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The static potential in the Gribov-Zwanziger Lagrangian

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We discuss the recent computation of the static potential in the Gribov-Zwanziger Lagrangian at one loop in the $\overline{\text{MS}}$ scheme.

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1. Introduction

Gribov's observation, [1], that a linear covariant gauge cannot be fixed uniquely globally in a non-abelian gauge theory leads to a profound effect on the infrared behaviour of the gluon propagator. For instance, using semi-classical methods Gribov demonstrated that the gluon propagator was suppressed and the Faddeev-Popov ghost propagator was enhanced in the zero momentum limit. The appearance of a non-perturbative mass parameter called the Gribov mass, which satisfies a gap equation, plays a key role in this zero momentum behaviour. In recent years, this behaviour has become known as the conformal or scaling solution in contrast to the decoupling solution. The latter, which has gluon propagator freezing and no Faddeev-Popov ghost enhancement, is more consistent with explicit lattice calculations, [2, 3]. Though the numerical debate is not closed, [4]. One drawback of the original Gribov analysis is the fact that it is based on a non-local operator defining the first Gribov region by the no-pole condition involving the Faddeev-Popov operator inverse, [1]. In order to probe the Gribov problem from a calculational point of view Zwanziger and others, [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15], managed to localize the operator defining the horizon condition. The introduction of extra (Zwanziger) ghost fields to implement the condition leads to a softly broken BRST renormalizable Lagrangian, [11, 16, 17]. One significance of this localized Lagrangian is that perturbative calculations can be performed. For example, the two loop correction to the Gribov gap equation is known in $\overline{\text{MS}}$, [18], and the gluon suppression verified to one loop, [19]. Consequently, the Kugo-Ojima confinement criterion holds to two loops. Whilst lattice studies indicate that the decoupling solution appears to be favoured, the conformal solution satisfies a confinement criterion. Therefore, it seems appropriate to compute a quantity which accesses confining behaviour such as the static potential between (heavy) coloured sources. Therefore, we report on some aspects of a recent computation, [20], of the one loop $\overline{\text{MS}}$ static potential in the Gribov-Zwanziger Lagrangian. This is effectively a simple application of the perturbative formalism of [21, 22, 23, 24, 25, 26, 27].

2. Formalism

The static potential starting point is the Wilson loop which is taken to be a rectangle of temporal and spatial extents of sizes T and r respectively where $T \gg r$. As the Wilson loop is a gauge invariant object one is dealing with a physical quantity. Indeed the two loop static potential has been shown to be independent of the linear gauge fixing parameter in non-Gribov QCD, [27]. Specifically the static potential, $V(r)$, is defined as

$$V(r) \;=\; -\lim_{T\to\infty} \frac{1}{iT} \ln \left\langle 0 \left| \text{Tr}\mathscr{P} \exp\left(ig \oint dx^\mu T^a A^a_\mu \right) \right| 0 \right\rangle \tag{2.1}$$

where the path ordering stems from the non-abelian property. In (2.1) the measure is taken to be Gribov's original non-local Lagrangian, [1, 6],

$$L^\gamma \;=\; -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} \;+\; \frac{C_A \gamma^4}{2} A^a_\mu \frac{1}{\partial^\nu D_\nu} A^{a\,\mu} \;-\; \frac{d N_A \gamma^4}{2g^2} \;. \tag{2.2}$$

The Gribov parameter, γ, is defined by the horizon condition

$$\left\langle A^a_\mu(x) \frac{1}{\partial^\nu D_\nu} A^{a\,\mu}(x) \right\rangle \;=\; \frac{d N_A}{C_A g^2} \tag{2.3}$$

where g is the coupling constant, d is the spacetime dimension and N_A is the adjoint dimension. The Gribov mass is not an independent object as (2.2) has no meaning as a gauge theory unless γ satisfies (2.3) which equates to the gap equation. To proceed to a calculation of the static potential one uses the localized Gribov-Zwanziger Lagrangian, [6, 7, 11, 12],

$$\begin{aligned} L^Z \;=\; & -\frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} \;-\; \frac{1}{2\alpha}(\partial^\mu A^a_\mu)^2 \;-\; \bar{c}^a\partial^\mu D_\mu c^a \;+\; i\bar{\psi}^{iI}\not{D}\psi^{iI} \\ & +\bar{\phi}^{ab\,\mu}\partial^\nu\left(D_\nu\phi_\mu\right)^{ab} \;-\; \bar{\omega}^{ab\,\mu}\partial^\nu\left(D_\nu\omega_\mu\right)^{ab} \;-\; gf^{abc}\partial^\nu\bar{\omega}^{ae}_\mu\left(D_\nu c\right)^b\phi^{ec\,\mu} \\ & -\frac{\gamma^2}{\sqrt{2}}\left(f^{abc}A^{a\,\mu}\phi^{bc}_\mu \;-\; f^{abc}A^{a\,\mu}\bar{\phi}^{bc}_\mu\right) \;-\; \frac{dN_A\gamma^4}{2g^2} \end{aligned} \tag{2.4}$$

where ϕ^{ab}_μ, $\bar{\phi}^{ab}_\mu$, ω^{ab}_μ and $\bar{\omega}^{ab}_\mu$ are the localizing (Zwanziger) ghosts with the latter pair being anti-commuting. In addition to rewriting the measure of the perturbative static potential formalism to accommodate the Zwanziger ghosts, which are regarded as completely internal fields, one can replace the path ordering by an external source, $J^a_\mu(x)$, coupled to the gluon which represents the two heavy particles which exchange the strong force quanta. The latter is regared as a spin-1 adjoint object which in the context of (2.4) is not necessarily the gluon due to the presence of the bosonic Zwanziger ghosts. Specifically, we take

$$J^a_\mu(x) \;=\; gv_\mu T^a\left[\delta^{(3)}\left(\mathbf{x}+\tfrac{1}{2}\mathbf{r}\right) \;-\; \delta^{(3)}\left(\mathbf{x}-\tfrac{1}{2}\mathbf{r}\right)\right] \tag{2.5}$$

where $v_\mu = \delta_{\mu 0}$. This results in additional Feynman rules but as they involve only the gluon these are the same as the non-Gribov static potential, [21, 22, 23, 24, 25, 26, 27]. Due to the presence of γ in the propagators we work in momentum space but one can map to coordinate space via

$$V(r) \;=\; \frac{1}{2\pi^2}\int_0^\infty dk\,k^2V(k)\frac{\sin(kr)}{kr} \tag{2.6}$$

after completing the angular integration of the Fourier transform, since we are working at zero spin and angular momentum.

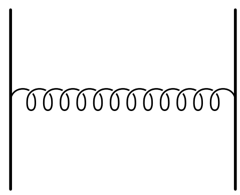

Figure 1: Leading order graph for the static potential.

We briefly summarize the calculational details. We use the symbolic manipulation language FORM, [28], and generate the diagrams using QGRAF, [29]. At leading order there is one graph, illustrated in Figure 1, and 31 one loop graphs. The single exchange set-up at one loop is given in Figure 2 primarily because there is a mixed gluon Zwanziger ghost propagator which plays a role. The main computational details are given in [20] and so we record the full one loop $\overline{\text{MS}}$ momentum

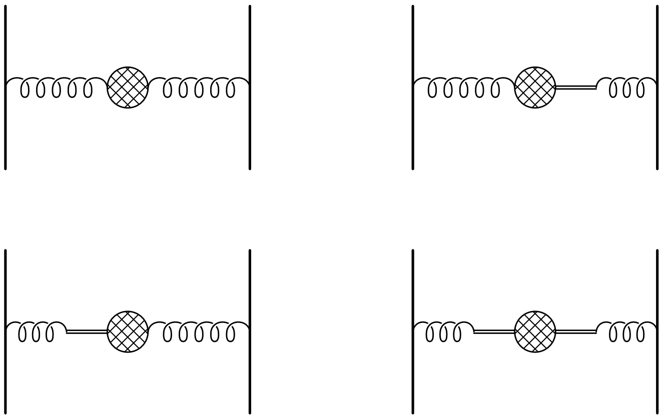

Figure 2: One loop single exchange corrections.

space potential is

$$
\begin{aligned}
\tilde{V}(\mathbf{p}) = & - \frac{C_F \mathbf{p}^2 g^2}{[(\mathbf{p}^2)^2 + C_A\gamma^4]} \\
& + \Bigg[\frac{\pi\sqrt{C_A}}{768\gamma^2} - \frac{1}{768\gamma^4} \tan^{-1} \Bigg[- \frac{\sqrt{4C_A\gamma^4 - (\mathbf{p}^2)^2}}{\mathbf{p}^2} \Bigg] \sqrt{4C_A\gamma^4 - (\mathbf{p}^2)^2} \\
& + \frac{\sqrt{2}}{\gamma^2} \Bigg[\frac{\sqrt{C_A}}{768} \eta_1(\mathbf{p}^2) - \frac{\sqrt{C_A}}{192\sqrt{2}} \tan^{-1} \Bigg[\frac{\sqrt{C_A}\gamma^2}{\mathbf{p}^2} \Bigg] \Bigg] \Bigg] + \frac{231\pi C_A^{3/2}\gamma^2}{128[(\mathbf{p}^2)^2 + C_A\gamma^4]} \\
& + \frac{\sqrt{2}C_A\gamma^2}{[(\mathbf{p}^2)^2 + C_A\gamma^4]} \Bigg[\frac{13\sqrt{C_A}}{24\sqrt{2}} \tan^{-1} \Bigg[\frac{\sqrt{C_A}\gamma^2}{\mathbf{p}^2} \Bigg] + \frac{79\sqrt{C_A}}{128} \eta_1(\mathbf{p}^2) \Bigg] \\
& + \frac{2\pi C_A^{3/2}\gamma^2}{[(\mathbf{p}^2)^2 - 4C_A\gamma^4]} - \frac{1201\pi C_A^{5/2}\gamma^6}{768[(\mathbf{p}^2)^2 + C_A\gamma^4]^2} + \frac{\sqrt{2}C_A^{3/2}\gamma^2}{[(\mathbf{p}^2)^2 + 16C_A\gamma^4]} \eta_1(\mathbf{p}^2) \\
& - \frac{685C_A^2\gamma^4}{768[(\mathbf{p}^2)^2 + C_A\gamma^4]^2} \tan^{-1} \Bigg[- \sqrt{\frac{4C_A\gamma^4 - (\mathbf{p}^2)^2}{\mathbf{p}^2}} \Bigg] \sqrt{4C_A\gamma^4 - (\mathbf{p}^2)^2} \\
& - \frac{395\sqrt{2}C_A^{5/2}\gamma^6\eta_1(\mathbf{p}^2)}{768[(\mathbf{p}^2)^2 + C_A\gamma^4]^2} + \frac{C_A}{\mathbf{p}^2} \Bigg[\frac{13}{96} \ln \Bigg[1 + \frac{(\mathbf{p}^2)^2}{C_A\gamma^4} \Bigg] - \frac{1}{192} \Bigg] \\
& + \frac{455C_A}{384[(\mathbf{p}^2)^2 + C_A\gamma^4]} \tan^{-1} \Bigg[- \frac{\sqrt{4C_A\gamma^4 - (\mathbf{p}^2)^2}}{\mathbf{p}^2} \Bigg] \sqrt{4C_A\gamma^4 - (\mathbf{p}^2)^2} \\
& + \frac{\pi C_A^{3/2}\gamma^2}{384(\mathbf{p}^2)^2} - \frac{C_A^{3/2}\gamma^2}{192(\mathbf{p}^2)^2} \tan^{-1} \Bigg[\frac{\sqrt{C_A}\gamma^2}{\mathbf{p}^2} \Bigg] \\
& - \frac{C_A}{[(\mathbf{p}^2)^2 - 4C_A\gamma^4]} \tan^{-1} \Bigg[\frac{\sqrt{4C_A\gamma^4 - (\mathbf{p}^2)^2}}{\mathbf{p}^2} \Bigg] \sqrt{4C_A\gamma^4 - (\mathbf{p}^2)^2} \\
& + \frac{C_A T_F N_f \gamma^4 \mathbf{p}^2}{[(\mathbf{p}^2)^2 + C_A\gamma^4]^2} \Bigg[\frac{4}{3} \ln \Bigg[\frac{\mathbf{p}^2}{\mu^2} \Bigg] - \frac{20}{9} \Bigg] - \frac{T_F N_f \mathbf{p}^2}{[(\mathbf{p}^2)^2 + C_A\gamma^4]} \Bigg[\frac{4}{3} \ln \Bigg[\frac{\mathbf{p}^2}{\mu^2} \Bigg] - \frac{20}{9} \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{C_A^2\gamma^4\mathbf{p}^2}{[(\mathbf{p}^2)^2+C_A\gamma^4]^2}\left[\frac{365}{72} - \frac{251}{192}\ln\left[\frac{C_A\gamma^4}{\mu^4}\right] - \frac{895\sqrt{2}}{3072}\eta_2(\mathbf{p}^2) - \frac{29}{96}\ln\left[\frac{\mathbf{p}^2}{\mu^2}\right]\right] \\
&+ \frac{C_A\mathbf{p}^2}{[(\mathbf{p}^2)^2+C_A\gamma^4]}\left[\frac{125}{64}\ln\left[\frac{C_A\gamma^4}{\mu^4}\right] - \frac{13}{48}\ln\left[\frac{[C_A\gamma^4+(\mathbf{p}^2)^2]}{\mu^4}\right] - \frac{31}{9}\right. \\
&\qquad\qquad \left. + \frac{167\sqrt{2}}{1536}\eta_2(\mathbf{p}^2) + \frac{29}{96}\ln\left[\frac{\mathbf{p}^2}{\mu^2}\right]\right] \\
&+ \left. \frac{\sqrt{2}C_A\mathbf{p}^2}{4[(\mathbf{p}^2)^2+16C_A\gamma^4]}\eta_2(\mathbf{p}^2) + \frac{\sqrt{2}\mathbf{p}^2}{3072\gamma^4}\eta_2(\mathbf{p}^2)\right]\frac{C_F g^4}{16\pi^2} + O(g^6)
\end{aligned} \tag{2.7}
$$

where we have introduced the intermediate functions $\eta_1(\mathbf{p}^2)$ and $\eta_2(\mathbf{p}^2)$ for compactness with

$$
\begin{aligned}
\eta_1(\mathbf{p}^2) = &- \ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}\right]\sqrt{-1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}} \\
&+ \ln\left[\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}\right]\sqrt{-1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}} \\
&- 2\ln\left[\sqrt{1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}}-\sqrt{2}\right]\sqrt{-1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}} \\
&- 2\tan^{-1}\left[\frac{\sqrt{2}}{\sqrt{-1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}}}\right]\sqrt{1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}}
\end{aligned} \tag{2.8}
$$

and

$$
\begin{aligned}
\eta_2(\mathbf{p}^2) = &\ln\left[\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}\right]\sqrt{1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}} - \ln\left[1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}\right]\sqrt{1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}} \\
&- 2\ln\left[\sqrt{1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}}-\sqrt{2}\right]\sqrt{1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}} \\
&+ 2\tan^{-1}\left[\frac{\sqrt{2}}{\sqrt{-1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}}}\right]\sqrt{-1+\sqrt{1+\frac{16C_A\gamma^4}{(\mathbf{p}^2)^2}}} \, .
\end{aligned} \tag{2.9}
$$

We use the notation that in four dimensions $p_\mu = (p_0,\mathbf{p})$ and note that μ is the mass scale introduced as a result of working in dimensional regularization in $d = 4 - 2\epsilon$ dimensions. Whilst this is not illuminating it does agree with the $\gamma \to 0$ expression of [21, 22, 23] which is

$$
\begin{aligned}
\lim_{\gamma\to 0}\tilde{V}(\mathbf{p}) = &- \frac{4\pi C_F\alpha_s(\mu)}{\mathbf{p}^2}\left[1 + \left[\left[\frac{31}{9} - \frac{11}{3}\ln\left[\frac{\mathbf{p}^2}{\mu^2}\right]\right]C_A + \left[\frac{4}{3}\ln\left[\frac{\mathbf{p}^2}{\mu^2}\right] - \frac{20}{9}\right]T_F N_f\right]a\right. \\
&\qquad\qquad \left. + O(a^2)\right]
\end{aligned} \tag{2.10}
$$

where $a = g^2/(16\pi^2)$. This was originally computed in the Feynman gauge, [21, 22, 23], and later in an arbitrary linear covariant gauge, [26, 27]. In the Gribov-Zwanziger set-up one is working purely in the Landau gauge, so the agreement is a non-trivial check on (2.7).

If we examine (2.7) in the zero momentum limit we find

$$\tilde{V}(\mathbf{p}) ~=~ -~ \frac{C_F \mathbf{p}^2 g^2}{C_A \gamma^4} ~-~ C_F \left[\frac{\pi \sqrt{C_A}}{32\gamma^2} ~+~ \left(\frac{13}{72} - \frac{3}{8} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right) \frac{\mathbf{p}^2}{\gamma^4} \right] \frac{g^4}{16\pi^2} ~+~ O((\mathbf{p}^2)^2; g^6) \quad (2.11)$$

which implies there is no net dipole whose Fourier transform would lead to a linear potential despite there being dipole-like terms in (2.7). However, this zero momentum limit assumes that $\gamma^2 > 0$ but if $\gamma^2 < 0$ then there would be a net dipole and a linearly rising potential. This does not mean a confining potential has been established since the higher order loop corrections would have to have no net triple or higher order poles in p^2 as $p^2 \to 0$. Otherwise these would out-compete a linear potential if their overall sign was positive after taking the Fourier transform. A final observation is that one can compute the next-to-high energy power correction to $V(\mathbf{p})$ in the $\gamma^2/\mathbf{p}^2$ expansion. Using the V-scheme definition $\tilde{V}(\mathbf{p}) = -\frac{4\pi C_F}{\mathbf{p}^2} \alpha_V(\mathbf{p})$ we have

$$\alpha_V(\mathbf{p}) ~=~ \alpha_V^{\mathrm{pert}}(\mathbf{p}) ~-~ \frac{C_A^{3/2} \gamma^2 \alpha_s^2(\mu)}{2\mathbf{p}^2} ~+~ O \left(\frac{\gamma^4}{(\mathbf{p}^2)^2} \right) \quad (2.12)$$

where

$$\alpha_V^{\mathrm{pert}}(\mathbf{p}) ~=~ \left[1 ~+~ \left[\left[\frac{31}{9} - \frac{11}{3} \ln \left[\frac{\mathbf{p}^2}{\mu^2} \right] \right] C_A ~+~ \left[\frac{4}{3} \ln \left[\frac{\mathbf{p}^2}{\mu^2} \right] - \frac{20}{9} \right] T_F N_f \right] a(\mu) ~+~ O(a^2) \right] \alpha_s(\mu) \quad (2.13)$$

and $\alpha_s = g^2/(4\pi)$. This appears to produce a dipole but this is misleading since there is no net dipole overall. Instead this dimension two correction is washed out as one approaches low energy, consistent with recent observations, [30].

3. Bosonic ghost enhancement

The absence of a dipole in (2.7) is not unexpected as the Gribov gap equation has not been used explicitly. The Faddeev-Popov ghost enhancement itself emerges as a result of the horizon condition leading to a dipole propagator in the infrared, [1]. However, this can never be the single exchange particle propagating between two coloured sources as the Faddeev-Popov ghost is Grassmann. Instead one requires a bosonic particle. Recently, a candidate for this was discussed in [31] where Schwinger Dyson methods showed that the propagator of the imaginary part of the ϕ^{ab}_μ field is enhanced. It is also possible to see this in the perturbative set-up, [20]. Though the analysis is more involved than the Faddeev-Popov ghost derivation as the ϕ^{ab}_μ propagator is entwined with that of A^a_μ. We have examined the implications of an enhanced bosonic ghost propagator for the static potential. However, the bosonic ghost has no direct coupling to the heavy colour sources so it can only become relevant via diagrams such as those of Figure 3. Focusing on the left hand graph where the sources are now in the adjoint representation it transpires that the enhanced ghost propagator does not lead to a dipole domination in the zero momentum limit. Whilst this is disappointing it does not exclude the linearly rising potential in the Gribov-Zwanziger Lagrangian. It merely

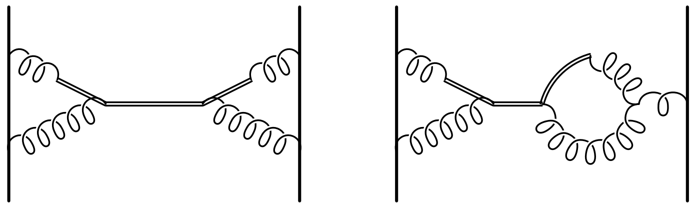

Figure 3: Some two loop single exchange corrections.

implies that either higher order loop corrections will resolve the behaviour or that the confining potential arises from a different set of topologies which are dominant in the infrared limit.

Finally, we remark on the decoupling solution in the static potential set-up. This solution can be accessed by the local composite operator (LCO) formalism by the condensation of the dimension two operator $\bar{\phi}^{ab}_{\mu} \phi^{ab\,\mu} - \bar{\omega}^{ab}_{\mu} \omega^{ab\,\mu}$, [32, 33]. In the context of (2.1) it is difficult to see how the LCO method can be applied here to the static potential formalism as it requires coupling the operator to an external source and the localizing ghosts are assumed to be completely internal localizing fields. However, the source of (2.1) is coupled to heavy coloured objects. Moreover, the starting point is (2.2) which has only one scale γ and not the two of the decoupling solution. One would therefore have to modify (2.2) to something such as

$$L^{\gamma} ~=~ -~ \frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} ~+~ \frac{C_A \gamma^4}{2} A^a_\mu \, \frac{1}{(\partial^\nu D_\nu - m^2)} A^{a\,\mu} ~-~ \frac{d N_A \gamma^4}{2g^2} \tag{3.1}$$

where m is the additional scale, before localizing. This operator is probably not unique and the presence of m does not destroy renormalizabilty of the localized Lagrangian. However, in this formulation it appears to require a second gap equation to define the mass scale m, if one overlooks the main difficulty of the loss of the no pole condition and the lack of consistency with the Kugo-Ojima confinement criterion. Aside from this, irrespective of how the decoupling solution is accommodated if one took the propagators of this situation then the absence of a Zwanziger ghost enhancement could not improve the zero momentum behaviour of our conformal solution calculation to produce a dipole in the single exchange diagrams of Figure 3.

4. Discussion

We have reviewed the recent computation of the one loop $\overline{\mathrm{MS}}$ static potential computation of [20] in the Gribov-Zwanziger Lagrangian. Whilst a net dipole does not arise there seems to be an intriguing possibility that the enhancement of the localizing ghost might play a central role. If one considers diagrams where there is single bosonic ghost exchange it could be the case that when higher loop corrections to the source ghost vertex are included then its zero momentum limit could be modified so that there is a net dipole exchange consistent with the underlying Kugo-Ojima criterion. The absence of a bosonic ghost enhancement for the decoupling solution appears to be a difficulty in extracting a linear potential in that case within the static potential formalism. However, even if the conformal solution of the Gribov-Zwanziger did produce a linearly rising potential, there would then be the subsequent problem of obtaining a slope or string tension commensurate with other methods.

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