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Relations between SU(2)- and SU(3)-LECs in chiral perturbation theory

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Chiral perturbation theory in the two–flavour sector allows one to analyse Green functions in QCD in the limit where the strange quark mass is considered to be large in comparison to the external momenta and to the light quark masses m_u and m_d. In this framework, the low–energy constants of $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$ depend on the value of the heavy quark masses. For the coupling constants which occur at order p^2 and p^4 in the chiral expansion, we worked out in Ref. [1] the dependence on the strange quark mass at two–loop accuracy, and provided in Ref. [2] analogous relations for some of the couplings c_i which are relevant at order p^6. This talk comments on the methods used, and illustrates implications of the results obtained.

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1. Introduction

At low energies and small quark masses, the Green functions of quark currents can be analysed in the framework of chiral perturbation theory (χPT) [3, 4, 5]. The method allows one to work out the momentum and quark mass dependence of the quantities of interest in a systematic and coherent manner. It is customary to perform the quark mass expansion either around $m_u = m_d = 0$, with the strange quark mass held fixed at its physical value (χPT_2), or to consider an expansion in all three quark masses around $m_u = m_d = m_s = 0$ (χPT_3). The corresponding effective Lagrangians contain low–energy constants (LECs) that parametrise the degrees of freedom which are integrated out. The two expansions are not independent: in a particular limit specified below, χPT_3 reduces to χPT_2. As a result of this, one can express the LECs in the two–flavour case through the ones in χPT_3, in a perturbative manner. The relations amount to a series expansion in the strange quark mass. Generically,

$$k^{\rm r} = \sum_{m\geq -m_0} d_m z^m\,, \; z = \frac{m_s B_0}{(4\pi F_0)^2}\,. \tag{1.1}$$

Here, $k^{\rm r}$ stands for any of the renormalized χPT_2 LECs, while F_0, B_0 are the LECs at order p^2 in χPT_3. The coefficients d_m (whose dependence on the chosen $k^{\rm r}$ is suppressed in the notation) contain renormalized LECs from χPT_3, and powers of the logarithm $\ln\frac{m_s B_0}{\mu^2}$, where μ denotes the standard renormalization scale. For $k^{\rm r}$ at order $p^{2(N+1)}$, one has $m_0 = N$, and the corresponding leading term $d_{m_0} z^{-m_0}$ is generated by tree graphs in χPT_3. The next-to-leading order term requires a one–loop calculation, etc. In the following, we refer to the relations (1.1) as *matching relations*, obtained by *matching* χPT_2 to χPT_3 in the specific limit mentioned. The matching relations are useful, because they provide i) additional information on the LECs in χPT_3, and ii) internal consistency checks.

For the LECs at order p^2 and p^4, the matching was performed to one loop (to two loops) in Ref. [5] (Ref. [1]), and for a subclass of LECs at order p^6 to two loops in Ref. [2].

We comment on related work which is available in the literature.

i) The strange quark mass expansion of the χPT_2 LEC B (F^2B) was provided at two–loop accuracy in Ref. [6] ([7]).

ii) Matching of the order p^6 LECs in the parity–odd sector was performed recently in Ref. [8].

iii) Analogous work was done in the baryon sector in Refs. [9, 10], and for electromagnetic interactions in Refs. [11, 12, 13, 14].

The outline of the talk is as follows. In Section 2, we illustrate the matching for the pion vector form factor at order p^4. In Section 3, we give a short description of the method used to obtain the matching relations in general. In Section 4 we display the structure of the results at order p^2 and p^4, and discuss in some detail the matching relation for $l_2^{\rm r}$ to illustrate its use, whereas Section 5 concerns the matching at order p^6. The final Section 6 contains concluding remarks. We refer the interested reader to Refs. [1, 2] for more details, and for the full results of the matching relations.

2. The pion vector form factor at order p^4

We first illustrate how the relations between the LECs emerge, and consider for this purpose the vector form factor of the pion,

$$\langle \pi^+(p')\,|\,\tfrac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)|\pi^+(p)\rangle = (p+p')_\mu F_V(t)\;;\; t=(p'-p)^2\,, \tag{2.1}$$

in the chiral limit $m_u = m_d = 0$. In the three–flavour case, at one–loop order, the form factor reads in d space–time dimensions

$$F_{V,3}(t) = 1 + \frac{t}{F_0^2}\left[\Phi(t,0;d) + \tfrac{1}{2}\Phi(t,M_K;d)\right] + \frac{2L_9 t}{F_0^2}\,. \tag{2.2}$$

The loop function Φ is given by

$$\Phi(t,M;d) = \frac{\Gamma(2-\frac{d}{2})}{2(4\pi)^{d/2}}\int_0^1 \mathrm{d}u\, u^2\left[M^2 - \tfrac{t}{4}(1-u^2)\right]^{\frac{d-4}{2}}\,. \tag{2.3}$$

It is generated by mesons of mass M running in the loop [M_K denotes the kaon mass at $m_u = m_d = 0$, at order p^2]. Furthermore, F_0 stands for the pion decay constant at $m_u = m_d = m_s = 0$, and L_9 is one of the LECs in χPT$_3$ at order p^4.

In χPT$_2$, the corresponding one–loop expression is

$$F_{V,2}(t) = 1 + \frac{t}{F^2}\Phi(t,0;d) - \frac{l_6 t}{F^2}\,, \tag{2.4}$$

where F denotes the pion decay constant at $m_u = m_d = 0, m_s \neq 0$, and where l_6 is one of the low–energy constants in χPT$_2$ at order p^4. If one identifies F with F_0 at this order, the expressions $F_{V,3}$ and $F_{V,2}$ still differ in the coefficient of the term proportional to t, and in the contribution $\Phi(t,M_K;d)$, which is absent in the two–flavour case, because kaons are integrated out in that framework.

To proceed, we note that the loop function $\Phi(t,M;d)$ is holomorphic in the complex t–plane, cut along the real axis for Re $t \geq 4M^2$. Therefore,$\Phi(t,0;d)$ develops a branch point at $t=0$, whereas $\Phi(t,M_K;d)$ reduces to a polynomial at $t/M_K^2 \ll 1$,

$$\Phi(t,M_K;d) = \sum_{l=0}^{\infty}\Phi_l(M_K,d)\left(\frac{t}{M_K^2}\right)^l\,. \tag{2.5}$$

Let us discard for a moment the terms of order t and higher in this expansion. It is then seen that $F_{V,3}$ reduces to $F_{V,2}$, provided that we set

$$l_6 = -2L_9 - \frac{1}{2}\Phi_0(M_K,d)\,. \tag{2.6}$$

At $d=4$, this relation reduces to the one between the renormalised LECs $l_6^{\rm r}$ and $L_9^{\rm r}$ worked out in Ref. [5],

$$l_6^{\rm r}(\mu) = -2L_9^{\rm r}(\mu) + \frac{1}{192\pi^2}\left(\ln\frac{B_0 m_s}{\mu^2}+1\right)\,. \tag{2.7}$$

This expressions is indeed of the form displayed in Eq. (1.1), with $d_{-1}=0$, whereas d_0 is simply the right hand side of Eq. (2.7), generated by the one–loop graphs considered here. We conclude

that, at low energies, the expression of the vector form factor in $\chi\mathrm{PT}_3$ reduces to the one in the two–flavour case, up to polynomial terms of order t^2 and higher. An analogous statement holds true for all Green functions of quark currents built from up and down quarks alone, see below.

We now come back to the higher–order terms in Eq. (2.5). We start with the observation that the term of order t^l contributes at order t^{l+1} to $F_{V,2}$ – those with $l \geq 1$ are thus of the same chiral order in $F_{V,2}$ as the ones generated by graphs with $l+1$ loops in $\chi\mathrm{PT}_2$. Apparently, one runs into a problem with power counting here: the low–energy expansion of the one–loop contribution in $\chi\mathrm{PT}_3$ amounts to terms of arbitrarily high orders in the $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$ expansion of $F_{V,2}$. Indeed, this is a rule rather than an exception: Because the strange quark mass is counted as a quantity of chiral order zero in $\chi\mathrm{PT}_2$, the counting of a quantity like t/M_K^2 is different in the two theories. As a result of this, higher–order loops in $\chi\mathrm{PT}_3$ in general start to contribute already at leading order in $\chi\mathrm{PT}_2$. A systematic and coherent scheme is obtained by counting n–loop contributions – and, in particular the relevant LECs – to be of order $\bar{h}^n$, and the strange quark mass to be of order $\bar{h}^{-1}$, see Refs. [1, 2].

3. Matching of generating functionals in $\chi\mathrm{PT}_2$ and $\chi\mathrm{PT}_3$

We have developed in Ref. [1] a generic method for the matching, which is based on the path integral formulation of χPT. The idea of this method is not to compare matrix elements that can be obtained in both formulations, but rather to restrict the three–flavour theory such that it only describes the same physics as the two–flavour formulation. Then, one compares their generating functionals containing all the Green functions and reads off the matching of the LECs.

The LECs do not depend on the light quark masses m_u and m_d. Since both theories are expansions around vanishing quark masses, we may set $m_u = m_d = 0$ for the purpose of the matching.

The comparison of the generating functionals is in fact a comparison of all possible Green functions, which depend on the external fields. Obviously, they can only be compared with each other if they depend on the *same* external fields. Therefore, the external fields of $\chi\mathrm{PT}_3$ need to be restricted to those of $\chi\mathrm{PT}_2$. We also have to assure that the heavy mesons K or η running in the loops do not have the possibility to go on–shell. Therefore, we consider in addition the case where all external momenta are small compared to the kaon mass. The physics of $\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L$ then reduces to the one of $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$. We refer to this limit as the *two–flavour limit*.

The LECs are the coefficients of local chiral operators in the effective Lagrangian. Once one evaluates the generating functional with the effective Lagrangian, besides the local terms also many non–local contributions are generated, both in $\chi\mathrm{PT}_2$ as well as in $\chi\mathrm{PT}_3$. However, the non–local contributions, appearing in $\chi\mathrm{PT}_3$ as the result of low-energy expansion, will be exactly canceled by $\chi\mathrm{PT}_2$ counterparts once the matching is performed. Therefore, to obtain the matching relations, it is sufficient to restrict oneself to the local parts in the evaluation of the generating functional of $\chi\mathrm{PT}_3$.

4. LECs at order p^2 and p^4

All the relations may be put in the form of Eq. (1.1). To render the formulae more compact, we found it convenient to slightly reorder the expansions, such that they become a series in the quantity

$\bar{M}_K^2$, which stands for the one–loop expression of the (kaonmass)^2 in the limit $m_u = m_d = 0$, see e.g. [5]. The result is

$$
\begin{aligned}
Y &= Y_0 \left[1 + a_Y\, x + b_Y\, x^2 + \mathscr{O}(x^3)\right] , \qquad Y = F\,, \Sigma\,, \\
l_i^{\rm r} &= a_i + x\, b_i + \mathscr{O}(x^2)\,, \quad (i \neq 7)\,, \qquad l_7 = \frac{F_0^2}{8B_0 m_s} + a_7 + x\, b_7 + \mathscr{O}(x^2)\,, \\
x &= \frac{\bar{M}_K^2}{N F_0^2}\,, \qquad N = 16\pi^2\,, \qquad \Sigma = F^2 B\,, \qquad \Sigma_0 = F_0^2 B_0\,.
\end{aligned}
\tag{4.1}
$$

We denote the contributions proportional to a_i (b_i) as NLO (NNLO) terms, generated by one–loop (two–loop) graphs in χPT$_3$. Note that l_7 receives a contribution at leading order (LO) as well, proportional to m_s^{-1}, in agreement with the remarks made in the Introduction. The LO and NLO terms were determined in Ref. [5] more than 25 years ago, whereas the NNLO terms b_i were only recently worked out [1]. They have the following structure,

$$
b = p_0 + p_1\, \ell_K + p_2\, \ell_K^2\,, \tag{4.2}
$$

where $\ell_K = \ln(\bar{M}_K^2/\mu^2)$ is the chiral logarithm, and where we have dropped for simplicity the index i. The polynomials p_j are independent of the strange quark mass, and their scale dependence is such that in combination with the logarithms it adds up to the scale independent quantity b. In other words, the scale dependence of l_i is exclusively generated by the one–loop contribution a_i. The explicit results for the polynomials p_j are displayed in tables 2-4 of Ref.[1].

Let us now illustrate, in the case of the low-energy constant $l_2^{\rm r}$, the strange quark mass dependence and the information one can obtain from the pertinent matching relation. We found [1] at two–loop order the result

$$
\begin{aligned}
l_2^{\rm r} = &-\frac{1}{24N}\,(\ell_K + 1) + 4L_2^{\rm r} + x\left\{\frac{1}{N}\left[\frac{433}{288} - \frac{1}{24}\ln\frac{4}{3} + \frac{1}{16}\,\rho_1\right] - 16N\left(2C_{13}^{\rm r} - C_{11}^{\rm r}\right)\right. \\
&\left. + \left[\frac{13}{24N} - 8L_2^{\rm r} - 2L_3\right]\ell_K + \frac{3}{8N}\ell_K^2\right\}\,,
\end{aligned}
\tag{4.3}
$$

where

$$
\rho_1 = \sqrt{2}\,{\rm Cl}_2(\arccos(1/3)) \cong 1.41602\,, \qquad {\rm Cl}_2(\theta) = -\frac{1}{2}\int_0^\theta {\rm d}\phi\ \ln\left(4\sin^2\tfrac{\phi}{2}\right)\,. \tag{4.4}
$$

On the right–hand side, the p^4 (p^6) LECs $L_{2,3}$ ($C_{11,13}^{\rm r}$) occur, aside from known quantities. Our definition of the C_i differs from the one of Ref. [15] by a factor of F_0^2. Explicitly, the $\mathscr{L}_6$–Lagrangian reads

$$
\mathscr{L}_6^{\rm SU(2)} = F^{-2}\sum_{k=1}^{57} c_k P_k\,, \qquad \mathscr{L}_6^{\rm SU(3)} = F_0^{-2}\sum_{k=1}^{94} C_k O_k \tag{4.5}
$$

for two and three flavours, respectively. [Note that the 57 terms in $\mathscr{L}_6^{\rm SU(2)}$ are not independent [16]. We adhere to the original notation used in Ref. [15] for later convenience.]

We now note that $l_2^{\rm r}$ was determined in Ref. [17] from a dispersive analysis to rather high precision, and $L_{2,3}$ are also quite well known [18]. As a result of this, the relation (4.3) allows

one to constrain the value of the combination $2C^{\rm r}_{13} - C^{\rm r}_{11}$ [2]. We introduce the scale independent quantity

$$\bar{l}_2 = 48\pi^2 l^{\rm r}_2(\mu) - \ln\frac{M_\pi^2}{\mu^2}, \tag{4.6}$$

with M_π=139.57 MeV, and illustrate the strange quark mass dependence of $\bar{l}_2$ in Fig.1 (left panel),

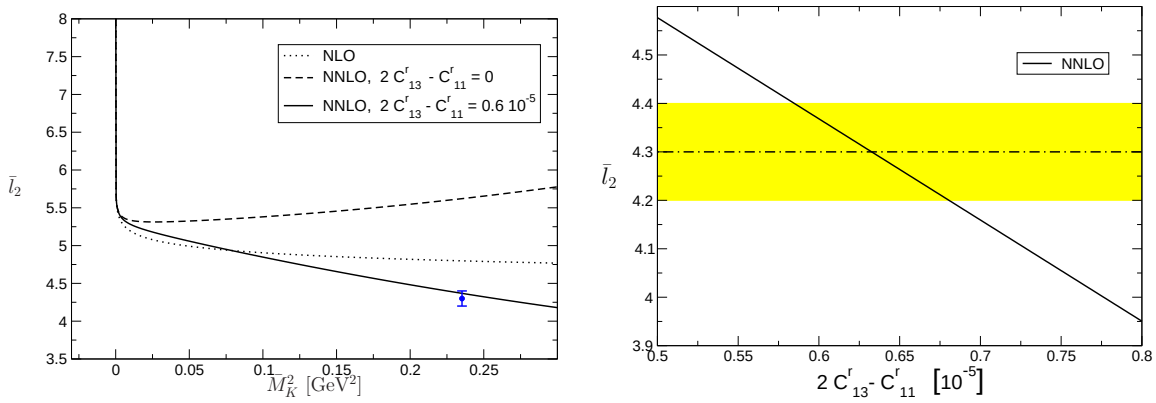

Figure 1: Left panel: Strange quark mass dependence of $\bar{l}_2$. As mentioned in the text, $\bar{M}_K$ denotes the kaon mass at one–loop accuracy in the limit $m_u = m_d = 0$. The physical value of m_s corresponds to $\bar{M}_K \approx 485$ MeV. We show the NLO (dotted line) as well as the NNLO result with two choices for $C^{\rm r}_{11,13}$: The dashed line corresponds to $2C^{\rm r}_{13} - C^{\rm r}_{11} = 0$, while the solid line is evaluated at $2C^{\rm r}_{13} - C^{\rm r}_{11} = 0.6 \times 10^{-5}$, which reproduces the prediction from the dispersive analysis (data point with small error bar). Right panel: Dependence of $\bar{l}_2$ on the p^6 LECs $2C^{\rm r}_{13} - C^{\rm r}_{11}$ at the physical value of m_s. The dashed–dotted line with the error band corresponds to the data point and its error bar in the left panel. The running scale is taken at $\mu = M_\rho = 770$ MeV, and $F_0 = F_\pi = 92.4$ MeV.

where $\bar{l}_2$ is shown as a function of $\bar{M}_K^2$, at $\mu = M_\rho = 770\,\text{MeV}, F_0 = F_\pi = 92.4\,\text{MeV}$. The dotted line stands for the NLO approximation, and the NNLO result is shown for two choices for $C^{\rm r}_{11,13}$: the dashed (solid) line displays the case $2C^{\rm r}_{13} - C^{\rm r}_{11} = 0$ ($2C^{\rm r}_{13} - C^{\rm r}_{11} = 0.6 \cdot 10^{-5}$). The solid line is constructed such that at the physical value of the strange quark mass, the LEC $\bar{l}_2$ agrees with the measured one [17] , within the uncertainties.

We shortly comment on the $C^{\rm r}_{11,13}$ that occur in this application. In Ref. [19], $C^{\rm r}_{13}$ is worked out from an analysis of scalar form factors. While the result is of the order $2C^{\rm r}_{13} - C^{\rm r}_{11} = 0.6 \cdot 10^{-5}$, its precise value depends considerably on the input used, see table 2 in Ref. [19] for more information. In Ref. [20, table 12 (published version)] estimates for both LECs $C^{\rm r}_{11,13}$ are provided: the authors find that these do not receive a contribution from resonance exchange at leading order in large N_C and therefore vanish at this order of accuracy. Because the scale at which this happens is not fixed a priori, that observation is not necessarily in contradiction with the above result.

The impact of these LECs on $\bar{l}_2$ is rather enhanced at physical strange quark masses. This is illustrated in Fig.1 (right panel). Taking the LECs $L^{\rm r}_j$ at face value, the window for a possible choice of the $C^{\rm r}_{11,13}$ is then very narrow to be in agreement with the data from a dispersive analysis. To pin down the $C^{\rm r}_{11,13}$ to good precision including an error analysis requires, however, a more thorough exploration. In particular, one has to take into account that in the fits performed in Ref. [18], an estimate of order p^6 counterterm contributions was already used.

5. LECs at order p^6

The evaluation of all matching relations at two–loop order for the LECs at order p^6 is very complicated. To ease the calculations, we did not deal with the full framework in Ref. [2], but rather switched off the sources s and p (while retaining m_s). This yields the following simplifications:

i) the solution of the classical EOM for the eta–field is trivial, $\eta = 0$;

ii) there is no mixing between the η and the π^0 fields.

Point i) greatly simplifies the transition from the χPT$_3$ building blocks of the monomials to those of two flavours, as it suppresses any effects from the eta, whereas point ii) eliminates many possible graphs and hence considerably reduces the requested labour. For example, in this restricted framework, the one–particle reducible graphs (two one–loop diagrams linked by a single propagator) do not contribute to the matching, see Ref. [2].

Aiming for the $\mathscr{L}_6$-monomials in the generating functional requires the evaluation of many graphs with sunset–like topology. In the two–flavour limit, where one is interested in the local contributions only, one can simplify the loop calculations by using a short distance expansion for the massive propagators. This simplifies drastically the involved loop integrals; however, the contributions from individual graphs are not chirally invariant. Collecting terms stemming from different graphs to obtain a manifestly chirally invariant result is rather cumbersome. Since we are interested in the local terms only, we use a shortcut which is based on gauge invariance[1]: one may choose a gauge such that at some fixed space–time point x_0, the totally symmetric combination of up to three derivatives acting on the chiral connection vanish,

$$\Gamma_\mu(x_0) = 0\,, \partial_{\{\mu}\Gamma_{\nu\}}(x_0) = 0\,, \partial_{\{\mu}\partial_\nu\Gamma_{\rho\}}(x_0) = 0\,, \partial_{\{\mu}\partial_\nu\partial_\rho\Gamma_{\sigma\}}(x_0) = 0\,. \tag{5.1}$$

Up to four ordinary derivatives are then indistinguishable from the fully symmetric combinations of covariant derivatives:

$$\partial_\mu f(x_0) = \nabla_\mu f(x_0)\,, \partial_\mu\partial_\nu f(x_0) = \tfrac{1}{2}\{\partial_\mu,\partial_\nu\} f(x_0) = \tfrac{1}{2}\{\nabla_\mu,\nabla_\nu\} f(x_0)\,, \text{etc.} \tag{5.2}$$

This allows us to write even intermediate results in a manifestly chiral invariant manner.

To check our calculations in one corner, we matched the available χPT$_2$– and χPT$_3$–results for the vector–vector correlator [21] and for the pion form factor, worked out in Refs. [22, 23]. In this manner, we found that our relations for $c^{\rm r}_{56}$ and $c^{\rm r}_{51} - c^{\rm r}_{53}$ agree with the results of Refs. [22, 23]. Needless to say that this is quite a non–trivial check.

As already stated in Ref. [16], the monomial P_{27} can be discarded from the p^6–Lagrangian for χPT$_2$. Therefore, the matching relations will certainly be a combination of some $c^{\rm r}_i$ and $c^{\rm r}_{27}$. Due to the restricted framework, only relations for LECs not involving monomials dependent on the sources s or p are nontrivial. In the restricted framework, there is an additional relation among the remaining SU(2)–monomials:

$$\begin{aligned}&\tfrac{4}{3}P_1 - \tfrac{1}{3}P_2 + P_3 - \tfrac{10}{3}P_{24} + \tfrac{4}{3}P_{25} + 2P_{26} - \tfrac{8}{3}P_{28} - \tfrac{1}{2}P_{29} + \tfrac{1}{2}P_{30} - P_{31} + 2P_{32} - \tfrac{1}{2}P_{33}\\ &+ \tfrac{4}{3}P_{36} - \tfrac{4}{3}P_{37} - \tfrac{11}{6}P_{39} + \tfrac{5}{6}P_{40} + \tfrac{7}{3}P_{41} - \tfrac{4}{3}P_{42} - \tfrac{3}{2}P_{43} + \tfrac{1}{2}P_{44} - \tfrac{1}{2}P_{45} - P_{51} - P_{53} = 0\,.\end{aligned} \tag{5.3}$$

[1] We are grateful to H. Leutwyler for pointing out this possibility to us.

i	x_i	i	x_i	i	x_i
1	$c_2^{\rm r}+\frac{1}{4}c_1^{\rm r}$	10	$c_{32}^{\rm r}-\frac{3}{2}c_1^{\rm r}-c_{27}^{\rm r}$	19	$c_{43}^{\rm r}+\frac{9}{8}c_1^{\rm r}+\frac{1}{4}c_{27}^{\rm r}$
2	$c_3^{\rm r}-\frac{3}{4}c_1^{\rm r}$	11	$c_{33}^{\rm r}+\frac{3}{8}c_1^{\rm r}+\frac{1}{4}c_{27}^{\rm r}$	20	$c_{44}^{\rm r}-\frac{3}{8}c_1^{\rm r}-\frac{1}{4}c_{27}^{\rm r}$
3	$c_{24}^{\rm r}+\frac{5}{2}c_1^{\rm r}$	12	$c_{36}^{\rm r}-c_1^{\rm r}$	21	$c_{45}^{\rm r}+\frac{3}{8}c_1^{\rm r}+\frac{1}{4}c_{27}^{\rm r}$
4	$c_{25}^{\rm r}-c_1^{\rm r}$	13	$c_{37}^{\rm r}+c_1^{\rm r}$	22	$c_{50}^{\rm r}$
5	$c_{26}^{\rm r}-\frac{3}{2}c_1^{\rm r}$	14	$c_{38}^{\rm r}$	23	$c_{51}^{\rm r}+\frac{3}{4}c_1^{\rm r}+\frac{1}{2}c_{27}^{\rm r}$
6	$c_{28}^{\rm r}+2c_1^{\rm r}-c_{27}^{\rm r}$	15	$c_{39}^{\rm r}+\frac{11}{8}c_1^{\rm r}+\frac{1}{4}c_{27}^{\rm r}$	24	$c_{52}^{\rm r}$
7	$c_{29}^{\rm r}+\frac{3}{8}c_1^{\rm r}+\frac{1}{4}c_{27}^{\rm r}$	16	$c_{40}^{\rm r}-\frac{5}{8}c_1^{\rm r}-\frac{1}{4}c_{27}^{\rm r}$	25	$c_{53}^{\rm r}+\frac{3}{4}c_1^{\rm r}+\frac{1}{2}c_{27}^{\rm r}$
8	$c_{30}^{\rm r}-\frac{3}{8}c_1^{\rm r}-\frac{1}{4}c_{27}^{\rm r}$	17	$c_{41}^{\rm r}-\frac{7}{4}c_1^{\rm r}-\frac{1}{2}c_{27}^{\rm r}$	26	$c_{55}^{\rm r}$
9	$c_{31}^{\rm r}+\frac{3}{4}c_1^{\rm r}+\frac{1}{2}c_{27}^{\rm r}$	18	$c_{42}^{\rm r}+c_1^{\rm r}$	27	$c_{56}^{\rm r}$

Table 1: The quantities x_i in Eq. (5.4)

Because the EOM is different in the full framework, this relation is no longer valid there. We used Eq. (5.3) to exclude the monomial P_1 from our consideration. As a result, we give the matching for the 27 combinations of $c_i^{\rm r}$ displayed in table 1. In the full framework, an additional matching relation (apart from the ones for the monomials involving the sources s and p) for $c_1^{\rm r}$ could be worked out, yielding the only missing piece in the matching for the 28 LECs worked out here.

The final result may be written in the form

$$x_i = p_i^{(0)} + p_i^{(1)}\ell_K + p_i^{(2)}\ell_K^2 + O(m_s)\,, \tag{5.4}$$

where x_i denotes one of the 27 linear combinations of the $c_i^{\rm r}$ displayed in table 1. The explicit expressions for the polynomials $p_i^{(n)}$ in the χPT$_3$–LECs are displayed in tables 2 and 3 of our article [2].

6. Summary

In this talk, we have discussed a general procedure [1, 2] to work out the matching relations between the LECs in χPT$_2$ and χPT$_3$ in a perturbative manner. For the LECs at order p^2 and p^4, and for a subset of those at order p^6, the relations are now available at two–loop order. The method could be used with only moderate adaption to work out more general matching relations, like the ones for χPT$_{N-1}$ to χPT$_N$. To obtain the matching relations of the the remaining LECs at order p^6 to the same accuracy would require, however, a very big amount of work.

We have in addition illustrated the use of the results in the case of $l_2^{\rm r}$: its precise knowledge, together with the known values of $L_2^{\rm r}, L_3$, in principle allows one to pin down the combination $2C_{13}^{\rm r}-C_{11}^{\rm r}$ rather precisely.

We refer the interested reader to our articles Refs. [1, 2] for the matching relations found, and for further details on the method used.

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