Strong Phase Measurements - Towards $\gamma$ at CLEO-c

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Strategies that utilise the interference effects within $B \to DK$ decays hold great potential for improving our sensitivity to the CKM angle $\gamma$. However, in order to exploit fully this potential, knowledge of parameters associated with the $D$ decay, such as strong-phase differences, are required. This essential information can be obtained from the unique quantum-correlated $\psi(3770)$ datasets at CLEO-c. Results of such analyses involving the decay modes $D \to K\pi, K\pi\pi^0, K\pi\pi\pi$ and $K^0_S\pi\pi$ will be presented.
1. Introduction

A theoretically clean method to extract the CKM-angle $\gamma$ is to exploit the interference present in $B^+ \rightarrow DK^{\pm}$, where the $D$ is a $D^0$ or $\bar{D}^0$ decaying to a common final state, $f$. Decay rates in these channels are sensitive to the following amplitude ratios

$$ \frac{A(B^- \rightarrow D^0 K^-)}{A(B^- \rightarrow D^0 K^0)} = r_B e^{i(\delta_B - \gamma)}, \quad \frac{A(B^+ \rightarrow D^0 K^+)}{A(B^+ \rightarrow D^0 K^0)} = r_B e^{i(\delta_B + \gamma)}. \quad (1.1) $$

which are functions of three parameters: the ratio of the absolute magnitudes of the amplitudes, $r_B$; a $CP$-invariant strong-phase difference, $\delta_B$; and the weak phase $\gamma$. A variety of $\gamma$ extraction strategies have been suggested depending on the $D$ final state. Final states that can be used are: two-body modes such as $K^+ K^-/\pi^+ \pi^-$ [1, 2], $K^0 \pi^\pm$ [3], as well as multi-body final states such as $K_0^0 \pi^+ \pi^- [4, 5]$ and $K^\pm \pi^\pm \pi^0 / K^\pm \pi^\pm \pi^0 [6]$.

In all cases, the measurement of $\gamma$ is affected by properties of the $D$ decay amplitude. In order to exploit fully the sensitivity to the $B$-specific parameters $(r_B, \delta_B$ and $\gamma$) it is, therefore, highly advantageous to have prior knowledge of the parameters associated with the $D$ decay. This is where CLEO-c plays a crucial role.

These proceedings describe three sets of measurements performed by CLEO-c of $D$-specific parameters relevant to the measurement of $\gamma$. Sec. 2 introduces the $D$ parameters of interest in the context of the $B$ decay rates. Sec. 3 then explains how one can exploit quantum-correlations at the $\psi(3770)$ in order to probe these $D$ parameters. Sec. 4 describes the CLEO-c experiment and data sets used for the analyses. Secs. 5, 6 and 7 describe the experimental procedure and results.

2. $D$ Parameters Associated with the ADS Method

In the case of the so-called ADS method [3], where $f = K^\pm \pi^\mp$, $D$-specific parameters contribute to the suppressed $B^\pm$ decay-rates as follows:

$$ \Gamma(B^\pm \rightarrow (K^\mp \pi^\pm)_DK^\pm) \propto r_B^2 + (r_D^K)^2 + 2r_B r_D^K \cos (\delta_B + \delta_D^K \pm \gamma), \quad (2.1) $$

where $r_D^K$ and $\delta_D^K$ are analogous to the $B^\pm$ parameters $r_B$ and $\delta_B$; $r_D^K$ is the absolute ratio of the doubly Cabibbo suppressed (DCS) to Cabibbo favoured (CF) amplitudes and $\delta_D^K$ is the corresponding $D$ strong-phase difference. Furthermore, the extended method [6], which considers multi-body ADS modes i.e. $f = \{K^\pm \pi^\mp \pi^0, \ K^\pm \pi^\mp \pi^\mp \pi^\mp \}$, introduces an additional $D$ parameter, $R_f$, the coherence factor:

$$ \Gamma(B^\pm \rightarrow (\bar{f})_DK^-) \propto r_B^2 + (r_D^K)^2 + 2r_B r_D^K R_f \cos (\delta_B + \delta_D^K \pm \gamma), \quad (2.2) $$

where $R_f$ satisfies the condition $\{R_f \in \mathbb{R} | 0 < R_f \leq 1 \}$. This dilution term results from accounting for the resonant sub-structure of the multi-body mode. For modes whose intermediate resonances interfere constructively, $R_f$ tends to unity, however if the resonances interfere destructively, then $R_f$ tends to zero.

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For a review of all these methods, and a summary of current and future $B^\pm \rightarrow DK^{\pm}$ $\gamma$ measurements, see Refs. [7] and [8].
3. Quantum Correlations at the $\psi(3770)$

Determination of strong-phase differences and coherence factors can be made from analysis of quantum-correlated $D^0\bar{D}^0$ pairs. Such an entangled state, with $C = -1$, is produced in $e^+e^-$ collisions at the $\psi(3770)$ resonance. To conserve this charge-conjugation state, the final state of the $D^0\bar{D}^0$ pair must obey certain selection rules. For example, both $D^0$ and $\bar{D}^0$ cannot decay to $CP$-eigenstates with the same eigenvalue. However, decays to $CP$-eigenstates of opposite eigenvalue are enhanced by a factor of two. More generally, final states that are accessible by both $D^0$ and $\bar{D}^0$ (such as $K^-\pi^+$) are subject to similar interference effects. Consequently, by considering time-integrated decay rates of double tag (DT) events, where both the $D^0$ and the $\bar{D}^0$ are reconstructed, one is sensitive to interference dependent parameters such as strong-phases and coherence factors. Furthermore, these decay rates are also sensitive to charm mixing. Charm mixing is described by two dimensionless parameters: $x \equiv (M_1 - M_2)/\Gamma$ and $y \equiv (\Gamma_1 - \Gamma_2)/2\Gamma$, where $M_{1,2}$ and $\Gamma_{1,2}$ are the masses and widths, respectively, of the neutral $D$ meson $CP$-eigenstates. The explicit dependence on the mixing parameters can be seen by considering the generalised, time-integrated, DT rate. That is, for a $D^0\bar{D}^0$ pair decaying to the final state $(f,g)$:

$$\Gamma(f|g) = Q_M|A_f\bar{A}_g - \bar{A}_fA_g|^2 + R_M|A_fA_g - \bar{A}_f\bar{A}_g|^2,$$

where $A_i \equiv \langle iD^0 \rangle$, $\bar{A}_i \equiv \langle i\bar{D}^0 \rangle$. The coefficients $Q_M$ and $R_M$ posses the dependence on the mixing parameters, where $Q_M \equiv 1 - (x^2 - y^2)/2$ and $R_M \equiv (x^2 + y^2)/2$ [11].

3.1 Probing strong-phases and coherence factors

Letting $f$ represent the signal $D$ decay of interest, it is possible to obtain access to strong-phases and coherence factors by considering specific states of the ‘tag’, $g$. As an example, we demonstrate here how sensitivity to strong-phases can be obtained by considering $g$ to be in a $CP$-eigenstate with eigenvalue $\lambda_{CP}$. For the purpose of this discussion, we simplify the problem by ignoring $D$-mixing effects, i.e. $x, y \to 0$. In this scenario, $Q_M \to 1$, $R_M \to 0$. Consequently, for $f = K^-\pi^+$, Eqn.(3.1) reduces to:

$$\Gamma(K^-\pi^+|CP) \propto |A_{K\pi}A_{CP} - \bar{A}_{K\pi}A_{CP}|^2 = |A_{K\pi}|^2|A_{CP}|^2(1 + \langle fK^\pi \rangle^2)^2 - 2\lambda_{CP}\Gamma_{K^\pi}^0 \cos(\delta_{K^\pi}^0)) \tag{3.2}.$$  

Therefore, with a knowledge of $|A_{K\pi}|$, $|A_{CP}|$ and $\Gamma_{K^\pi}^0$, the observed asymmetry between the rates for $\lambda_{CP} = +1$ and $\lambda_{CP} = -1$ provides direct sensitivity to $\cos(\delta_{K^\pi}^0)$. When a multi-body signal mode is considered, such as $f = \{K^+\pi^-\pi^0,\ K^\pm\pi^\mp\pi^\mp\}$, the amplitude $A_f$ must be integrated over all phase-space. This has the effect of modifying Eqn. (3.2) through the transformation $\cos(\delta_{K^\pi}^0) \to R_f \cos(\delta_{K^\pi}^0)$. Therefore, for $f = K^-\pi^+\pi^0$:

$$\Gamma(K^-\pi^+\pi^0|CP) = |A_{K\pi\pi}|^2|A_{CP}|^2(1 + \langle fK^\pi\pi^0 \rangle^2)^2 - 2\lambda_{CP}\Gamma_{K^\pi\pi^0}^0 \Gamma_{K^\pi\pi^0}^0 \cos(\delta_{K^\pi\pi^0}^0)) \tag{3.3}.$$  

To give a more concrete overview, expressions from evaluating Eqn. (3.1) are listed in Table 1 for various tag modes against $f = K^-\pi^+$. As is demonstrated in Ref.[9], while $|A_{K\pi}|^2$ has direct correspondence to the CF branching fraction $\langle \overline{B}_{K^\pi}^C \rangle$, $|\bar{A}_{K\pi}|^2$ and $|A_{CP}|^2$ possess dependence on the mixing parameters $x$ and $y$, i.e. $|A_{K\pi}|^2 = \overline{B}_{K^\pi}^C(1 + \theta(x,y))$. Consequently, a linear dependence on $x$ and $y$ is observed in some of the quantum correlated branching fractions quoted in Table 1.
This information is gathered by averaging results of single-tagged yields at the mixing parameters are used as constraints. All correlations amongst the inputs are accounted for. The signal is identified using two kinematic variables: the beam-constrained mass, $M$, described in Ref. [12]. The analysis finds a result of $\delta_{D}^{K\pi} = (22_{-12}^{+11}+9)^{\circ}$ from using 281 pb$^{-1}$ of data, which is the first direct determination of this phase [13]. An updated result following analysis of the full 818 pb$^{-1}$ dataset is in preparation.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Relative Correlated Branching Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+\pi^+ + K^-\pi^+$</td>
<td>$R_M$</td>
</tr>
<tr>
<td>$K^+\pi^+ + K^+\pi^-$</td>
<td>$(1 + R_W)^2 - 4R_W \cos \delta_D^{K\pi}(r \cos \delta_D^{K\pi} + y)$</td>
</tr>
<tr>
<td>$K^-\pi^+ + CP\pm$</td>
<td>$1 + R_{WS} \pm 2r \cos \delta_D^{K\pi} + y$</td>
</tr>
<tr>
<td>$K^-\pi^+ + e^-$</td>
<td>$1 - r y \cos \delta_D^{K\pi} - rx \sin \delta_D^{K\pi}$</td>
</tr>
<tr>
<td>$CP\pm + CP\pm$</td>
<td>$0$</td>
</tr>
<tr>
<td>$CP^+ + CP^-$</td>
<td>$4$</td>
</tr>
<tr>
<td>$CP\pm + e^-$</td>
<td>$1 \pm y$</td>
</tr>
</tbody>
</table>

Table 1: Correlated ($C = -1$) effective $D^0\bar{D}^0$ branching fractions to leading order in $x$, $y$ and $r^2$. The rates are normalised to the multiple of the uncorrelated branching fractions. Some rates show dependence to the wrong-sign rate ratio, $R_{WS} = r^2 + ry^2 + R_M$, where $y' = (y \cos \delta_D^{K\pi} - x \sin \delta_D^{K\pi})$.

4. CLEO-c

All measurements presented are made with $e^+e^- \rightarrow \psi(3770)$ data accumulated at the Cornell Electron Storage Ring (CESR). The CLEO-c detector was used to collect these data. Details of the experiment can be found elsewhere [10]. The total integrated luminosity of the data is 818 pb$^{-1}$, however, only 281 pb$^{-1}$ have been used so far for the measurement of $\delta_{D}^{K\pi}$ presented in Sec. 5.

5. Measurement of the strong-phase difference in $D \rightarrow K^-\pi^+$

The first analysis presented is that of the strong-phase difference in $D \rightarrow K^-\pi^+$. Implementing the method described in Ref. [11], this analysis has performed the first measurements of $y$ and $\cos(\delta_D^{K\pi})$ in quantum-correlated $\psi(3770)$ data. By comparing the correlated event yields, whose rates are listed in Table 1, with the uncorrelated expectations, we are able to extract $r^2$, $r \cos(\delta_D^{K\pi})$, $y$ and $x^2$. To achieve this, a knowledge of the relevant uncorrelated branching-ratios are needed. This information is gathered by averaging results of single-tagged yields at the $\psi(3770)$ with external measurements using incoherently-produced $D^0$ mesons. In addition, to extract $\cos(\delta_D^{K\pi})$ from $r \cos(\delta_D^{K\pi})$, knowledge of $r$ is required. This necessary information is obtained by including $R_{WS}$ and $R_M$ as external inputs to the least-squares fit. Furthermore, external measurements of the mixing parameters are used as constraints. All correlations amongst the inputs are accounted for.

The analysis has considered a total of seven CP-eigenstates reconstructed against the $K^\pm\pi^\mp$ signal mode: $K^+K^-, \pi^+\pi^-, K^0\pi^0, K^0\omega, K^0\pi^0\pi^0, K^0\eta$ and $K^0\pi^0$. In those DTs without a $K^0\pi^0$, the signal is identified using two kinematic variables: the beam-constrained mass, $M \equiv \sqrt{E_{Beam}^2 - p_D^2}$, and $\Delta E \equiv E_D - E_{Beam}$, where $E_{Beam}$ is the beam energy, $p_D$ and $E_D$ are the $D^0$ candidate momentum and energy, respectively. The reconstruction of $K^0\pi^0$ events utilises the missing-mass technique described in Ref. [12]. The analysis finds a result of $\delta_{D}^{K\pi} = (22_{-12}^{+11}+9)^{\circ}$ from using 281 pb$^{-1}$ of data, which is the first direct determination of this phase [13]. An updated result following analysis of the full 818 pb$^{-1}$ dataset is in preparation.
6. Measurement of the coherence factor and average strong-phase difference in $D \rightarrow K^{\pm} \pi^{+} \pi^{0}$ and $D \rightarrow K^{\pm} \pi^{+} \pi^{+} \pi^{-}$

Determination of the average strong-phase difference and associated coherence factors for the modes $f = \{K\pi\pi^{0}, K3\pi\}$ have been made using an analogous technique to that described in Sec. 5 [14]. As shown in Eqn.(3.3), $CP$-tagged multi-body rates provide sensitivity to the product $R_{f} \cos(\delta^{f}_{D})$. A means of decoupling these parameters fortunately comes from considering the rate $\Gamma_{f}$. Evaluating Eqn.(3.1) for $g = f$, one obtains:

$$\Gamma_{f} = Q_{M}|A_{f}|^{2}|\bar{A}_{f}|^{2} \left(1 - (R_{f})^{2}\right) + |A_{f}|^{4}R_{M} \left(1 - 2(R_{f})^{2} + (R_{f})^{4}\right).$$

(6.1)

In the case of the two-body mode, $f = K^{\pm} \pi^{\mp}$, $R_{f} = 1$ and Eqn.(6.1) reduces to $|A_{f}|^{4}R_{M}$ as quoted in Table 1. However, for multi-body final states, one observes that $(1 - R_{f}^{2})$ is the leading term in Eqn.(6.1). Consequently, the rate $\Gamma_{f}$ provides direct sensitivity to $R_{f}$ and allows for a decoupling of the parameters. All the $CP$-tags listed in Sec. 5 are employed in this analysis, as well as $K_{0}^{0}\phi$, $K_{S}^{0}\eta'$ and $K_{L}^{0}\omega$.

As was done in the $K^{\pm} \pi^{\mp}$ analysis, a least-squares fit has been used to extract both mixing and strong-phase parameters. Likelihood contours in $R_{f}$, $\delta^{f}_{D}$ parameter space are shown in Fig. 1(a) for $f = K\pi\pi^{0}$, and Fig. 1(b) for $f = K3\pi$. The best-fit values of the coherence factors and average strong-phases are $R_{K\pi\pi^{0}} = 0.84 \pm 0.07$, $\delta^{K\pi\pi^{0}} = (227^{+14}_{-17})^{\circ}$, $R_{K3\pi} = 0.33^{+0.20}_{-0.23}$ and $\delta^{K3\pi} = (114^{+28}_{-23})^{\circ}$. The uncertainties quoted are a combination of statistical and systematic errors.

![Figure 1](image-url)  
**Figure 1:** The limits determined on (a) $(R_{K\pi\pi^{0}}, \delta^{K\pi\pi^{0}})$ and (b) $(R_{K3\pi}, \delta^{K3\pi})$ at the 1, 2 and 3$\sigma$ levels.

The results show significant coherence for $D^{0} \rightarrow K\pi\pi^{0}$, but much less so for $D^{0} \rightarrow K3\pi\pi$. These results will improve the measurement of $\gamma$ and the amplitude ratio $r_{B}$ in $B^{\pm} \rightarrow DK^{\pm}$, where the $D$ decays to $K\pi\pi^{0}$ and $K3\pi\pi$. Earlier preliminary results of $R_{K3\pi}$ and $\delta^{K3\pi}$ [15] combined with CLEO-c’s measurement of $\delta^{K3\pi}$ were shown to improve the expected sensitivity to $\gamma$ at LHCb in a combined ADS analysis of $K\pi$ and $K3\pi\pi$ final states by up to 40% [16].

7. Measurement of strong-phase variations in $D \rightarrow K_{S}^{0} \pi^{+} \pi^{-}$

The current best constraints on $\gamma$ come from measurements in $B^{\pm} \rightarrow D(K_{S}^{0}\pi^{+}\pi^{-})K^{\pm}$ and re-
lated modes [17, 18] by performing likelihood fits to the $K_S^0\pi^+\pi^-$ Dalitz plot [4]. These fits require
models to represent the $D^0 \rightarrow K_S^0\pi^+\pi^-$ resonant amplitude structure. Since these models are based
on certain assumptions, an inherent systematic uncertainty is associated with this technique. Current estimates predict this error to be between $5^\circ$ and $9^\circ$, meaning the $\gamma$ measurement would soon become systematically limited at the next generation of flavour-physics experiment. However, an alternative model-independent method has been proposed where events are counted in specified regions of the $K_S^0\pi^+\pi^-$ Dalitz plot [4, 5], thus eliminating the model-uncertainty. This method relies on necessary strong-phase parameters having been determined at CLEO-c.

As Dalitz plot variables we use the invariant-mass squared of the $K_S^0\pi^-$ and $K_S^0\pi^+$ pairs, which we label as $s_-$ and $s_+$, respectively. The strong-phase at a given point in the $K_S^0\pi^+\pi^-$ Dalitz plot is then $\delta_D (s_-, s_+)$. For the phase difference between $D^0 \rightarrow K_S^0\pi^+\pi^-$ and $\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-$ at the same point in the Dalitz plot, we define

$$\Delta \delta_D \equiv \delta_D (s_-, s_+) - \delta_D (s_+, s_-). \quad (7.1)$$

The quantities measured by CLEO-c that provide input to the model-independent $\gamma$ determination are the averages of $\cos (\Delta \delta_D)$ and $\sin (\Delta \delta_D)$ in the $i$th Dalitz plot bin. We denote these terms $c_i$ and $s_i$, respectively. In a completely analogous manner to the analyses presented in Secs. 5 and 6, $c_i$ can be determined from $CP$-tagged decay rates, while $s_i$ is extracted from considering the double Dalitz plot of $K_S^0\pi^+\pi^-$ vs. $K_S^0\pi^+\pi^-$. Furthermore, additional constraints on $c_i$ and $s_i$ are obtained through $K_0^L\pi^+\pi^-$ events.

The choice of Dalitz plot binning affects the statistical precision of the analysis. It has been demonstrated in Ref. [5] that it is beneficial to choose bins such that $\Delta \delta_D$ varies as little as possible across each bin. The binning used in this analysis, with eight-pairs of bins uniformly dividing $\Delta \delta_D$ over the range $[0, 2\pi]$, is shown in Fig. 2(a). The location of these bins in phase space are chosen based on the BaBar isobar model given in Ref. [19].
The values of \(c_i\) and \(s_i\) from the combined analysis of \(K_0^0\pi^+\pi^-\) and \(K_L^0\pi^+\pi^-\) tagged events are shown graphically in Fig 2(b). When used as input to the \(\gamma\) measurement, these results are expected to replace the current model uncertainty of \(5^\circ \sim 9^\circ\) with an uncertainty due to the statistically dominated error on \(c_i\) and \(s_i\) of \(1.7^\circ\) [20].

8. Conclusion

The importance of CLEO-c’s quantum-correlated \(\psi(3770)\) dataset in the context of measuring the CKM angle \(\gamma\) has been described. Analysis of a variety of two- and multi-body \(D^0\) decays with these data have provided vital measurements of \(D^0\) strong-phases, and associated parameters, for model-independent \(\gamma\) measurements at LHCb. In addition to the modes presented here, results are in preparation for other promising final states, such as \(D^0 \to K_S^0K^+K^-\).

References