Renormalization group equations
in resonance chiral theory: the $\pi\pi$ vector form-factor

Juan José Sanz-Cillero[*][†]

Grup de Física Teòrica and IFAE, Univ. Autònoma de Barcelona, 08193 Barcelona, Spain

The use of the equations of motion and meson field redefinitions allows the simplification of the subleading operators required in the one-loop resonance chiral theory calculation of the $\pi\pi$– vector form-factor. The study of the renormalization group equations of the relevant parameters shows the existence of an infrared fixed point for all the couplings. It is important to remark that this result does not rely on the high-energy form-factor constraints, which are often considered in other works. The possibility of developing a perturbative $1/N_C$ expansion in the slow running region around the fixed point is shown here.

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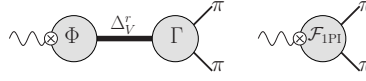
Figure 1: 1PI-topologies contributing to the pion VFF.

Resonance chiral theory (RχT) is a description of the Goldstone-resonance interactions within a chiral invariant framework [2]. The pseudo-Goldstone fields ϕ are introduce through the exponential realization $u(\phi) = \exp\left(i\phi/\sqrt{2}F\right)$. The standard effective field theory momentum expansion is not valid in the presence of heavy resonance states and an alternative perturbative counting is required. RχT takes then the formal $1/N_C$ expansion as a guiding principle [3]: at leading order (LO) the interaction terms in the lagrangian with a number k of meson fields (and their corresponding couplings) scale as $\sim N_C^{1-\frac{k}{2}}$ [3]. At large N_C, the resonance fields become classified in $U(n_f)$ multiplets, with n_f the number of light quark flavours.

The interacting operators of the leading RχT lagrangian relevant for the analysis of the $\pi\pi$ vector form-factor (VFF) in the chiral limit are given by [1, 2, 4]

$$\mathscr{L}_{\mathrm{LO}}^{\mathrm{GB}} = \frac{F^2}{4}\langle u_\mu u^\mu \rangle, \qquad \mathscr{L}_{\mathrm{LO}}^{V} = \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle, \tag{1}$$

The Goldstones fields, given by $u(\phi)$, enter into play through the covariant tensor $u_\mu = i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\}$. Likewise, it is convenient to define $f_\pm^{\mu\nu} = uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u$, with $F_{L,R}^{\mu\nu}$ the left and right field strength tensors [2]. Here, the antisymmetric tensor field $V^{\mu\nu}$ will be considered for the description of the spin–1 mesons [2, 5].

The VFF can be decomposed in the one-particle irreducible (1PI) topologies of Fig. 1:

$$\mathscr{F}(q^2) = \mathscr{F}(q^2)_{\mathrm{1PI}} + \frac{\Phi(q^2)\Gamma(q^2)}{F^2}\frac{q^2}{M_V^2 - q^2 - \Sigma(q^2)}, \tag{2}$$

with $q \equiv p_1 + p_2$, $\Sigma(q^2)$ the vector self-energy, and $\mathscr{F}(q^2)_{\mathrm{1PI}}$, $\Phi(q^2)$ and $\Gamma(q^2)$ being provided, respectively, by the 1PI vertex-functions for $J_V^\mu \to \pi\pi$, $J_V^\mu \to V$ and $V \to \pi\pi$. The one-loop calculation produces a series of ultraviolet divergences that require of subleading operators in $1/N_C$ (X_Z, X_F, X_G, $\widetilde{L}_9$) to fulfill the renormalization of the vertex functions [4]:

$$\begin{aligned}
\mathscr{L}_{\mathrm{NLO}}^{\mathrm{GB}} &= -i\widetilde{L}_9\langle f_+^{\mu\nu}u_\mu u_\nu \rangle, \\
\mathscr{L}_{\mathrm{NLO}}^{V} &= X_Z\langle V_{\lambda\nu}\nabla^\lambda\nabla_\rho\nabla^2 V^{\rho\nu}\rangle + X_F\langle V_{\mu\nu}\nabla^2 f_+^{\mu\nu}\rangle + 2iX_G\langle V_{\mu\nu}\nabla^2[u^\mu, u^\nu]\rangle.
\end{aligned} \tag{3}$$

However, since the subleading $\mathscr{L}_{\mathrm{NLO}}^{V}$ operators are proportional to the equations of motion, one finds that $\mathscr{L}_{\mathrm{NLO}}^{V}$ can be fully transformed into the M_V, F_V, G_V and $\widetilde{L}_9$ terms and into other operators that do not contribute to the VFF by means of a convenient meson field redefinition [1, 4].

Taking now the one-loop expressions for the 1PI topologies (simplified after the field redefinitions) [1] and setting $\mu^2 = Q^2$, one gets then the simple form-factor structure [1],

$$\mathscr{F}(q^2) = -\frac{2Q^2\widetilde{L}_9(Q^2)}{F^2} + [1 + \Delta_t(q^2)]\left[1 - \frac{F_V(Q^2)G_V(Q^2)}{F^2}\frac{Q^2}{M_V^2(Q^2) + Q^2}\right], \tag{4}$$

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[1]The dilogarithmic contribution Δ_t from the triangle with the t–channel vector exchange can be found in Ref. [1] and, for the energies we are going to study ($|q^2| \lesssim 1$ GeV^2), it has little numerical impact.
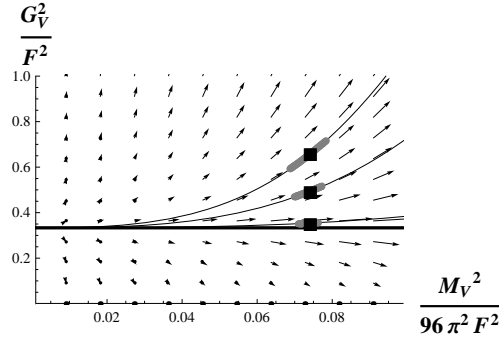
Figure 2: Renormalization group flow for M_V^2 and G_V^2. The points $M_V(\mu_0) = 775$ MeV and $G_V(\mu_0) = 75, 65, 55$ MeV are plotted with filled squares, together with their trajectories for $n_f = 2$ (thin black lines). The horizontal line represents the G_V–fixed point at $G_V^2 = F^2/3$.

with the evolution of the remaining couplings (after the field redefinition) with the Euclidean squared momentum $Q^2 \equiv -q^2$ prescribed by the renormalization group equations (RGE) [1],

$$\frac{1}{M_V^2} \frac{\partial M_V^2}{\partial \ln \mu^2} = \frac{n_f}{2} \frac{2G_V^2}{F^2} \frac{M_V^2}{96\pi^2 F^2}, \tag{5}$$

$$\frac{\partial G_V}{\partial \ln \mu^2} = G_V \frac{n_f}{2} \frac{M_V^2}{96\pi^2 F^2} \left(\frac{3G_V^2}{F^2} - 1 \right),$$

$$\frac{\partial F_V}{\partial \ln \mu^2} = 2 G_V \frac{n_f}{2} \frac{M_V^2}{96\pi^2 F^2} \left(\frac{F_V G_V}{F^2} - 1 \right),$$

$$\frac{\partial \widetilde{L}_9}{\partial \ln \mu^2} = \frac{n_f}{2} \frac{1}{192\pi^2} \left(\frac{F_V G_V}{F^2} - 1 \right) \left(1 - \frac{3G_V^2}{F^2} \right).$$

The RGE solutions for M_V and G_V form a closed system with the trajectories given by

$$G_V^2 = \frac{F^2}{3} \left(1 + \kappa^3 M_V^6 \right), \qquad \frac{1}{M_V^2} + \kappa f(\kappa M_V^2) = -\frac{2}{3} \frac{n_f}{2} \frac{1}{96\pi^2 F^2} \ln \frac{\mu^2}{\Lambda^2}, \tag{6}$$

with $f(x) = \frac{1}{6} \ln \left(\frac{x^2 + 2x + 1}{x^2 - x + 1} \right) - \frac{1}{\sqrt{3}} \arctan \left(\frac{2x - 1}{\sqrt{3}} \right) - \frac{\pi}{6\sqrt{3}} = \mathcal{O}(x)$, and κ and Λ integration constants. Since $-\frac{2\pi}{3\sqrt{3}} \leq f(x) \leq 0$, the term $\kappa f(\kappa M_V^2)$ in (6) becomes negligible for very low momentum, $\mu \ll \Lambda$, producing a logarithmic running.

The parameters M_V and G_V show then an infrared fixed point at $M_V = 0$ and $G_V = F/\sqrt{3}$. The corresponding flow diagram is shown in Fig. 2. The same happens for F_V and $\widetilde{L}_9$, which freeze out when $\mu \to 0$. F_V tends to the infrared fixed point $\sqrt{3}F$ (and hence $F_V G_V \xrightarrow{\mu \to 0} F^2$) and $\widetilde{L}_9(\mu)$ goes to a constant value $\widetilde{L}_9(0)$.

Notice that the value of the resonance couplings at the infrared fixed point, $F_V G_V = F^2$ and $3G_V^2 = F^2$, coincides with those obtained if one demands at large–N_c the proper high energy behaviour of, respectively, the VFF [4, 6] and the partial-wave scattering amplitude [7]. This interplay between fixed points and short-distance behaviour is explained with more detail in [1]. This is also related there with value obtained from the requirement that our one-loop form factor (4) vanishes when $Q^2 \to \infty$ [6, 8, 9]. It leads to the same values: the constraints $F_V G_V = F^2$ and $3G_V^2 = F^2$

are required to freeze out the running of $\widetilde{L}_9$ and $F_V G_V$, killing the $Q^2 \ln(Q^2)$ and $Q^0 \ln(Q^2)$ short-distance behaviour in (4); additionally, $\widetilde{L}_9 = 0$ is needed in order to remove the remaining $\mathcal{O}(Q^2)$ terms at $Q^2 \to \infty$.

Independently of any possible high energy matching [6, 9], what becomes clear from the RGE analysis in [1] is the existence of a region in the RGE space of parameters (around the infrared fixed point at $\mu \to 0$) where the RχT loops produce small logarithmic corrections. Thus, although we start with a formal expansion in $1/N_C$, the perturbative description for the renormalized RχT amplitude only makes sense in the momentum range in the proximity of the fixed point. In an analogous way, although the fixed order perturbative QCD cross-section calculations are formally correct for arbitrary μ (and independent of it), perturbation theory can only be applied at high energies.

One of the aims of [1] was to show how the potentially dangerous higher power corrections arising at next-to-leading order (NLO) [4] actually correspond to a slow logarithmic running of the couplings of the LO lagrangian. We made use of the equations of motion of the theory and meson field redefinitions to remove analytical corrections going like higher powers of the momenta. This left in the vertex functions just the problematic log terms $Q^4 \ln(Q^2)$, which were minimized by means of the convenient choice of scale $\mu^2 = Q^2$. Their evolution was then controlled by the RGE (5), giving place to a slow logarithmic running for M_V, F_V, G_V and $\widetilde{L}_9$.

These considerations are expected to be relevant for the study of other QCD matrix elements. In particular, they may play an important role in the case of scalar resonances. The width and radiative corrections are usually rather sizable in the spin–0 channels. The possible presence of fixed points and slow–running regions in other amplitudes (e.g. the pion scalar form-factor) will be studied in future analyses.

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