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Consistency checks between OPE condensates and low-energy couplings

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One of the most controversial issues in the phenomenological analyses of tau decay data is the determination of the OPE condensates in the $V-A$ two-point correlator. The fact that the discrepancies only arise when predicting high energies (there is excellent agreement at low energies), makes it a very subtle issue that has not been resolved so far. Here I address the problem from an entirely different perspective by using a minimal meromorphic ansatz for the correlator and fixing its parameters entirely from known properties of QCD. This way one can establish relations between OPE condensates (high energies) and ChPT parameters (low energies) which can then be compared with existing phenomenological determinations. Following this approach one is led to the relation $\xi_8 = -8\, f_\pi^{-4} L_{10}\, \xi_6^2$, which not only predicts $\xi_8 > 0$ but also turns out to be satisfied with excellent accuracy by all phenomenological determinations consistent with $\xi_8 > 0$. The same strategy is used to make a prediction for the magnetic susceptibility of the quark condensate, leading to $\chi_0 = (7.5 \pm 2.8)$ GeV^{-2}.

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1. Introduction

The use of meromorphic ansätze for correlators in the determination of non-perturbative observables is now a mature field which was initially motivated by the large-N_c limit of QCD. One of the key ingredients in this approach is to ensure, through matching, that the correlator under study complies with high and/or low energy properties of QCD. Since this information is typically scarce, in practice one fixes the poles of the ansatz *ab initio* at the values of physical resonance masses and determines the residues by matching. It turns out that this approach can be justified from a mathematical point of view within the theory of Padé-type approximants to meromorphic functions.

Obviously, once the free parameters from the ansatz are determined, one can use the ansatz as an extrapolator to make predictions for the correlator in a local way. For instance, one can predict relations between low and high-energy parameters which can then be used to test the existing phenomenological determinations, especially in instances where there are conflicting results.

Here I will concentrate on two such instances, namely (i) the determination of the dimension-six and dimension-eight OPE condensates in Π_{LR}, which are related to the electroweak penguin contribution to $K \to \pi\pi$; and (ii) the magnetic susceptibility of the quark condensate, which recently has been shown to have an impact in the determination of the muon $(g-2)$.

However, I will depart from Padé-type approximants and instead will leave the poles to be determined fully by the matching equations. This *maximal matching* strategy on a meromorphic ansatz is known in the mathematical literature as a regular Padé approximant. Padé-type approximants are extremely useful for *global* applications (for instance, when dealing with moments of correlators). However, for the *local* low to high-energy extrapolation I am interested in here, Padé approximants fare better.

Throughout the article I will follow closely Ref. [1], where further details can be found.

2. OPE condensates in Π_{LR}

We start with the correlator

$$\Pi_{LR}^{\mu\nu}(q) = i\int \mathrm{d}^4x\, e^{iq\cdot x}\langle 0\,|\,T\{\,L_\mu(x)R_\nu^\dagger(0)\,\}|\,0\,\rangle = (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi_{LR}(q^2)\ , \tag{2.1}$$

whose spectrum has been measured with remarkable precision from hadronic tau decays [2]. From a phenomenological point of view, its high and low energies can be parameterised (in the chiral limit) as

$$\begin{aligned} \lim_{q^2\to(-\infty)} \Pi_{LR}(q^2) &= \frac{\xi_6}{q^6} + \frac{\xi_8}{q^8} + \cdots\ , \\ \lim_{q^2\to 0} \Pi_{LR}(q^2) &= \frac{f_\pi^2}{q^2} - 8\,L_{10} + 16\,C_{87}\,q^2 + \cdots \end{aligned} \tag{2.2}$$

The meromorphic ansatz for the correlator with minimal hadronic content is

$$\Pi_{LR}(q^2) = \frac{f_\pi^2}{q^2} + \frac{f_V^2}{-q^2+m_V^2} - \frac{f_A^2}{-q^2+m_A^2}\ . \tag{2.3}$$

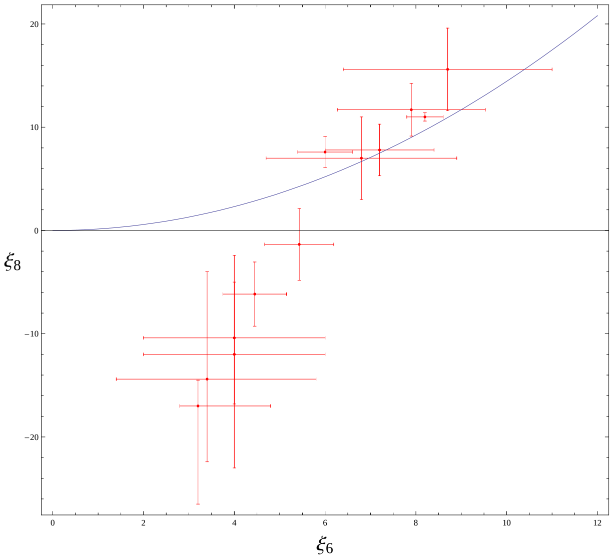

Figure 1: Values of the OPE condensates for the different phenomenological analyses [3] and their comparison with Eq. (2.5).

Its poles and residues can be determined by solving the following system of equations:

$$\begin{aligned} f_A^2 - f_V^2 &= -f_\pi^2 \,, \\ f_A^2 m_A^2 - f_V^2 m_V^2 &= 0 \,, \\ f_A^2 m_A^4 - f_V^2 m_V^4 &= \xi_6 \,, \\ f_A^2 m_A^6 - f_V^2 m_V^6 &= \xi_8 \,. \end{aligned} \tag{2.4}$$

The solution for masses and decay constants in terms of f_π, ξ_6 and ξ_8 is quite complicated, but it leads to a remarkably simple expression for the low energy parameter L_{10}:

$$L_{10} = \frac{1}{8}\left[\frac{f_A^2}{m_A^2} - \frac{f_V^2}{m_V^2}\right] = -\frac{1}{8}\frac{\xi_8}{\xi_6^2} f_\pi^4 \,. \tag{2.5}$$

Since $L_{10} < 0$, the previous expression neatly predicts ξ_8 to be positive. The most remarkable thing however is that the previous relation is satisfied by the whole set of phenomenological determinations consistent with $\xi_8 > 0$ (see figure 1).

3. The magnetic susceptibility of the quark condensate

Consider the following two-point correlator:

$$\Pi^{VT}_{\mu;\nu\rho}(q) = i\int d^4x\, e^{iq\cdot x}\langle 0|\, T\{V_\mu(x)\, T^\dagger_{\nu\rho}(0)\}|0\rangle = i\,(q^\rho g^{\mu\nu} - q^\nu g^{\mu\rho})\,\Pi_{VT}(q^2) \,. \tag{3.1}$$

As in the previous section, we can work out its high and low energy expansion, yielding

$$\begin{aligned} \lim_{q^2\to(-\infty)} \Pi_{VT}(q^2) &= 2\frac{\langle\bar\psi\psi\rangle}{q^2} - \frac{2g_s}{3}\frac{\langle\bar\psi\hat{G}\psi\rangle}{q^4} + \cdots \,, \\ \lim_{q^2\to 0} \Pi_{VT}(q^2) &= -\chi_0\langle\bar\psi\psi\rangle + \cdots \end{aligned} \tag{3.2}$$

The minimal meromorphic ansatz for the correlator in this case reads

$$\Pi_{VT}(q^2) = \frac{\hat{f}_V^{\perp}\,\hat{f}_V\,\hat{m}_V}{-q^2+\hat{m}_V^2}\,. \tag{3.3}$$

As before, its free parameters can be determined by expanding the ansatz at large values of $(-q^2)$ and matching to the OPE. The resulting system of equations is easily solvable, and it predicts the following value for the magnetic susceptibility:

$$\chi_0 = -\frac{\hat{\xi}_V\,\hat{f}_V^2}{\langle\bar{\psi}\psi\rangle\hat{m}_V} = \frac{6}{m_0^2}\,, \qquad m_0^2 = \frac{g_s\langle\bar{\psi}\hat{G}\psi\rangle}{\langle\bar{\psi}\psi\rangle} \tag{3.4}$$

For typical values of m_0^2 one finds $\chi_0 = (7.5\pm 2.8)$ GeV^{-2}, which is in excellent agreement with [4].

4. Concluding remarks

New relations can be found between the low energy and high energy regime of correlators if one: (a) starts from a minimal meromorphic ansatz, and (b) constrains its free parameters (masses and decay constants) in a maximal way. This is especially useful if one of the two energy regimes is known with enough accuracy. This is the case for the Π_{LR} and Π_{VT} correlators. In the former, the low energies are known to high accuracy through the parameter L_{10} and a prediction is available for the OPE condensates. In contrast, for Π_{VT} the known input is the short distance parameter m_0^2 (the ratio between the mixed and quark condensates), from which a prediction for the magnetic susceptibility of the quark condensate χ_0 is possible. Interestingly, our results are compatible in a non-trivial way with some existing phenomenological determinations, namely those which predict $\xi_8 > 0$ for Π_{LR} and those which find χ_0 in the window $7-9$ GeV^{-2} rather than $2-3$ GeV^{-2}.

The method used here can be embedded in the framework of the theory of Padé approximants to meromorphic functions. This is especially interesting because the ansatz, as one includes more and more terms, is ensured to converge to the physical correlator (except probably on a set of null measure that includes the physical axis). Obviously, the relevant issue for physical applications is to know the onset of the asymptotic regime, a question that most probably depends on the physical observable.

However, a strong indication that the asymptotic regime might already have been achieved in Π_{LR} is precisely the accuracy with which the results of existing phenomenological analyses are accomodated by Eq. (2.5). Without being a proof, it is certainly suggestive evidence.

References

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