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Theoretical overview of $b \to s$ hadronic decays

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A wealth of data on hadronic $b \to s$ transitions is available from the B-factories and will be improved at the LHCb experiment and possible future super-B-factories. I review the theory of these decays as it pertains to the search for physics beyond the Standard Model and various puzzles in the present data.

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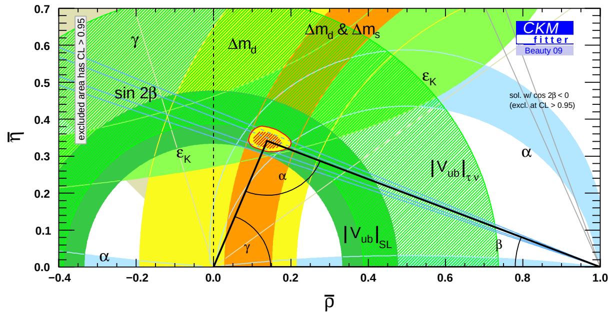

Figure 1: Global CKM fit as shown at this conference [1]

1. Introduction

Charmless hadronic $b \to s$ transitions are a rich source of information about the physics of the weak and/or TeV scales. Their sensitivity to short-distance physics derives from the CKM hierarchy and a GIM cancellation which combine to suppress contributions at tree-level in the weak interaction or through light-quark loops. As a consequence, the Standard-Model (SM) amplitudes are governed by the combination

$$V_{ts}^* V_{tb} \times \frac{1}{16\pi^2} \times \frac{m_B^2}{M_W^2} \sim 10^{-6}. \tag{1.1}$$

The resulting rareness of these modes makes them sensitive to contributions of new particles with TeV-scale masses, so we should expect deviations from the Standard Model. The task is to disentangle SM and new-physics (NP) contributions in a given mode, such that a possible NP signal can be recognized, and to identify those observables, or combinations of them, where this is best possible. More ambitiously, one may want to quantify a signal in terms of NP-model parameters.

It is worth contrasting the $b \to s$ transitions with the $b \to d$ ones. Here, the CKM hierarchy is different, such that tree-level contributions involving $V_{ub} \propto (\bar\rho - i\bar\eta)$ can compete with or dominate over loop contributions involving $V_{td} \propto (1-\bar\rho - i\bar\eta)$. Indeed, $b \to d$ hadronic decays, together with $b \to u$ semileptonic and, by now, purely leptonic $B^+ \to \tau\nu_\tau$ decays, provide the main input to the global CKM fit (Figure 1). The two dominant inputs are the ratio of B_d and B_s mass differences (orange ring) and the mixing-induced CP violation in $B_d \to J/\psi K_S$ (blue wedge) derive from the $B-\bar B$ mixing amlitudes, which are again loop processes. Two constraints in a plane will generically intersect in a discrete set of points, and the most significant consistency check is through the "α" measurements in $B_d \to \pi\pi, \pi\rho, \rho\rho$ transitions (shown as light blue "half moon" in the Figure). Hence the consistency of the CKM fit at present allows $\mathcal{O}(10\%)$ NP effects. Beyond this level, NP contributions to different observables would have to conspire to maintain the observed level of agreement.[1] On the other hand, the combination $V_{ts}^* V_{tb}$ relevant to $b \to s$ transitions is very weakly dependent on $\bar\rho$ and $\bar\eta$. Hence these processes are determined, in principle, with a small parametric

[1]The picture may change as progress in lattice QCD makes more precise predictions for B meson mixing, $B \to \tau\nu$, and ε_K possible. Interestingly, a significantly improved calculation of B_K [2] indicates a tension with the aforementioned CKM determinations at about the 2σ level [3].

uncertainty in the SM. Moreover, the consistency of the CKM fit has little to say about new physics in $b \to s$ transitions. Indeed, several puzzles have shown up in recent years in the data, notably

1. time-dependent CP violation in $b \to s$ decays of B_d^0 mesons to a CP eigenstate. In the SM, one expects to measure $-\eta_{CP} S \approx \sin 2\beta$, but some modes show a deviation (Figure 2). None of these is very significant at the moment, but this might change with more precise data becoming available from LHCb and, eventually, a super-B factory.

2. The time-dependent CP violation in $B_s \to J/\psi\phi$, in combination with lifetime difference and semileptonic asymmetry, determines the phase of the mixing amplitude to be [4]

$$\phi_{B_s} \in (-168, -102)^\circ \cup (-78, -11)^\circ, \tag{1.2}$$

   about 2.2σ from the SM, with much better statistics ahead at Tevatron and LHCb. The theory is reviewed in a separate talk at this conference [5].

3. Direct CP asymmetries in $B \to \pi K$ decays. These modes have received attention for several years. It has been stressed that $A_{\rm CP}(B^+ \to \pi^0 K^+) \neq A_{\rm CP}(B^0 \to \pi^- K^+)$ at 5σ significance [6]. The verdict is less clear, since the SM does *not* predict identical asymmetries.

2. Hadronic decay amplitudes

Interpreting items 1 and 3 requires knowledge about hadronic decay amplitudes, which always involve nonperturbative QCD. As the latter is generally under limited control, approximations are necessary, either neglecting some small parameter or expanding in it.

For any $b \to s$ transition to a final state f, we can write

$$\mathscr{A}_f \equiv \mathscr{A}(B \to f) = V_{us} V_{ub}^* T_f + V_{cs} V_{cb}^* P_f + P_f^{\rm NP}, \tag{2.1}$$

$$\mathscr{A}_{\bar f} \equiv \mathscr{A}(\bar B \to \bar f) = V_{us}^* V_{ub} T_f + V_{cs}^* V_{cb} P_f + P_{\bar f}^{\rm NP}, \tag{2.2}$$

where T_f and C_f are CP-even "strong" amplitudes and $P_f^{\rm NP}$, $P_{\bar f}^{\rm NP}$ are new-physics contributions. CKM unitarity has been used to eliminate the combination $V_{ts} V_{tb}^*$ ($V_{ts}^* V_{tb}$). Branching fractions and CP asymmetries are functions of the magnitudes and relative phases of the strong amplitudes, as well as magnitudes and phases of the CKM elements. For instance, if f is a CP eigenstate, $|\bar f\rangle = \eta_{\rm CP}(f) |f\rangle$, then the time-dependent CP asymmetry is given as

$$A_{\rm CP}(f;t) \equiv \frac{\Gamma(\bar B(t) \to f) - \Gamma(B(t) \to f)}{\Gamma(B(t) \to f) + \Gamma(B(t) \to f)} \equiv -C_f \cos \Delta m_d\, t + S_f \sin \Delta m_d\, t, \tag{2.3}$$

$$C_f = \frac{1 - |\xi|^2}{1 + |\xi|^2}, \qquad S_f = \frac{2 {\rm Im} \xi}{1 + |\xi|^2}, \qquad \xi = e^{-i2\beta} \frac{\mathscr{A}(\bar B \to f)}{\mathscr{A}(B \to f)} = -\eta_{\rm CP}(f) e^{-i2\beta} \frac{V_{cs}^* V_{cb} + \dots}{V_{cs} V_{cb}^* + \dots}. \tag{2.4}$$

Here the dots are proportional to the ratio T_f/P_f, multiplied by CKM factors of $\mathscr{O}(\lambda^2)$. If the tree amplitudes are neglected, then $-\eta_{\rm CP}(f) S_f = \sin(2\beta)$ results to very good approximation. While experimentally (Figure 2) the various modes are in reasonable agreement with each other and the determination of $\sin 2\beta$ from $b \to c\bar c s$ transitions, the suggestive pattern of the central values begs the question whether it could be caused by the neglected SM tree amplitudes, or one has to invoke

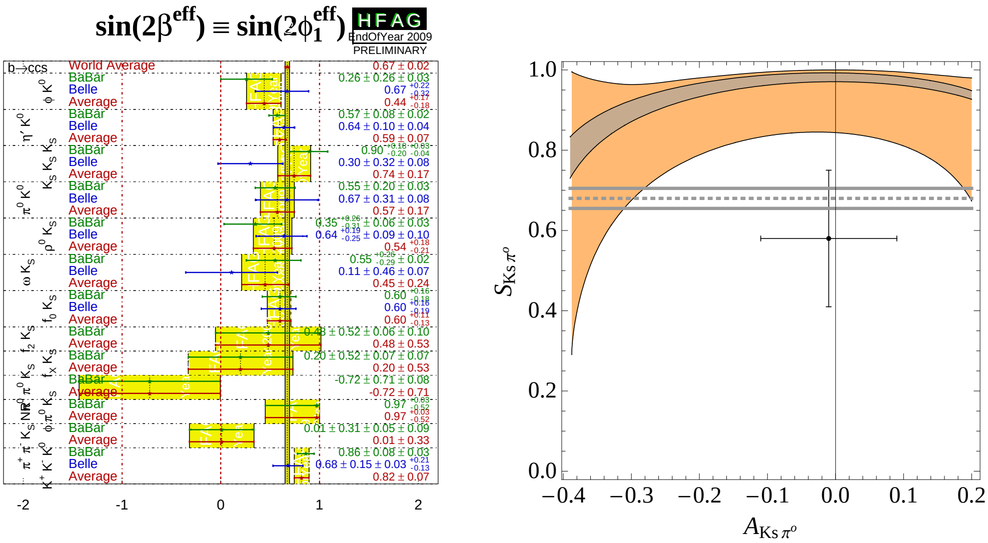

Figure 2: Left: Measurements of mixing-induced CP asymmetries in $b \to s$ penguin transitions as compiled by the HFAG [7]. Right: Constraints from decay rate data in the (A,S) plane for $B \to \pi^0 K_S$ $(A = -C)$ [23]

NP terms $P_f^{\rm NP}$. Quantitative information on the amplitudes derives from (i) flavour-$SU(3)$ (and isospin) relations [8] together with measurements of $b \to d$ transitions and (ii) the heavy-quark expansion in $\Lambda_{\rm QCD}/m_b$ (QCDF [9] and its effective-field-theory formulation in SCET [10, 11, 12], and the somewhat different "pQCD" approach [13]). Guidance on the relative importance of amplitudes follows from (iii) Cabibbo counting and (iv) the large-N expansion [14]. (i), (ii), and (iv) involve the subdivision of the "physical" tree and penguin amplitudes into several "topological" amplitudes,

$$T_{M_1M_2} = \Big[A_{M_1M_2}(\alpha_1(M_1M_2)+\alpha_2(M_1M_2)+\alpha_4^u(M_1M_2)) \tag{2.5}$$

$$+B_{M_1M_2}(b_1(M_1M_2)+b_2(M_1M_2)+b_3^u(M_1M_2)+b_4^u(M_1M_2))+\mathcal{O}(\alpha)\Big] + (M_1 \leftrightarrow M_2)\,,$$

$$P_{M_1M_1} = \Big[A_{M_1M_2}\alpha_4^c(M_1M_2)+B_{M_1M_2}(b_3^c(M_1M_2)+b_4^c(M_1M_2))+\mathcal{O}(\alpha)\Big] + (M_1 \leftrightarrow M_2)\,, \tag{2.6}$$

where we employ the notation of [15, 9], which is general but is particularly suited for the heavy-quark expansion. $A_{M_1M_2}$ and $B_{M_1M_2}$ are normalization factors which by convention contain certain form factors and decay constants. The α_i and b_i denote the different topological amplitudes. Often, $A\alpha_1$ and $A\alpha_2$ are written as T and C, $A\alpha_4^c$ as P_{ct}, etc., or variations thereof. Table 1 summarizes the counting in the various small parameters. At the quantitative level, the leading-power amplitudes α_1, α_2, ...can be factorized [9] into products of "hard kernels" that can be computed order by order in perturbation theory and include all strong (rescattering) phase information, and nonperturbative normalization factors such as $f_+^{B\pi}(0)f_K$ or $f_B f_K f_\pi$ (usually factored out into $A_{M_1M_2}$ and $B_{M_1M_2}$). This statement holds up to generally incalculable Λ/m_b corrections. Certain amplitudes (annihilation amplitudes b_i) are altogether power-suppressed and not calculable. See [16] for more details. Over the last years, a number of higher-order (NNLO) calculations of the kernels have been performed [17]. The main phenomenological findings can be summarized as follows.

Table 1: Hierarchies among topological amplitudes from expansions in the Cabibbo angle λ, in $1/N_c$, and in $\Lambda_{\rm QCD}/m_b$. (Some amplitudes, such as electroweak penguins, are omitted from the list.)

	α_1 (T)	α_2 (C)	α_4^u (P_{ut})	α_4^c (P_{ct})	α_{3EW} $(P_{\rm EW})$	α_{4EW} $(P_{\rm EW}^{\rm C})$	b_3^c	b_4^c	b_1 (E)	b_2 (A)
Cabibbo $(b \to d)$	all amplitudes are $\mathscr{O}(\lambda^3)$									
Cabibbo $(b \to s)$	λ^4	λ^4	λ^4	λ^2	λ^2	λ^2	λ^2	λ^2	λ^4	λ^4
$1/N$	1	$\frac{1}{N}$	$\frac{1}{N}$	$\frac{1}{N}$	1	$\frac{1}{N}$	$\frac{1}{N}$	$\frac{1}{N}$	$\frac{1}{N}$	1
Λ/m_b	1	1	1	1	1	1	Λ/m_b	Λ/m_b	Λ/m_b	Λ/m_b

- The colour-allowed trees α_1 are well behaved in perturbation theory, with overall uncertainties at the few-percent level (not counting the nonperturbative normalization).

- The colour-suppressed trees α_2 show cancellations within the (well-behaved) perturbative part, and suffer from a large uncertainty in the normalization and sensitivity to power corrections. Attaching an $\mathscr{O}(1)$ uncertainty to the (small) theoretical prediction using the power-correction model of [9] would still imply $|\alpha_2/\alpha_1| < \mathscr{O}(1/2)$.

- The topological (QCD) penguin amplitudes are also under good control (but only a subset of NNLO corrections is known), but are phenomenologically indistinguishable from the incalculable (formally power-suppressed) penguin annihilation amplitudes.

- The colour-allowed and colour-suppressed electroweak penguins amplitudes behave qualitatively like the colour-allowed and colour-suppressed trees, respectively.

Further recent work focussing on phenomenological issues can be found in [18], and a new take on long-distance charm penguins in [19].

3. Phenomenological applications

Table 2: Predictions for ΔS defined in the text for several penguin-dominated modes. *From [16]; see therein for details, in particular the meaning and comparison of errors.*

mode	QCDF/BBNS [20]	SCET/BPRS [11, 21]	pQCD [22]	experiment [7]
ϕK_S	$0.01 \dots 0.05$	0 / 0	$0.01 \dots 0.03$	-0.23 ± 0.18
ωK_S	$0.01 \dots 0.21$	$-0.25 \dots -0.14$ / $0.09 \dots 0.13$	$0.08 \dots 0.18$	-0.22 ± 0.24
$\rho^0 K_S$	$-0.29 \dots 0.02$	$0.11 \dots 0.20$ / $-0.16 \dots -0.11$	$-0.25 \dots -0.09$	-0.13 ± 0.20
ηK_S	$-1.67 \dots 0.27$	$-0.20 \dots 0.13$ / $-0.07 \dots 0.21$		
$\eta' K_S$	$0.00 \dots 0.03$	$-0.06 \dots 0.10$ / $-0.09 \dots 0.11$		-0.08 ± 0.07
$\pi^0 K_S$	$0.02 \dots 0.15$	$0.04 \dots 0.10$		-0.10 ± 0.17

Several authors have estimated the tree "pollution" in the hadronic $b \to s$ penguins combining experimental data and heavy-quark-expansion calculations in different ways. Their results, compared in Table 2, are in general agreement with each other (as they should) and can be compared to Figure 2. Clearly, the SM does not produce the pattern of experimental (central) values. While the significance of the measured ΔS values is low for all modes, in the case of $\pi^0 K_S$ one can perform a combined analysis of all $B \to \pi K$ decay data to get a somewhat stronger "signal". The method discussed here [23] (see also [24]) invokes the well-known isospin symmetry relation

$$\sqrt{2}\mathscr{A}(B^0 \to \pi^0 K^0) + \mathscr{A}(B^0 \to \pi^- K^+) = -\left[(\hat{T} + \hat{C})e^{i\gamma} + \hat{P}_{\rm ew}\right] \equiv 3A_{3/2}. \tag{3.1}$$

This relation, and a similar one for the CP conjugates, allows to fix all four complex decay amplitudes from the four decay rates *if* the isospin-3/2 amplitudes are known, up to a four-fold ambiguity. The latter can indeed be obtained as

$$3A_{3/2} = -R_{T+C}|V_{us}/V_{ud}|\sqrt{2}|A(B^+ \to \pi^+\pi^0)|\left(e^{i\gamma} - 0.66\frac{0.41}{R_b}R_q\right), \tag{3.2}$$

where R_b is a side of the unitarity triangle and $R_{T+C} = 1.23^{+0.02}_{-0.03}$ and $R_q = (1.02^{+0.27}_{-0.22})e^{i(0^{+1}_{-1})^\circ}$ quantify $SU(3)$ breaking, with uncertainties obtained in a QCDF calculation. Fixing the ambiguity by a (minimal) usage of either QCDF or $SU(3)$, one obtains a prediction of $S_{\pi^0 K_S}$ (Figure 2) from the remaining data. This is one of many ways of visualizing the tension in the πK system, distinguished perhaps by a particularly limited use of uncertain theoretical predictions or assumptions. A future perspective on the uncertainty is also indicated (thin band). For more on NP in $B \to \pi K$, see [25].

One can also attempt to compute directly the difference in direct CP asymmetries. Unfortunately, this involves the uncertain colour-suppressed tree amplitude, and the significance of this discrepancy is currently difficult to quantify. Making no assumptions about C, one still has the relation [26] $A_{\rm CP}(K^+\pi^-) + A_{\rm CP}(K^0\pi^+) \approx A_{\rm CP}(K^+\pi^0) + A_{\rm CP}(K^0\pi^0)$, which is satisfied by the current experimental data [24], and expected to hold (in general) to few-percent level.

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