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Shear and bulk viscosities of the gluon plasma in the stochastic-vacuum approach

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The contribution of stochastic background fields to the shear viscosity η of the gluon plasma in SU(3) YM theory is calculated. The result for the ratio of this contribution to the entropy density, proportional to the squared chromo-magnetic gluon condensate and the fifth power of the correlation length of the chromo-magnetic vacuum, falls off with the increase of temperature. Numerically, it is of the order of the conjectured lower bound of $1/(4\pi)$, achievable in $\mathcal{N}=4$ SYM theory. As a by-product of the calculation, we find a particular form of the two-point correlation function of gluonic field strengths, which is the only one consistent with the Lorentzian shape of the shear-viscosity spectral function. By the same method, we calculate the contribution of the background fields to the bulk viscosity ζ.

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1. Introduction and preliminary estimates

The RHIC data on collective-expansion dynamics of the hot and dense QGP-fireball formed in ultrarelativistic nucleus-nucleus collisions can be described by the assumption that this dynamics is governed by the laws of relativistic hydrodynamics. Particles of different mass are emitted from the fireball with a common fluid velocity, that is a signature of a hydrodynamic-type behavior. Furthermore, an agreement between the experimental data [1, 2] and the predictions of relativistic hydrodynamics can be reached if the flow of the QGP-fluid is treated as almost non-viscous [3]. This leads to an indication that, in the vicinity of the deconfinement phase transition, the quark-gluon plasma (QGP) produced in the RHIC experiments behaves more like an ideal quantum liquid rather than a weakly interacting gas. The mean free path $L_{\rm mfp}$ of a parton, which traverses such a liquid, is much smaller than the thermal wavelength $\beta = 1/T$, i.e. $(L^{\rm liq}_{\rm mfp}/\beta) \ll 1$.

One can consider for comparison a weakly interacting dilute-gas model of the QGP. There, $L^{\rm gas}_{\rm mfp} \sim (\rho\sigma_t)^{-1}$ with ρ and σ_t standing for the particle-number density and the Coulomb transport cross section, respectively. Using the standard estimates $\rho \sim T^3$ and $\sigma_t \sim g^4\beta^2 \ln g^{-1}$, where $g = g(T)$ is the perturbative finite-temperature QCD coupling, one obtains $(L^{\rm gas}_{\rm mfp}/\beta) \sim 1/(g^4 \ln g^{-1}) \gg 1$, that contradicts the above-mentioned experimental results. One can check [4] that these results could have only been reproduced by the dilute-gas model if the perturbative transport cross section, σ_t, were larger by an order of magnitude. This inconsistency of the weakly interacting QGP with the RHIC data initiated recent calculations of kinetic coefficients in the *strongly* interacting relativistic plasmas.

Among these coefficients, the one whose values define whether the plasma can be considered as weakly or strongly interacting is the shear viscosity η. It is related to the above $(L_{\rm mfp}/\beta)$-ratio via the estimate $(\eta/s) \sim (L_{\rm mfp}/\beta)$, where s is the entropy density. According to this relation, the shear-viscosity to the entropy-density ratio, η/s, becomes smaller when the plasma interacts stronger. For instance, for $T \sim 200\,{\rm MeV}$ and the estimated typical mean free path $L_{\rm mfp} \sim 0.1\,{\rm fm}$, one has $(\eta/s) \sim 0.1$. On the other hand, since the mean momentum change Δp of a parton, which propagates through the plasma over the distance $L_{\rm mfp}$, is of the order of T, the Heisenberg uncertainty principle forbids the ratio $(L_{\rm mfp}/\beta) \sim L_{\rm mfp} \cdot \Delta p$ (and therefore also η/s) to be vanishingly small. Up to now, the minimal *temperature-independent* value of $1/(4\pi) \simeq 0.08$ for the shear-viscosity to the entropy-density ratio has been found in $\mathcal{N} = 4$ SYM theory [5]. It is thus challenging to find other QCD-motivated models where this ratio would be that small.

In this talk, we demonstrate that a *temperature-dependent* η/s of this order of magnitude is produced by soft stochastic background fields present in the gluon plasma of SU(3) YM theory. We obtain this contribution to the shear viscosity by means of the Kubo formula, which relates the corresponding spectral density $\rho(\omega)$ to the Euclidean correlation function of the $(1,2)$-component of the energy-momentum tensor $T_{12}(\mathbf{x}, x_4)$. This method, proposed in Ref. [6], has been explored in Refs. [7, 8] with the aim to simulate shear viscosity on the lattice. Here we work in the continuum limit and parametrize the Euclidean correlation function of the energy-momentum tensors by means of the stochastic vacuum model [9]. This model generalizes QCD sum rules by assuming the existence of not only the gluon condensate $\left\langle g^2 (F^a_{\mu\nu})^2 \right\rangle$ but also of the finite vacuum correlation length μ^{-1}. This assumption is justified by the lattice results on the exponential fall-off at large distances of the two-point correlation function of gluonic field strengths [10, 11], $\left\langle F^a_{\mu\nu}(0) F^b_{\lambda\rho}(x) \right\rangle \sim {\rm e}^{-\mu|x|}$.

By virtue of this finding, the model manages to quantitatively describe confinement; for instance, the string tension reads $\sigma \propto \mu^{-2}\langle g^2(F^a_{\mu\nu})^2\rangle$.

Below we will use a finite-temperature generalization of the stochastic vacuum model, accessible by implementing the Euclidean periodicity of the x_4-coordinate. In the deconfinement phase of interest, such a generalization yields for the spatial string tension $\sigma_s(T)$ a formula [12] similar to its above-quoted vacuum counterpart. This formula reads $\sigma_s(T) \propto \mu_T^{-2}\langle g^2(F^a_{ij})^2\rangle_T$, where μ_T^{-1} is the correlation length of the chromo-magnetic vacuum, and $\langle g^2(F^a_{ij})^2\rangle_T$ is the chromo-magnetic gluon condensate, which survives the deconfinement phase transition. The temperature dependence of the two main ingredients of the model, μ_T and $\langle g^2(F^a_{ij})^2\rangle_T$, can be extracted from the results of the lattice simulations [10, 13].

Since $T_{12} = g^2F^a_{1\mu}F^a_{2\mu}$, one *a priori* expects from the Kubo formula, where the $\langle T_{12}(0)T_{12}(x)\rangle$-correlator enters, that the corresponding contribution to the shear viscosity is $\eta \propto \langle g^2(F^a_{ij})^2\rangle_T^2$. This is a general prediction of the stochastic vacuum model for *all* the kinetic coefficients, for example for the jet quenching parameter $\hat{q}$ [14]. In fact, according to the Kubo formula, all the kinetic coefficients are proportional to the total scattering cross section of the propagating parton, which itself is proportional to $\langle g^2(F^a_{ij})^2\rangle_T^2$ in the stochastic vacuum model [15, 16]. Since the shear viscosity η and the bulk viscosity ζ have the dimensionality of [mass]^3, one can on entirely dimensional grounds expect for the contribution of stochastic background fields to these quantities the following result:

$$\eta \propto \zeta \propto \mu_T^{-5}\langle g^2(F^a_{ij})^2\rangle_T^2. \tag{1.1}$$

At temperatures larger than the temperature of dimensional reduction, $T > T_*$, μ_T and $\langle g^2(F^a_{ij})^2\rangle_T^2$ are proportional to the corresponding power of the only dimensionful parameter present in the YM action at such temperatures, g^2T, i.e.

$$\mu_T \propto g^2T, \quad \langle g^2(F^a_{ij})^2\rangle_T \propto (g^2T)^4.$$

On the other hand, the entropy density $s(T) \propto T^3$, so that one would get

$$\frac{\eta}{s} \propto \frac{\zeta}{s} \propto g^6(T) \quad \text{at} \quad T > T_*. \tag{1.2}$$

Thus, the stochastic vacuum model predicts a monotonic fall-off with temperature of both η/s and ζ/s, where η and ζ are the contributions of stochastic background fields to the shear and bulk viscosities, respectively. However, much as for thermodynamic quantities [17], one expects that the full contribution to kinetic coefficients consists of the part produced by the background fields *and* the part produced by the so-called valence gluons. The latter can be confined by the background fields at large *spatial* separations, and go over to perturbatively interacting gluons at small spatial separations. At temperatures $T \gg T_*$, valence gluons interact perturbatively, and should reproduce known perturbative contributions to kinetic coefficients. The following striking difference between the two viscosities then occurs. Namely, $\frac{\zeta_{\rm pert}}{s} \propto g^4(T)$ [18] is as monotonically decreasing as the $\mathcal{O}(g^6)$-contribution to $\frac{\zeta}{s}$, Eq. (1.2), produced by stochastic background fields. Instead, $\frac{\eta_{\rm pert}}{s} \propto \frac{1}{g^4(T)}$ [19], so that the full $\frac{\eta}{s}$ is eventually *increasing* with temperature. Here, we calculate only the contribution to $\frac{\eta}{s}$ produced by stochastic background fields, Eqs. (1.1), (1.2). The calculation of the valence-gluons' contribution, which should reproduce $\frac{\eta_{\rm pert}}{s}$ at $T \gg T_*$, is postponed for future studies. Fortunately (see for details the original paper [20]), in our approach based on the Kubo formula, the perturbative contribution to viscosities can be isolated simultaneously with the perturbative contribution to the corresponding correlation function of the energy-momentum tensors. This fact allows us to say that, at least at temperatures $T \gg T_*$, where perturbatively interacting valence gluons play the main role, their contribution is clearly separated from the contribution of stochastic background fields, which is explored below.

Our study aims at the *quantitative* calculation of relations (1.1) for η and ζ, and a numerical comparison of the result for η/s with the $1/(4\pi)$-threshold. In Section 2, by assuming an exponential fall-off for the two-point correlation function of the energy-momentum tensors $\langle T_{12}(0)T_{12}(x)\rangle$, we obtain from the Kubo formula an integral equation for the spectral density $\rho(\omega)$ of η. Also in Section 2, by using for $\rho(\omega)$ a Lorentzian-type *ansatz*, with the width equal to the correlation length of $\langle T_{12}(0)T_{12}(x)\rangle$, we explore this equation for the cases of large and small $|k|$'s, where k is the number of a Matsubara mode. The solution in the large-$|k|$ limit yields the range of variation of the numerical parameter α, which enters the initial parametrization of $\langle T_{12}(0)T_{12}(x)\rangle$. The solution in the small-$|k|$ limit can only coincide with the large-$|k|$ solution for a single value of α from this range. This fixes α completely and makes further calculations straightforward. In Section 3, we first calculate η analytically, and then use this result to find the ratio η/s numerically. In Section 4, we obtain by the same method the bulk-viscosity to the entropy-density ratio, ζ/s. In Section 5, we summarize the results of our study.

2. Shear viscosity from the Kubo formula

Shear viscosity η can be defined through the relation

$$\eta = \pi \frac{d\rho}{d\omega}\bigg|_{\omega=0}, \tag{2.1}$$

where the spectral density $\rho(\omega)$ is a solution to the following integral equation, called Kubo formula [6, 8]

$$\int_0^\infty d\omega\,\rho(\omega)\,\frac{\cosh\left[\omega\left(x_4-\frac{\beta}{2}\right)\right]}{\sinh(\omega\beta/2)} = \int d^3x \sum_{n=-\infty}^{+\infty}\left\langle T_{12}(0)T_{12}(\mathbf{x},x_4-\beta n)\right\rangle. \tag{2.2}$$

The correlation function on the RHS of this equation is Euclidean, the sum runs over winding modes, and the temperature dependence of η and ρ is for brevity suppressed. Fourier decomposition of the integral kernel,

$$\frac{\cosh\left[\omega\left(x_4-\frac{\beta}{2}\right)\right]}{\sinh(\omega\beta/2)} = 2T\omega\sum_{k=-\infty}^{+\infty}\frac{e^{i\omega_k x_4}}{\omega^2+\omega_k^2},$$

where $\omega_k = 2\pi T k$ is the k-th Matsubara frequency, suggests to solve Eq. (2.2) in terms of its Fourier coefficients. One can show (see for details the original paper [20]) that, if at $T=0$,

$$\left\langle T_{12}(0)T_{12}(x)\right\rangle = N(\alpha)\left\langle G^2\right\rangle^2\cdot\frac{K_{2-\alpha}(M|x|)}{(M|x|)^{2-\alpha}}, \tag{2.3}$$

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where $K_{2-\alpha}$ is the MacDonald function, then at $T > T_c$ the equation for the Fourier coefficients reads

$$\int_0^\infty d\omega\,\rho(\omega)\,\frac{\omega}{\omega^2+\omega_k^2} = \pi^2 2^\alpha \Gamma(\alpha) N(\alpha) \left\langle G^2\right\rangle_T^2 \cdot \frac{M_T^{2\alpha-4}}{(\omega_k^2+M_T^2)^\alpha}. \tag{2.4}$$

Henceforth, we denote $\left\langle G^2\right\rangle \equiv \left\langle g^2(F_{\mu\nu}^a)^2\right\rangle$, $\left\langle G^2\right\rangle_T \equiv \left\langle g^2(F_{ij}^a)^2\right\rangle_T$, $M = 2\mu$, $M_T = 2\mu_T$. Furthermore, $\alpha > 0$ is a numerical parameter, and $N(\alpha) > 0$ is a coefficient, which will be determined. Equation (2.4) is the central object of the subsequent analysis.

To solve this equation, we assume for the spectral density a Lorentzian-type form (cf. Refs. [6, 8, 21])

$$\rho(\omega) = C(T)\cdot\frac{\omega}{(\omega^2+M_T^2)^{\alpha+\frac12}},$$

which guarantees that both sides of Eq. (2.4) have the same large-$|k|$ behavior. This *ansatz* is consistent with the interpretation of M_T as a momentum scale below which perturbation theory breaks down. Shear viscosity can be obtained by means of Eq. (2.1) as

$$\eta = \frac{\pi C(T)}{M_T^{2\alpha+1}}. \tag{2.5}$$

We now solve Eq. (2.4) subsequently for $|k| \gg 1$ and $|k| \sim 1$, and find both α and $C(T)$. For $|k| \gg 1$, one can expand

$$\text{LHS of Eq. (2.4)} = \frac{C(T)}{\omega_k^{2\alpha}}\left[\frac{\pi}{2\sin(\pi\alpha)} + \mathscr{O}\left(\frac{M_T^2}{\omega_k^2}\right) + \sum_{i=2}^\infty c_i\left(\frac{M_T}{\omega_k}\right)^{i-2\alpha}\right], \tag{2.6}$$

so that the leading term in the brackets is k-independent only for $\alpha < 1$. Using further the expansion

$$\text{RHS of Eq. (2.4)} = \pi^2 2^\alpha \Gamma(\alpha) N(\alpha) \frac{\left\langle G^2\right\rangle_T^2}{\omega_k^{2\alpha}} M_T^{2\alpha-4}\cdot\left[1+\mathscr{O}\left(\frac{M_T^2}{\omega_k^2}\right)\right], \tag{2.7}$$

we obtain

$$\eta(T)\Big|_{|k|\gg1} \simeq \pi^2 2^{\alpha+1}\Gamma(\alpha)N(\alpha)\sin(\pi\alpha)\frac{\left\langle G^2\right\rangle_T^2}{M_T^5}.$$

Rather, for $|k| \sim 1$, terms of the order $\mathscr{O}(\omega_k^{2i}/M_T^{2i})$, where $i \ge 1$, can be disregarded, and we obtain

$$\eta(T)\Big|_{|k|\sim1} \simeq \pi^{5/2} 2^{\alpha+1}\Gamma\left(\alpha+\frac12\right)N(\alpha)\frac{\left\langle G^2\right\rangle_T^2}{M_T^5}.$$

In particular, at $T > T_*$, where only the $(k=0)$-mode should be considered, this result is exact. As one can now readily check, the ratio of the two results obtained,

$$\frac{\eta(T)\Big|_{|k|\gg1}}{\eta(T)\Big|_{|k|\sim1}} = \frac{\Gamma(\alpha)\sin(\pi\alpha)}{\sqrt{\pi}\,\Gamma\left(\alpha+\frac12\right)} \quad \text{for} \quad 0<\alpha<1$$

is equal to 1 at $\alpha = 1/2$. That is, at

$$\alpha = \frac12,$$

our results for the shear viscosity become k-independent, as they should be. Remarkably, for $\alpha = 1/2$, the purely Lorentzian form of $\rho(\omega)$ is recovered.

3. Calculation of the ratio η/s

We should now determine the coefficient $N(\alpha)$ in Eq. (2.3) for $\alpha = 1/2$. To this end, we impose the Gaussian-dominance hypothesis [9], which disregards the connected part of the correlation function

$$\left\langle T_{12}(0)T_{12}(x)\right\rangle = \left\langle g^4 F^a_{1\mu}(0)F^a_{2\mu}(0)F^b_{1\nu}(x)F^b_{2\nu}(x)\right\rangle.$$

The stochastic vacuum model parametrizes confining self-interactions of the background fields in the remaining two-point functions as

$$\left\langle g^2 F^a_{\mu\nu}(x)F^b_{\lambda\rho}(0)\right\rangle = (\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda})\cdot\frac{\left\langle G^2\right\rangle}{12(N_c^2-1)}\delta^{ab}D(x), \tag{3.1}$$

so that

$$\langle T_{12}(0)T_{12}(x)\rangle \simeq \frac{\left\langle G^2\right\rangle^2}{72(N_c^2-1)}D^2(x). \tag{3.2}$$

The dimensionless function $D(x)$ is usually chosen in the form

$$D(x) = \mathrm{e}^{-\mu|x|}. \tag{3.3}$$

Plugging this expression into the formula for the string tension in the fundamental representation,

$$\sigma_{\rm f} = \frac{\left\langle G^2\right\rangle}{144}\int d^2x D(x), \tag{3.4}$$

one can define the gluon condensate in terms of $\sigma_{\rm f}$ and the vacuum correlation length μ^{-1} as follows [14, 16]:

$$\left\langle G^2\right\rangle = \frac{72}{\pi}\sigma_{\rm f}\mu^2. \tag{3.5}$$

To obtain for the correlator $\langle T_{12}(0)T_{12}(x)\rangle$ the functional form given by the RHS of Eq. (2.3), we modify parametrization (3.3) to

$$D(x) = \mathscr{A}(\alpha)\sqrt{\frac{K_{2-\alpha}(2\mu|x|)}{(2\mu|x|)^{2-\alpha}}}, \tag{3.6}$$

where $\mathscr{A}(\alpha)$ is a numerical normalization factor. At $|x| \gtrsim \mu^{-1}$, the new function (3.6) falls off with the same exponent as Eq. (3.3). To find the normalization factor $\mathscr{A}(\alpha)$, we plug Eq. (3.6) into relation (3.4), which holds for any function $D(x)$. Using further expression (3.5), we obtain

$$\mathscr{A}(\alpha) = \frac{4}{\int_0^\infty dz\sqrt{z^\alpha K_{2-\alpha}(z)}}. \tag{3.7}$$

The correlator (3.2) now reads

$$\langle T_{12}(0)T_{12}(x)\rangle = \frac{\mathscr{A}^2(\alpha)}{576}\left\langle G^2\right\rangle^2\cdot\frac{K_{2-\alpha}(2\mu|x|)}{(2\mu|x|)^{2-\alpha}}, \tag{3.8}$$

where the function $\mathscr{A}(\alpha)$ is given by Eq. (3.7), and we have fixed $N_c = 3$. Comparing Eq. (3.8) with the original definition (2.3), we conclude that

$$N(\alpha) = \frac{\mathscr{A}^2(\alpha)}{576}.$$

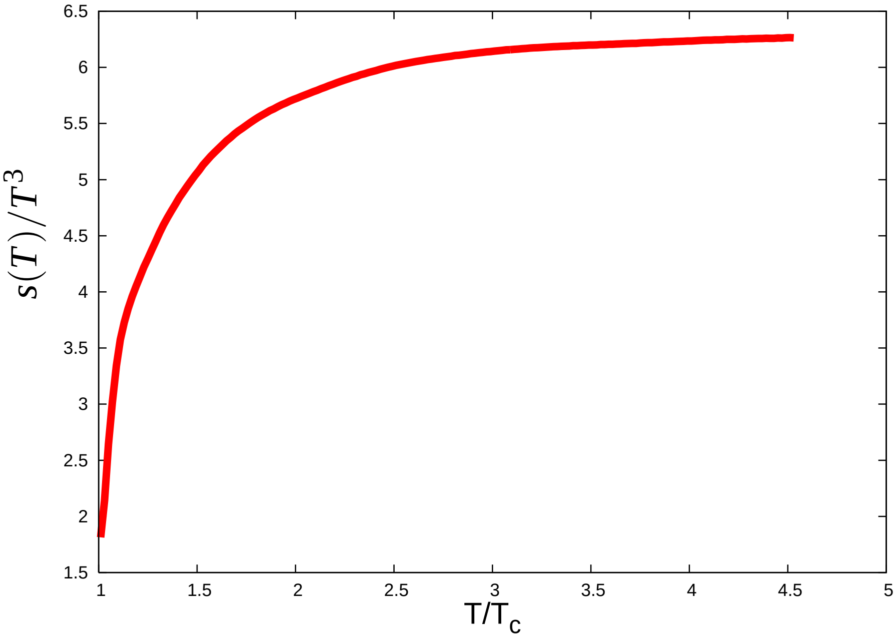

Figure 1: Entropy density $s(T)$ in the units of T^3 obtained from the lattice values for the pressure $p_{\rm lat}$ [13] (courtesy of F. Karsch).

This yields our principal analytic result:

$$\eta(T) = \frac{\pi^{5/2}}{4608\sqrt{2}} \frac{[\mathscr{A}(1/2)\left\langle G^2\right\rangle_T]^2}{\mu_T^5}, \tag{3.9}$$

where $\mathscr{A}(1/2) \simeq 1.05$. The parametric dependence of this expression on $\left\langle G^2\right\rangle_T$ and μ_T is indeed the one following from the elementary dimensional analysis made in Introduction. The corresponding function $C(T)$ entering the spectral density reads [cf. Eq. (2.5)]

$$C(T) = \left(\frac{\pi}{2}\right)^{3/2} \cdot \frac{\mathscr{A}^2(1/2)}{576} \frac{\left\langle G^2\right\rangle_T^2}{\mu_T^3}.$$

Remarkably, for $\alpha = 1/2$, the function $D(x)$ is expressible in terms of elementary functions:

$$D(x) = \mathscr{A}(1/2)\sqrt{\frac{K_{3/2}(2\mu|x|)}{(2\mu|x|)^{3/2}}} = \mathscr{A}(1/2)\cdot\frac{\pi^{1/4}}{2^{5/4}}\cdot\frac{{\rm e}^{-\mu|x|}}{\mu|x|}\sqrt{1+\frac{1}{2\mu|x|}}. \tag{3.10}$$

We evaluate now the (η/s)-ratio numerically. The value of the deconfinement critical temperature in SU(3) YM theory, which we assume, is $T_c = 270\,{\rm MeV}$ [13]. We use the two-loop running coupling [13]

$$g^{-2}(T) = 2b_0\ln\frac{T}{\Lambda} + \frac{b_1}{b_0}\ln\left(2\ln\frac{T}{\Lambda}\right), \text{ where } b_0 = \frac{11N_c}{48\pi^2},\ b_1 = \frac{34}{3}\left(\frac{N_c}{16\pi^2}\right)^2,\ \Lambda = 0.104T_c,$$

and $N_c = 3$ for the case under study. We also assume for μ_T and for the spatial string tension in the fundamental representation, $\sigma_{\rm f}(T)$, the following parametrizations [12, 14]: $\mu_T = \mu\cdot f(T)$,

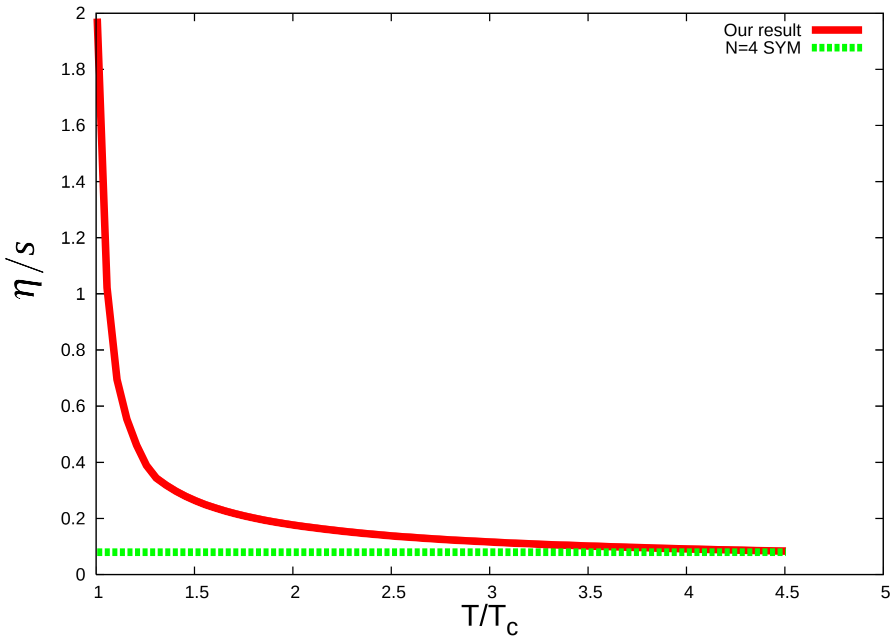

Figure 2: Calculated values of the ratio η/s as a function of temperature. Also shown is the conjectured lower bound of $1/(4\pi)$ for this quantity, realized in $\mathcal{N}=4$ SYM.

$\sigma_{\rm f}(T)=\sigma_{\rm f}\cdot f^2(T)$, where $\mu=894\,{\rm MeV}$ [10], $\sigma_{\rm f}=(0.44\,{\rm GeV})^2$, and

$$f(T)\equiv\begin{cases}1 \ \text{ at } \ T_c<T<T_*, \\ \frac{g^2(T)T}{g^2(T_*)T_*} \ \text{ at } \ T>T_*.\end{cases}$$

We note that, with these parametrizations adopted, the approximation $|k|\gg 1$, used in Eqs. (2.6) and (2.7), means in practice $|k|\ge 3$. In fact, $\frac{M_T}{\omega_3}<0.35$ for any $T>T_c$, while the terms disregarded in those equations are of the order of $\mathcal{O}\left(M_T^2/\omega_k^2\right)$.

Equation (3.5), extrapolated to finite temperatures, yields the chromo-magnetic gluon condensate $\left\langle G^2\right\rangle_T$ [12, 14]:

$$\left\langle G^2\right\rangle_T=\frac{72}{\pi}\sigma_{\rm f}(T)\mu_T^2=\left\langle G^2\right\rangle\cdot f^4(T)$$

The value of T_* can be obtained from the equation $\sigma_{\rm f}(T_*)=\sigma_{\rm f}$, where $\sigma_{\rm f}(T)=[0.566g^2(T)T]^2$ is the high-temperature parametrization of the fundamental spatial string tension [13]. Solving this equation numerically, one gets

$$T_*=1.28T_c.$$

The entropy density $s=s(T)$ can be obtained by the formula $s=dp_{\rm lat}/dT$, where we use for the pressure $p_{\rm lat}$ the corresponding lattice values from Ref. [13]. In Fig. 1, we plot $s(T)$ in the units of T^3.

In Fig. 2, we plot the ratio η/s, with η given by Eq. (3.9), as a function of temperature. The temperature dependence of this ratio is determined by the function $\left\langle G^2\right\rangle_T^2/[\mu_T^5 s(T)]$. One can check numerically that, at $T\gtrsim 2T_c$ where s/T^3 is nearly constant, $\left\langle G^2\right\rangle_T^2/[\mu_T^5 s(T)]=\mathcal{O}\left(g^6(T)\right)$, as has been mentioned in Introduction. Instead, at $T_c<T\lesssim 2T_c$, the calculated η/s falls off much more

rapidly, due to the strong variation of the entropy density $s(T)$ at such temperatures (cf. Fig. 1). Also in Fig. 2, we plot the conjectured lower bound for the (η/s)-ratio, $\frac{1}{4\pi}\simeq 0.08$, which is realized in $\mathcal{N}=4$ SYM [5]. This bound is indeed not reached by our values, although they get very close to it at the highest temperature $T=4.54T_c$ where the lattice data for the pressure (and therefore also for s) are available. However, as has been discussed in Introduction, it is expected that the yet unknown contribution of valence gluons should lead to an increase of the full η/s at $T\gtrsim 2T_c$. Eventually, at $T\gtrsim(5\div 10)T_c$, the full η/s should merge $\eta_{\rm pert}/s$, where $\eta_{\rm pert}$ in the next-to-leading logarithmic approximation reads [19]

$$\eta_{\rm pert}=\frac{T^3}{g^4}\cdot\frac{27.126}{\ln\frac{2.765}{g}}.$$

Thus, the calculated contribution to the (η/s)-ratio produced by stochastic background fields is anyhow subdominant at sufficiently high temperatures.

4. Bulk-viscosity to the entropy-density ratio, ζ/s

The other coefficient at the first-order derivatives of the velocity of energy transport in the energy-momentum tensor of a non-ideal liquid is the already mentioned in Introduction bulk viscosity ζ. It describes the degree of non-conformality of the QGP, and vanishes in any conformal field theory, including $\mathcal{N}=4$ SYM. Similarly to η, bulk viscosity is defined by its spectral density $\rho_{\rm bulk}(\omega)$ as [22]

$$\zeta=\frac{\pi}{9}\frac{d\rho_{\rm bulk}}{d\omega}\bigg|_{\omega=0}.$$

The spectral density obeys the Kubo formula

$$\int_0^\infty d\omega\,\rho_{\rm bulk}(\omega)\,\frac{\cosh\left[\omega\left(x_4-\frac{\beta}{2}\right)\right]}{\sinh(\omega\beta/2)}=\int d^3x\sum_{n=-\infty}^{+\infty}\left\langle T_{\mu\mu}(0)T_{\nu\nu}(\mathbf{x},x_4-\beta n)\right\rangle.$$

Here, $T_{\mu\mu}(x)=\frac{\beta(g)}{2g}[F^a_{\mu\nu}(x)]^2$ is the nonperturbative contribution to the trace of the YM energy-momentum tensor. In the one-loop approximation, where $\beta(g)\simeq -b_0g^3$, $b_0=\frac{11}{16\pi^2}$, one can express the correlator $\left\langle T_{\mu\mu}(0)T_{\nu\nu}(x)\right\rangle$ in terms of the four-point function of gluonic field strengths, $\left\langle g^4F^a_{\mu\nu}(0)F^a_{\mu\nu}(0)F^b_{\lambda\rho}(x)F^b_{\lambda\rho}(x)\right\rangle$. Using further again the Gaussian-dominance hypothesis, one can approximate this correlation function as follows:

$$\left\langle g^4F^a_{\mu\nu}(0)F^a_{\mu\nu}(0)F^b_{\lambda\rho}(x)F^b_{\lambda\rho}(x)\right\rangle\simeq\left\langle G^2\right\rangle^2+2\left\langle g^2F^a_{\mu\nu}(0)F^b_{\lambda\rho}(x)\right\rangle^2. \tag{4.1}$$

The renormalized spectral density is defined by the subtraction from the full $\rho_{\rm bulk}(\omega)$ of an infinite contribution produced by the first term on the RHS of Eq. (4.1). The corresponding nonperturbative contribution to the bulk viscosity can readily be obtained, and reads

$$\zeta=\frac{\pi^{5/2}b_0^2\mathscr{A}^2(1/2)}{6912\sqrt{2}}\cdot\frac{\left\langle G^2\right\rangle_T^2}{\mu_T^5}. \tag{4.2}$$

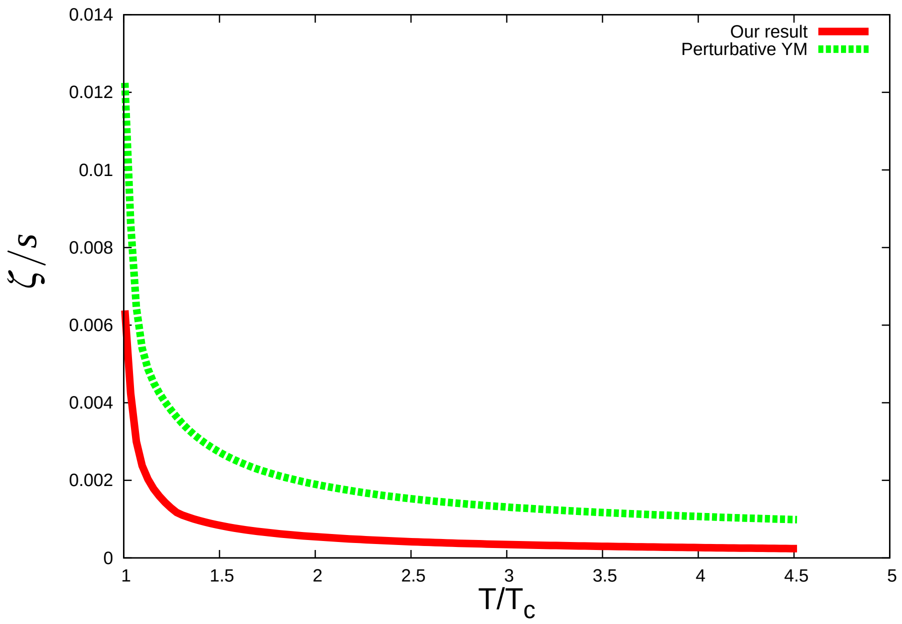

Figure 3: Calculated ratio ζ/s as a function of temperature. Also shown for comparison are perturbative values $\zeta_{\rm pert}/s$ extrapolated down to $T = T_c$.

The bulk-viscosity to the entropy-density ratio ζ/s as a function of temperature is plotted in Fig. 3. For comparison, in the same Fig. 3, we plot the ratio $\zeta_{\rm pert}/s$, where the perturbative bulk viscosity,

$$\zeta_{\rm pert} = \frac{0.443\alpha_s^2 T^3}{\ln(7.14/g)}, \tag{4.3}$$

has been obtained in Ref. [18] in the leading logarithmic approximation. For illustration, in Fig. 3, we extrapolate this weak-coupling formula, valid at $T \gtrsim (5 \div 10)T_c$, down to $T = T_c$. We note once again that, at temperatures $T \gg T_*$, Eq. (4.2) yields $(\zeta/s) \propto g^6$, whereas Eq. (4.3) yields $(\zeta_{\rm pert}/s) \propto g^4$, in a qualitative agreement with the corresponding curves in Fig. 3.

5. Concluding remarks and outlook

The main result reported in this talk is the contribution produced by stochastic background fields to the shear viscosity η in SU(3) YM theory. As has been expected (cf. Introduction), the calculated contribution to η turns out to be $\propto \mu_T^{-5}\langle g^2(F_{ij}^a)^2\rangle_T^2$, where the corresponding proportionality coefficient given by Eq. (3.9) is the main result of our study. The ratio of η to the entropy density as a function of temperature is plotted in Fig. 2. Surprisingly, our results for the contribution to η/s produced by stochastic background fields are of the order of $1/(4\pi)$, that is the conjectured lower bound for this ratio achievable in $\mathcal{N} = 4$ SYM. Moreover, unlike that theory, our results are temperature-dependent. The rapid variation at temperatures $T_c < T \lesssim 2T_c$ of the calculated contribution to η/s, visible in Fig. 2, could mean that stochastic background fields drive the full η/s towards a minimum, which occurs in this temperature range. At higher temperatures ($T \gtrsim 2T_c$), the calculated values should become subdominant compared to the contribution produced by valence gluons, which should provide an increase of the full η/s towards the known perturbative result.

We would also like to emphasize an interesting fact, which has been realized by the end of the calculation. We have started with the general α-dependent Lorentzian-type *ansatz* for the spectral density $\rho(\omega)$. By using it in the Kubo formula, we have come to the conclusion that only for the single value, $\alpha = 1/2$, this *ansatz* provides the Matsubara-mode independence of $\rho(\omega)$. For this value of α, the spectral density takes the conventional Lorentzian form. In this way, also the function $D(x)$ in the correlator (3.1) is defined unambiguously. Moreover, it turns out to be expressible in terms of elementary functions, cf. Eq. (3.10).

Furthermore, we have calculated the contribution of stochastic background fields to the bulk viscosity ζ. Its ratio to the entropy density, ζ/s, is plotted in Fig. 3, in comparison with the known perturbative result [18] extrapolated down to $T = T_c$. By using the same approach, one can also calculate the contribution of *non-confining* self-interactions of stochastic background fields, parametrized by the so-called function $D_1(x)$ [9, 16]. This work is currently in progress. Still, the main problem is to calculate the contribution of valence gluons to both η and ζ, that should provide an interpolation with the known perturbative results at sufficiently high temperatures.

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