $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ processes. [6] The major differences between Ref. [6] and much of previous work is that in the former we try to perform a coupled channel analysis in the strongly interacting I=0 s-wave – hence information from $\bar{K}K$ channel is also taken into account, at least in principle. We also fit Belle data up to 1.4GeV, which is certainly useful in fixing the d-waves. A better determination to the d-waves turns out to be very important in studying the low energy s-waves as well, where d-waves serve as a background contribution.

The dispersion representation of $\gamma\gamma \to \pi\pi, \bar{K}K$ amplitudes, $F(s)$, takes the following form: [7]

$$F(s) = F_B + D(s)[Ps - \frac{s^2}{\pi}\int_{4m_\pi^2} \frac{\mathrm{Im}D^{-1}(s')F_B(s')}{s'^2(s'-s-i\varepsilon)}ds'] \ , \tag{1.1}$$

where F_B denotes the Born term, P is a two dimensional (subtraction) constant array. The 2×2 matrix function $D(s)$ obeys the following equation:

$$D(s) \ = D(0) + \frac{s}{\pi}\int_{4m_\pi^2} \frac{D(s')\rho(s')T^*(s')}{s'(s'-s-i\varepsilon)}ds' \ , \tag{1.2}$$

where $\rho = \mathrm{diag}(\rho_1, \rho_2)$ and $\rho_1 = \sqrt{1-4m_\pi^2/s}$, $\rho_2 = \sqrt{1-4m_K^2/s}$, respectively; $T(s)$ denotes the 2×2 partial wave $\pi\pi, \bar{K}K$ scattering amplitudes. Numerical solution of Eq. (1.2) can be searched for. In the degenerate case of single channel problem, function D in Eq. (1.2) has a well-known analytic representation – the Omnés solution:

$$D(s) = \exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\delta(s')ds'}{(s'-s)s'}\right) \ . \tag{1.3}$$

The s-wave T matrix in Eq. (1.2) is obtained by fitting a coupled channel K matrix [8] to data. [9, 10] The relevant poles are listed in table 1. We notice from table 1 that the $f_0(980)$

pole	sheet–II	sheet–III
σ	$0.549-0.230i$	-
$f_0(980)$	$0.999-0.021i$	$0.977-0.060i$ i

Table 1: The pole locations on the $\sqrt{s}$–plane, in units of GeV.

resonance may consist of two poles – one locates on sheet II, while the other on sheet III, though the latter is found not quite stable in the numerical fit. Though the twin-pole phenomenon with respect to $f_0(980)$ was mentioned long time ago, [11] in $\gamma\gamma \to \pi\pi$ process one discovers further evidence in support of the idea that the $f_0(980)$ resonance could be a coupled channel Breit–Wigner resonance. [6] Similar phenomenon may occur in the situation of $X(3872)$ particle. [13]

PoS(CD09)084

The two I=0 d-wave and the I=2 s-wave amplitudes are attained through single channel approximation and the corresponding $\pi\pi$ scattering T matrices are borrowed from Refs. [12, 14]. With these T matrices the Omneś solution is used to determine the corresponding D functions. Other partial waves are tiny and have been approximated by their Born terms. Then the $\gamma\gamma\to\pi^+\pi^-,\pi^0\pi^0$ cross-sections can be fitted and the di-photon coupling of $f_0(600)$, $f_0(980)$, $f_2(1270)$ resonances can be extracted. We refer to Ref. [6] for the numerical results and related discussions.

By re-analyzing the whole process, the above estimates can be advanced, especially *at lower energies*. An improved I=0 s-wave single channel $\pi\pi$ scattering T matrix [14] provides a better analyticity property than that of a usual K matrix formalism, and gives a σ pole location in nice agreement with the Roy equation analysis. [15] The extracted di-photon width $\Gamma(\sigma\to 2\gamma)\simeq 2.1\text{keV}$ – a number significantly smaller than the value one expects for a naive $\bar{q}q$ meson. Therefore the result indicates the non-$\bar{q}q$ nature of the $f_0(600)$ resonance.

In the calculation as described by the last paragraph, as a byproduct when extracting the di-photon coupling one also gets the $\sigma\pi\pi$ coupling:

$$g^2_{\sigma\pi\pi}=(-0.20-0.13i)\text{GeV}^2\ . \tag{1.4}$$

It could be surprising to notice that the real part of the coupling strength, $\text{Re}[g^2_{\sigma\pi\pi}]$, is negative. A narrow resonance with such a property is not allowed, since it would be a ghost rather than a particle. [1] In the next section we devote to the discussion on physics behind this (once again) odd property of the $f_0(600)$ or σ meson.

2. What does a negative $\text{Re}[g^2_{\sigma\pi\pi}]$ tell us?

The negative value of $\text{Re}[g^2_{\sigma\pi\pi}]$ is related to the large width of $f_0(600)$ meson. To initiate the investigation let us recall the PKU dispersive representation for a partial wave elastic scattering S matrix element: [14, 16]

$$S^{phy.}=\prod_i S^{R_i}\cdot S^{cut}\ , \tag{2.1}$$

where S^{R_i} denotes the i-th resonances on sheet II and S^{cut} stands for the cut contribution. In each S^{R_i} pole residue is a function of the pole location, and hence if we neglect every pole and cut contribution other than the $f_0(600)$ pole, we can obtain its coupling strength to two pions, $g^2_{\sigma\pi\pi}=(-0.18-0.20i)\text{GeV}^2$, which is found not much different from the value given by Eq. (1.4). This implies that the $\sigma\pi\pi$ coupling is mainly of a kinematical effect, *i. e.*, largely affected by the σ pole location. In Fig. 1 we draw the region where the residue contains a negative real part based on the above approximation, *i. e.*, considering only single pole contribution. In the following, however, by studying the solvable $O(N)$ σ model, we will be able to learn more lessons on physics of negative coupling strength.

The bare IJ=00 channel $\pi\pi$ scattering amplitude takes the following form: [17]

$$T^{00}(s)=\frac{1}{32\pi}\frac{s-m_\pi^2}{f_\pi^2-(s-m_\pi^2)\left(\frac{1}{\lambda_0}+\widetilde{B}_0(s)\right)}$$

[1] The value, and especially the sign given in Eq. (1.4) is in qualitative agreement with that of Ref. [5] and especially Ref. [4]. Notice that in Ref. [4] there is a sign difference in the definition of coupling strength.

where

$$\widetilde{B}_0\left(p^2\right) = \frac{-i}{2}\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\frac{1}{q^2-m_\pi^2}\frac{1}{(p+q)^2-m_\pi^2}$$

is a divergent integral and can be made finite by redefining the renormalized coupling constant as, [18]

$$\frac{1}{\lambda(M)} = \frac{1}{\lambda_0} - \frac{i}{2}\int \frac{d^4 q}{(2\pi)^4}\frac{1}{(q^2+i\varepsilon)(q^2-M^2+i\varepsilon)}\,, \tag{2.2}$$

$$\frac{1}{\lambda(M)} + \tilde{B}(p^2;M) = \frac{1}{\lambda_0} + \tilde{B}_0(p^2)\,,$$

where

$$\tilde{B}(s;M) = \frac{1}{32\pi^2}\left[1+\rho(s)\log\frac{\rho(s)-1}{\rho(s)+1} - \log\frac{m_\pi^2}{M^2}\right]\,.$$

To define the theory one can set

$$\frac{1}{\lambda(M)} = 0\,,$$

where M denotes the scale when perturbation expansion fails, though above the scale M the theory can still be fine. The RGE of coupling constant λ becomes exact,

$$\mu^2\frac{d\lambda}{d\mu^2} = \frac{\lambda^2(\mu^2)}{32\pi^2}\,. \tag{2.3}$$

The true problem of such a theory (herewith called as $O(N)$ v1) is that a tachyon appears at m_t^2, and hence the theory only works when $|s| << |m_t^2|$. [18]

If one does not like the tachyon a sharp momentum cutoff at Λ can be used to make the theory finite. In this way one avoids the tachyon, but a spurious cut (at $4\Lambda^2$) and a spurious physical sheet pole near the spurious cut occur, instead. By this mean we define a cutoff version of the effective theory. Setting for example

$$\frac{1}{\lambda(\Lambda)} = 0\,,$$

defines another version of $O(N)$ model (called as $O(N)$ v2 hereafter).

The region where $\mathrm{Re}[g^2_{\sigma\pi\pi}] < 0$ is plotted in Fig. 1 both for $O(N)$ model v1 and v2, which are, however, almost identical. The σ pole trajectories with respect to varying the defining scale of two models are also plotted. Clearly, seen from Fig. 1, it is actually very difficult for $O(N)$ models to reach the 'realistic' σ pole location. In model v1, one has to decrease the scale M to face a situation that the tachyon pole mass and the σ pole mass are comparable in magnitude, and hence breaks down the validity of the effective theory. In model v2 similar things happen; in order to get the σ pole deep inside the region where $\mathrm{Re}[g^2_{\sigma\pi\pi}] < 0$, one has to decrease the cutoff parameter Λ facing the situation that the σ mass is comparable in magnitude with Λ, and thus also results in breaking the validity of the effective theory. The conclusion is that QCD interaction in the scalar sector becomes so strong that, the $O(N)$ toy model even fails to handel the situation when the σ pole gets as light and broad as it is determined from reality. A more 'realistic' calculation also leads to a similar conclusion. [19]

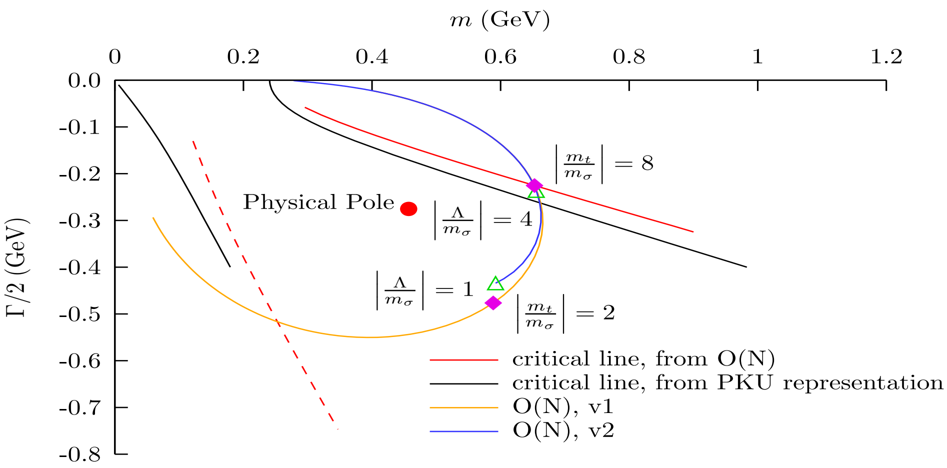

Figure 1: Region on the $\sqrt{s}$-plane with $\mathrm{Re}[g^2_{\sigma\pi\pi}] < 0$.

Another way to look at the non-perturbative nature of the σ meson is through examining the renormalization group equation, Eq. (2.3). To get the 'realistic' σ pole location, one finds $\lambda(\mu)$ blows up at $\mu \simeq 0.55$MeV.

It is certainly an extremely hard and non-perturbative task to predict a pole from an effective lagrangian inside which the pole does not have a corresponding field. Such kind of poles are sometimes called as 'dynamically generated' resonances. Once the existence of the σ pole was firmly established, it is wondered whether one should add the σ field explicitly into the low energy effective lagrangian. However the blow up of the the coupling constant λ at very low energy indicates that, even if the explicit σ degrees of freedom is added into the effective lagrangian, one still face a strongly non-perturbative problem.

To summarize, the σ pole manifests the maximal 'non-perturbativity' that QCD could offer.

This work is supported in part by National Nature Science Foundation of China under Contract Nos. 10875001, 10721063, 10647113 and 10705009.

References

[1] T. Mori *et al.* (Belle Collaboration), Phys. Rev. **D75**(2007)051101.

[2] M. R. Pennington, invited talk at YKIS Seminar on *New Frontiers in QCD: Exotic Hadrons and Hadronic Matter*, Kyoto, Japan, 20 Nov - 8 Dec 2006. Prog. Theor. Phys. Suppl. **168**(2007)143.

[3] Here we are only able to provide an incomplete list of references:
D. Morgan, M. R. Pennington, Z. Phys. **C48**(1990)623;
G. Mennessier, Z. Phys. **C16** (1983) 241;
A. V. Anisovich, V. V. Anisovich, Phys. Lett. **B467** (1999) 289;
L. V. Fil'kov, V. L. Kashevarov, Phys. Rev. **C72** (2005) 035211;
N. N. Achasov, G. N. Shestakov, arXive: 0712.0885 [hep-ph];
J. A. Oller, L. Roca, C. Schat, Phys. Lett. **B659** (2008) 201.

[4] J. Bernabeu, J. Prades, Phys. Rev. Lett. **100** (2008) 241804.

[5] G. Mennessier, S. Narison, W. Ochs, Phys. Lett. **B665** (2008) 205.

[6] Y. Mao et al., Phys. Rev. **D79** (2009) 116008.

[7] O. Babelon et al., Nucl. Phys. **B113**(1976)445; O. Babelon et al., Nucl. Phys. **B114**(1976)252.

[8] K. L. Au, D. Morgan and M. R. Pennington, Phys. Rev. **D35**(1987)1633.

[9] W. Ochs, Ph.D. thesis, Munich Univ., 1974;
D. H. Cohen *et al*., Phys. Rev. **D22**(1980)2595;
A. Etkin *et al*., Phys. Rev. **D25**(1982)1786;
A. D. Martin, E. N. Ozmutlu, Nucl. Phys. B158(1979)520;
G. Costa *et al*., Nucl. Phys. **B175**(1980)402;
V. A. Polychronakos *et al*., Phys. Rev. **D19**(1979)1317.

[10] S. Pislak *et al*., Phys. Rev. **D67**(2003)072004.

[11] Y. Fujii, M. Kato, Nuovo Cimento **13A**(1973)311;
D. Morgan, Nucl. Phys. **A543**(1992)632.

[12] J. J. Wang, Z. Y. Zhou, H. Q. Zheng, JHEP **0512**(2005)019.

[13] O. Zhang et al., arXiv: 0901.1553 [hep-ph], to appear in PLB.

[14] Z. Y. Zhou *et al*., JHEP 0502(2005)043.

[15] I. Caprini, G. Colangelo, H. Leutwyler, Phys. Rev. Lett. 96, 132001 (2006).

[16] Z. Y. Zhou, H. Q. Zheng, Nucl. Phys.**A775**(2006)212;
H. Q. Zheng et al., Nucl. Phys. **A733**(2004)235.

[17] S. Coleman, R. Jackiw and H. D. Politzer, Phys. Rev. **D10**(1974)2491.

[18] R. Sekhar Chivkula and M. Golden, Nucl. Phys. **B372**(1992)44 and references therein.

[19] M. X. Su, L. Y. Xiao, H. Q. Zheng, Nucl. Phys. **A792**(2007)288.