

Few-Body Systems and the Pionless Effective Field Theory

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The pionless effective field theory (EFT) is the appropriate low-energy EFT for short-range interactions that display a large scattering length. It has been successfully applied in atomic, nuclear and particle physics. We give an overview over recent calculations employing the pionless effective field theory and lay emphasis on applications in the three- and four-body sector where the most exciting developments have occurred.

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1. Introduction

As the lightest exchange particle of the internuclear interaction the pion plays a central role in the conventional description of the nucleon-nucleon (NN) interaction. However, at sufficiently low energies the pionic degrees are irrelevant to the description of the interaction and the NN interaction appears pointlike. In this regime the effective range parameters can be used to achieve an excellent description of two-nucleon scattering data. In the case of nucleons but also in many atomic systems, the effective range parameters display furthermore a separation of scales between the large two-body scattering length a and the range R of the interaction which provides an excellent starting point for the construction of an effective field theory (EFT) in which quantities are expanded in the small parameters a/R and kR (k denoting the momentum scale of the process under consideration). The resulting EFT which in nuclear physics is known as the pionless EFT, is a framework which facilitates the model-independent calculation of low-energy observables in systems with short-range interactions and a large scattering length. In the two-nucleon system it has been used successfully to calculate electroweak observables.

In the three-body sector this EFT has provided a new perspective on findings made in the 1970s. At this time Vitaly Efimov discovered that the zero-range limit of the 3-body problem for nonrelativistic particles with short-range interactions shows discrete scale invariance. If $a = \pm\infty$, there are infinitely many 3-body bound states with an accumulation point at the 3-atom scattering threshold. These *Efimov states* or *Efimov trimers* have a geometric spectrum [1]. Furthermore, he pointed out that these results were also valid for finite scattering length as long as $a \gg R$. The EFT analysis has showed that this discrete scale invariance is associated with a particular renormalization group behavior of the three-body problem (limit cycle). The phenomena associated with the implications of these results are generally known as Efimov physics [2] (see Ref. [3] for a summary of recent developments).

Another promising arena for the pionless EFT are halo nuclei which are nuclei consisting of a tightly bound halo and a small number of additional nucleons that are weakly bound to the core and form the *halo*. One characteristic of a halo nucleus is that the radius of the halo nucleus significantly larger than the radius of the core of the halo. This indicates a large scattering length and therefore a fine-tuned interaction between core and halo nucleons. The pionless EFT can offer in this context a new perspective on conventional cluster models which have been used frequently to describe such systems.

In the following section we will outline briefly the key ingredients, Lagrangian and power-counting, of the pionless EFT. In section 3, we will summarize the results when this EFT is applied to the three- and four-body system and in section 4 we report on current efforts to understand the impact of finite range corrections on predictions for few-body observables. In section 5 we will discuss briefly recent results obtained for halo systems and we will end with a short summary.

2. The Pionless EFT

The pionless EFT is the appropriate low-energy theory for reactions between particles interacting through short-range interactions of range R at momenta with $kR \ll 1$. The most general

Lagrangian describing such systems is given by

$$\mathcal{L} = \psi^\dagger \left[i\partial_t + \frac{\nabla^2}{2m} \right] \psi - \frac{C_0}{4} (\psi^\dagger \psi)^2 - \frac{D_0}{36} (\psi^\dagger \psi)^3 - \frac{E_0}{576} (\psi^\dagger \psi)^4 \dots, \quad (2.1)$$

where the ellipses represent operators of higher dimension which means terms with more derivatives and/or more fields. Here, we have neglected relativistic effects which are suppressed by factors of $(p/M)^2$. D_0 and E_0 denote the leading three- and four-body interactions.

Depending on the relative size of the effective range parameters, different powercountings have to be employed for the calculation of observables. For example if the scattering length is of natural size ($a \sim R$) the powercounting is completely perturbative and only a finite number a number of diagrams has to be evaluated at every order in the EFT expansion. However, if the scattering length is large compared to the range of the interaction ($a \gg R$) the problem becomes nonperturbative and all connected diagrams that include only the C_0 vertex have to be summed up at leading order in the EFT expansion. Here, we will focus on the latter case which is of more interest in the few-body sector. An overview over calculations in the two-body sector can be found in Ref. [4].

3. Few-Body Systems

3.1 The Three-Body System

It was shown by Bedaque, Hammer and van Kolck how the pionless EFT is applied to a three-body system of identical bosons [5, 6]. Using an auxiliary field T , they rewrote the Lagrangian given in Eq. (2.1)

$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi + \Delta T^\dagger T - \frac{g}{\sqrt{2}} (T^\dagger \psi \psi + \text{h.c.}) + h T^\dagger T \psi^\dagger \psi \dots \quad (3.1)$$

The Lagrange density above is equivalent to the density in Eq. (2.1) if the low-energy constants are chosen to be $2g^2/\Delta = C_0$ and $-18hg^2/\Delta^2 = D_0$ (and the four-body force terms has been omitted). The advantage of using this formulation is that it turns the three-body problem in an effective two-body problem. Feynman rules derived from Eq. (3.1) can be used to derive an integral equation for particle-dimer scattering. After S -wave projection and multiplication with wave function renormalization factors, the fully-off-shell equation takes the form

$$t(p, k; E) = \frac{8\pi}{ma} \left[\frac{1}{pk} \ln \left(\frac{p^2 + pk + k^2 - mE}{p^2 - pk + k^2 - mE} \right) + \frac{2H(\Lambda)}{\Lambda^2} \right] + \frac{2}{\pi} \int_0^\Lambda dq q^2 \frac{t(q, k; E)}{-1/a + \sqrt{3}q^2/4 - mE - i\epsilon} \left[\frac{1}{pq} \ln \left(\frac{p^2 + pq + q^2 - mE}{p^2 - pq + q^2 - mE} \right) + \frac{2H(\Lambda)}{\Lambda^2} \right]. \quad (3.2)$$

Here a cutoff Λ has been introduced to make the integral equation well-defined and $h = 2mg^2H(\Lambda)/\Lambda^2$. Equation (3.2) is then related to the atom-dimer phase shift via

$$t_0(k, k) = \frac{3\pi}{m} \frac{1}{k \cot \delta_{AD} - ik}. \quad (3.3)$$

Equation (3.2) (without the three-body force) is also known as the Skorniakov-Ter-Martirosian (STM) equation, named after the first ones to derive an integral equation for the three-body problem

with zero-range two-body interactions [7]. The three-body force has to be included however, since without it observables display strong cutoff dependence. The resulting running of the three-body force with the cutoff Λ shows limit cycle behavior [5, 6].

3.2 The Four-Body System

Since the three-body system requires an additional three-body datum for renormalization it is natural to ask whether the same will happen in the four-body system and a new four-body observable will be required for consistent renormalization. This question was answered in Ref. [8], where an analysis of four-body observables showed that one two-body and one three-body input are sufficient to obtain model-independent predictions for four-body observables. This implies that for fixed scattering length the binding energy of a four-body bound state will only depend on the value of the three-body observable used for the renormalization of the three-body system. The pionless EFT explains therefore the well-known correlation between triton and α -particle binding energy also known as the Tjon line.

This approach was used furthermore for a more detailed analysis of the four-boson system with large positive and large negative scattering length [9]. Results in this analysis also lead to the conclusion that every trimer state is tied to two universal tetramer states with binding energies related to the binding energy of the next shallower trimer:

$$E_{4,0} \sim 5E_T \quad \text{and} \quad E_{4,1} \sim 1.01E_T \quad \text{for} \quad \gamma \sim 0, \quad (3.4)$$

where $E_{4,0}$ denotes the binding energy of the deeper of the two tetramer states and $E_{4,1}$ the shallower of the two.

A recent calculation by von Stecher, d’Incao and Greene [10] supports the findings made in [8, 9]. The authors of this work extended previous results to higher numerical accuracy. They furthermore considered the relation between universal three- and four-body bound states in the exact unitary limit ($a \rightarrow \infty$). They found

$$E_{4,0} \approx 4.57E_T \quad \text{and} \quad E_{4,1} \approx 1.01E_T, \quad (3.5)$$

which agree with the results obtained in Ref. [9] and given in Eq. (3.4).

The results obtained by Hammer and Platter in Ref. [9] were furthermore presented in the form of an extended Efimov plot, shown in Fig. 1. Four-body states have to have a binding energy larger than the one of the deepest trimer state. The corresponding threshold is denoted by lower solid line in Fig. 1. The threshold for decay into the shallowest trimer state and an atom is indicated by the upper solid line. At positive scattering length, there are also scattering thresholds for scattering of two dimers and scattering of a dimer and two particles indicated by the dash-dotted and dashed lines, respectively. The vertical dotted line denotes infinite scattering length. A similar but extended version of this four-body Efimov plot was also presented by Stecher, d’Incao and Greene in Ref. [10]. They computed also the scattering lengths at which the binding energies of the tetramer states become zero and found

$$a_{4,0}^* \approx 0.43a_* \quad \text{and} \quad a_{4,1}^* \approx 0.92a_*. \quad (3.6)$$

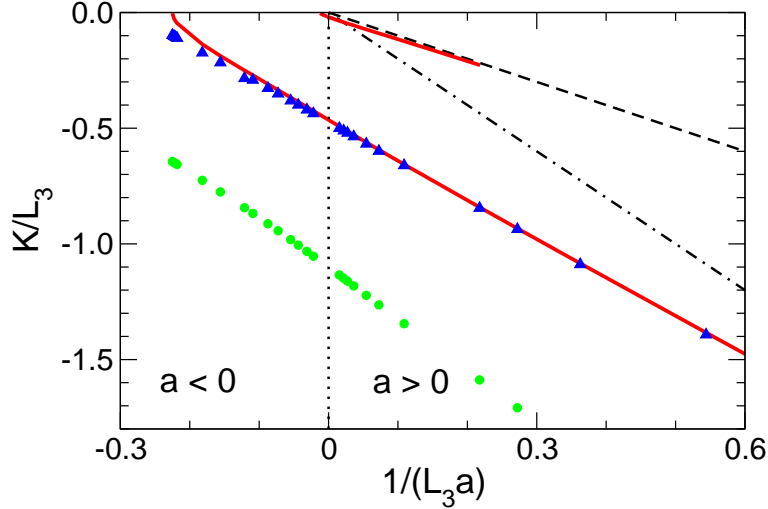


Figure 1: The a^{-1} - K plane for the four-body problem. The circles and triangles indicate the four-body ground and excited state energies $B_4^{(0)}$ and $B_4^{(1)}$, while the lower (upper) solid lines give the thresholds for decay into a ground state (excited state) trimer and a particle. The dash-dotted (dashed) lines give the thresholds for decay into two dimers (a dimer and two particles). The vertical dotted line indicates infinite scattering length. All quantities are given in units of the three-body parameter L_3 .

The authors concluded that at these values of the two-body scattering length the existence of the universal tetramer states should become visible as loss features due to recombination processes in systems of ultracold atoms.

Ferlaino *et al.* recently studied the four-body problem with short-range interactions experimentally [12]. Using ultracold ^{133}Cs atoms in the lowest hyperfine state at a temperature of 50 nK, they found loss features at scattering lengths $-730a_0$ and $-410a_0$ which were interpreted as the four-body loss features predicted by Stecher, d’Incao and Greene [10]. With the triatomic Efimov resonance measured at $-870a_0$, this gives for the ratios of the four- and three-body resonance position

$$a_{4,0}^*/a_* \approx 0.47 \quad \text{and} \quad a_{4,1}^*/a_* \approx 0.84. \quad (3.7)$$

These experimental results are in fact surprisingly close to the zero-range prediction made in [10] since finite range effects are expected to be important at these values of the scattering length. The range of the Cs-Cs interaction (which is set by the van-der Waals length scale) is approximately $200a_0$.

4. Higher Order Corrections

The promise of EFTs is that observables can be calculated to high accuracy. To deliver that promise is has first to be understand which operators have to be taken into account at what order. While the required two-body operators follow trivially from the effective range expansion, it is not a priory clear at what order the next three-body force enters. Hammer and Mehen calculated the phaseshift of neutron-deuteron scattering up to next-to-leading order (NLO) perturbatively and demonstrated that no additional three-body parameter is required [13] (as long as the scattering

remains fixed as we will discuss below). An analysis of the necessity of three-body forces in higher partial waves was carried out by Grißhammer [14]. Different conclusion have been reached for next-to-next-to-leading (N2LO) order. In Ref. [15] it was found that a new energy-dependent three-body force is required. A renormalization group analysis of the large cutoff behavior of the three-body amplitude lead the authors of Ref. [16] to the conclusion that an energy-dependent counterterm would be required at N3LO. In both references, the kernel of the three-body integration was modified to account for the effects of the effective range. The reason for the disagreement between both results might therefore simply be the fact that the cutoff dependence of the three-body amplitude is different for large cutoffs than for natural cutoffs ($\Lambda \sim 1/R$).

In a recent work [17] the next-to-leading order correction was reconsidered using a perturbative approach. The fact that the three-body bound state wave function is known exactly for infinite scattering length was used to calculate the NLO shift exactly in this limit. It was shown that the bound state spectrum receives in this case no correction and that the discrete scale invariance of the three-body wave function has therefore a direct effect on the size of higher order corrections. A future publication will discuss the need of an additional energy-independent three-body counterterm appearing at NLO that is proportional to the inverse scattering length [18]. This counterterm will only be relevant for the NLO analysis of problems where the scattering length is variable such as in experiments that measure the three-body recombination rate around a Feshbach resonance.

The impact of range corrections in the four-body sector was discussed by Kirschner *et al.* in Ref. [19].

5. Halo Nuclei

Halo nuclei are another possible application of the EFT for short-range interactions. The weak binding of the halo nucleons to the core nucleus indicates a separation of scales which might be understood in terms of a large core-nucleon scattering length. The first application of the short-range EFT to halo nuclei was carried out in Refs. [20, 21]. In these works the authors considered the one-neutron halo ${}^5\text{He}$ and calculated phaseshifts and cross sections for elastic α -nucleon scattering. A further example of a nuclear two-body cluster that has been considered is the $2\text{-}\alpha$ system [22].

Recently, Canham and Hammer [23] performed the first EFT calculation for two-neutron halos, i.e. the three-body case. In their work they calculated the binding energies and radii of halos such ${}^{11}\text{Li}$ and ${}^{20}\text{C}$. Canham and Hammer also addressed the question whether any of the considered systems supports an excited Efimov state. Fig. 2 shows a parametric plot $((E_{nc}/B_3^{(n)})^{1/2}$ versus $(E_{nn}/B_3^{(n)})^{1/2}$) which describes the region in the two-body parameter space that supports a three-body state above $B_3^{(n)}$. They found that the ${}^{20}\text{C}$ system might exhibit an excited Efimov state close to the threshold.

6. Summary

We have discussed recent applications of the pionless EFT to few-body systems. Predictions for few-body observables can be made with this EFT provided one three-body datum is know. However, even in the absence of such an input, the pionless EFT is capable of explaining correlations between few-body observables (e.g. the Tjon line). Its success demonstrates that it is the ideal

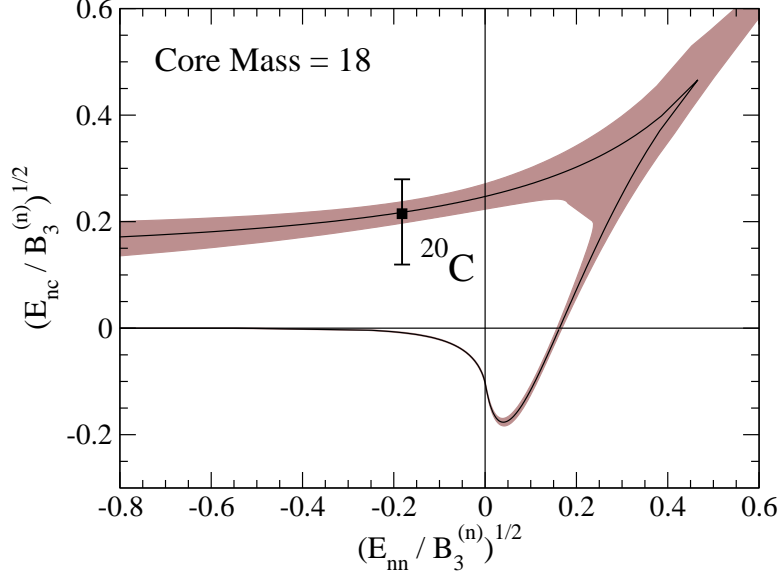


Figure 2: Boundary curve in the $\sqrt{E_{nc}/B_3^{(n)}}$ vs. $\sqrt{E_{mn}/B_3^{(n)}}$ plane with leading order error bands. Boundary curve shown for a core mass of $A = 18$ with the experimental data for ^{20}C from Ref. [24]. Figure taken from Ref. [23]

tool to analyze the universal properties of systems with a large scattering whether in the atomic or nuclear context.

It is furthermore a small parameter expansion that promises high accuracy for electroweak observables of wide interest. Form factors [25, 26], thermal neutron capture on the deuteron [27, 28] and triton photo-dissociation [29] have already been considered. However, other observables such as triton β -decay or electroweak reactions in the four-body sector remain to be calculated.

Halo nuclei are a relatively new application of the pionless EFT. Here it can provide answers to questions such as whether Halo nuclei are examples of Efimov physics and whether these states might display additional excited states belonging to an Efimov spectrum. Since the EFT for short-range interactions also significantly simplifies the complexity of this problem (for a example in the case of ^{20}C a 20-particle problem is reduced to a 3-body problem) one can hope that the EFT treatment of halo systems will also facilitate a calculation of scattering observables.

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References

- [1] V. Efimov, Phys. Lett. **33B**, 563 (1970).
- [2] E. Braaten and H. W. Hammer, Phys. Rept. **428**, 259 (2006).

- [3] L. Platter, *Few Body Syst.* **46**, 139 (2009).
- [4] P. F. Bedaque and U. van Kolck, *Ann. Rev. Nucl. Part. Sci.* **52**, 339 (2002).
- [5] P. F. Bedaque, H. W. Hammer and U. van Kolck, *Phys. Rev. Lett.* **82**, 463 (1999).
- [6] P. F. Bedaque, H. W. Hammer and U. van Kolck, *Nucl. Phys. A* **646**, 444 (1999).
- [7] G.V. Skorniakov and K.A. Ter-Martirosian, *Sov. Phys. JETP* **4**, 648 (1957).
- [8] L. Platter, H.-W. Hammer and U.-G. Meißner, *Phys. Rev. A* **70**, 052101 (2004).
- [9] H.-W. Hammer and L. Platter, *Eur. Phys. J. A* **32**, 113 (2007).
- [10] J. von Stecher, J. P. D’Incao, and C. H. Greene, *Nature Physics* **5**, 417 (2009).
- [11] J. von Stecher, J. P. D’Incao, and C. H. Greene, *Phys. Rev. Lett.* **103**, 033004 (2009).
- [12] F. Ferlaino, S. Knoop, M. Berninger, W. Harm, J. P. D’Incao, H.-C. Nägerl, R. Grimm, *Phys. Rev. Lett.* **102**, 140401 (2009).
- [13] H.-W. Hammer and T. Mehen, *Phys. Lett. B* **516**, 353 (2001).
- [14] H. W. Griebhammer, *Nucl. Phys. A* **760**, 110 (2005).
- [15] P. F. Bedaque, G. Rupak, H. W. Griesshammer and H.-W. Hammer, *Nucl. Phys. A* **714**, 589 (2003).
- [16] L. Platter and D. R. Phillips, *Few Body Syst.* **40**, 35 (2006).
- [17] L. Platter, C. Ji and D. R. Phillips, *Phys. Rev. A* **79**, 022702 (2009).
- [18] C. Ji, D. R. Phillips and L. Platter, *in preparation*.
- [19] J. Kirscher, H. W. Griesshammer, D. Shukla and H. M. Hofmann, arXiv:0903.5538 [nucl-th].
- [20] C. A. Bertulani, H.-W. Hammer and U. Van Kolck, *Nucl. Phys. A* **712**, 37 (2002).
- [21] P. F. Bedaque, H.-W. Hammer and U. van Kolck, *Phys. Lett. B* **569**, 159 (2003).
- [22] R. Higa, H.-W. Hammer and U. van Kolck, *Nucl. Phys. A* **809**, 171 (2008).
- [23] D. L. Canham and H.-W. Hammer, *Eur. Phys. J. A* **37**, 367 (2008).
- [24] TUNL nuclear data evaluation project, <http://www.tunl.duke.edu/NuclData/>.
- [25] L. Platter and H.-W. Hammer, *Nucl. Phys. A* **766**, 132 (2006) [arXiv:nucl-th/0509045].
- [26] H. Sadeghi, arXiv:0908.2052 [nucl-th].
- [27] H. Sadeghi and S. Bayegan, *Nucl. Phys. A* **753**, 291 (2005).
- [28] H. Sadeghi, S. Bayegan and H. W. Griebhammer, *Phys. Lett. B* **643**, 263 (2006).
- [29] H. Sadeghi and S. Bayegan, arXiv:0908.2220 [nucl-th].