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Nonperturbative QCD Methods for B-Physics: Status and Prospects

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I briefly overview the nonperturbative QCD calculations of hadronic matrix elements for exclusive B-decays, concentrating on $B \to \tau\nu_\tau$ and $B \to \pi l \nu_l$. Currently, there is some tension between the decay constant f_B calculated in QCD and the one extracted from the experimental width $B \to \tau\nu_\tau$, if $|V_{ub}|$ determined from $B \to \pi l \nu_l$ is used. In this respect, a potentially interesting channel is $B \to \pi\tau\nu_\tau$.

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1. Introduction

In this short talk it is impossible to cover all important applications of nonperturbative QCD to heavy-flavour physics. I will only discuss the leptonic and exclusive semileptonic B-meson decays. The data on these processes obtained in recent years at B factories (see, e.g. [1, 2]), provide essential information on flavour-changing weak transitions. In what follows, I will mainly discuss the calculation of hadronic matrix elements relevant for the two important channels: $B \to \tau\nu_\tau$ and $B \to \pi\ell\nu_\ell$, where the $b \to u$ transition is probed.

The B-meson is characterized by an interplay of two different scales: the b-quark mass $m_b \gg \Lambda_{QCD}$ and the "binding energy" of the b quark, $\bar{\Lambda} = m_B - m_b$, of order of a few hundreds of MeV. Since $\alpha_s(\bar{\Lambda})$ is too large, perturbative QCD is not an adequate tool for the B-meson and its exclusive transitions. Moreover, since the valence light quark in the B is relativistic, it is not possible to introduce a quark-antiquark potential and/or wave function. The B-meson is a bound state of a heavy quark and light quark-antiquark-gluon "cloud", and its properties are essentially determined by long-distance, i.e., by nonperturbative QCD dynamics. Hence, in order to calculate, for example, the $B \to \tau\nu$ decay constant f_B defined via the hadronic matrix element of the $b \to u$ weak current

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 b|B(p_B)\rangle = ip_{B_\mu} f_B\,, \tag{1.1}$$

one needs nonperturbative QCD methods. In fact, long-distance dynamics is important not only for the initial B-state in the hadronic matrix element (1.1), but also for the final vacuum (hadronless) state. The QCD vacuum contains fluctuations of quark-antiquark and gluon fields, with nonvanishing vacuum averages, such as

$$\langle 0|\bar{q}q|0\rangle \neq 0 \;\; (q = u,d,s)\,, \qquad \langle 0|G^a_{\mu\nu}G^{a\mu\nu}|0\rangle \neq 0\,, \qquad \langle 0|\bar{q}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^{a\mu\nu}q|0\rangle \neq 0\,, \tag{1.2}$$

known as quark-, gluon- and quark-gluon-condensate densities, respectively.

A well established and continuously developing approach to nonperturbative quark-gluon dynamics is provided by the simulation of QCD in a discretized 3$\oplus$1-dimensional space with fixed spacing, known as *lattice QCD*. This approach allows one to calculate various hadronic amplitudes in a form of Euclidean path integrals, evaluated numerically using the Monte-Carlo methods. Recent progress in the lattice QCD computations of f_B and other heavy-light hadronic matrix elements is overviewed in [3, 4, 5]. In [5] a detailed discussion of uncertainties of these calculations can be found.

Turning to "non-lattice" QCD tools, I will discuss in more detail *QCD sum rules* [6]. With this method, an approximate analytical calculation of f_B is possible, combining the perturbative expansion with universal nonperturbative input in a form of vacuum condensates (1.2). *Light-cone sum rules (LCSR)* [7] is a similar approach with a different nonperturbative input, allowing one to calculate hadron $\to$ hadron transition matrix elements, such as $B \to \pi$ form factors.

Beyond the scope of this talk remain the applications to B decays of various effective theories derived from QCD in a form of expansions in some inverse large or small mass/energy scale, such as HQET (heavy-quark effective theory), SCET (soft-collinear effective theory) and ChPT (chiral perturbation theory).

PoS(BEAUTY 2009)045

2. f_B from correlation function

Let me briefly outline the calculation of the B-meson decay constant from QCD sum rule. In this approach, one employs a specially "designed" correlation function of two $b \to u$ currents. A convenient choice is the quark current $j_5 = (m_b + m_u)\bar{u}i\gamma_5 b$ (the divergence of the weak axial current in (1.1)). Correspondingly, the correlation function is defined as

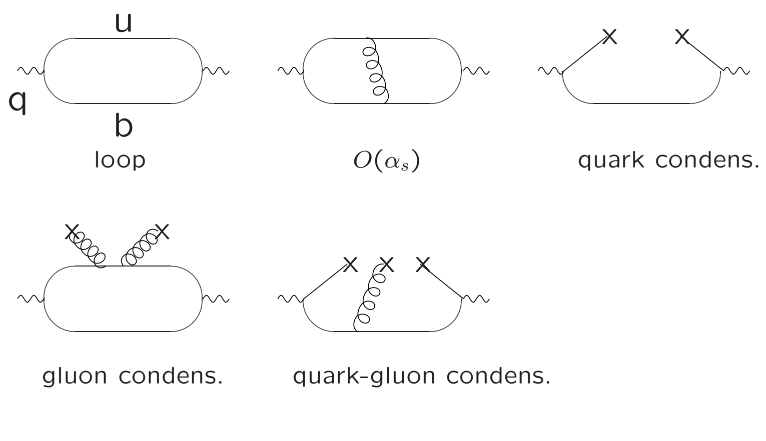
$$\Pi_5(q^2) = i\int d^4x\, e^{iqx}\langle 0|T\{j_5(x)j_5^\dagger(0)\}|0\rangle\,. \tag{2.1}$$

This function of external 4-momentum squared q^2 can be interpreted as a quantum amplitude of emitting and absorbing a $\bar{u}b$ pair in QCD vacuum.

At timelike $q^2 = m_B^2$, the amplitude (2.1) describes the emission and absorption of a real, on-shell B-meson, and has a resonance (pole) form with a residue proportional to f_B^2. (Note that $\langle 0|j_5|B\rangle = m_B^2 f_B$.) Increasing q^2 above m_B^2, one encounters excited and multiparticle hadronic states with the quantum numbers of B . Altogether, the sum of hadronic states contributing to $\Pi_5(q^2)$ is cast in a form of the dispersion relation, schematically:

$$\Pi_5(q^2) = \frac{m_B^4 f_B^2}{m_B^2 - q^2} + \sum_{B_{exc}} \frac{\langle 0|j_5|B_{exc}\rangle\langle B_{exc}|j_5^\dagger|0\rangle}{m_{B_{exc}}^2 - q^2}\,. \tag{2.2}$$

A calculation of the correlation function $\Pi_5(q^2)$ in QCD is possible at spacelike $q^2 \ll m_b^2$, where the propagating quarks are highly virtual and the integration in (2.1) is concentrated at short distances $x \sim 1/\sqrt{m_b^2 - q^2}$. Due to smallness of the running quark-gluon coupling α_s at short distances, the quarks in the correlation function are quasi-free. The correlation function is evaluated applying the *operator product expansion (OPE)*. In terms of QCD diagrams, OPE includes the loop diagram, perturbative $O(\alpha_s)$, $O(\alpha_s^2)$ corrections (where α_s is normalized at a large scale of order $\sqrt{m_b\Lambda}$) and the diagrams of quark and gluon condensates suppressed by inverse powers of $m_b^2 - q^2$. Different types of diagrams contributing to $\Pi_5(q^2)$ are shown in the following figure:



Calculated in terms of m_b, α_s and condensate densities, the sum of these diagrams determine the l.h.s. of the dispersion relation (2.2). The sum over excited states on r.h.s. of (2.2) is estimated using quark-hadron duality (for more details see e.g., the reviews [8, 9]). Finally, one obtains an

method	ref.	f_B [MeV]	f_{B_s} [MeV]	Group
QCD SR	[10]	210 ± 19	244 ± 21	
	[11]	206 ± 20	-	
		190 ± 13	231 ± 15	HPQCD I
lattice QCD	[4]	195 ± 11	243 ± 11	Fermilab/MILC
		203 ± 17	247 ± 16	ETMC
exp.average $\oplus$ $\lvert V_{ub}\rvert$	[1]	$280 \pm [30]_{exp} \pm [30]_{Vub}$	-	BABAR$\oplus$ Belle

Table 1: QCD results for f_B compared with the value extracted from experiment.

approximate analytic expression for f_B. The duality approximation introduces a sort of systematic error in this calculation, which is put under control by fixing the measured mass of B meson from the same sum rule. The results for f_B and f_{B_s} (including the s-quark mass effects) presented in Table 1 have been obtained from QCD sum rules with $O(\alpha_s^2)$ accuracy quite some time ago [10, 11].

The advantage of the sum rule method is its accessibility: using the formulae in [10] I was able to reproduce the same result. Some minor improvements and updates of input parameters are still possible, e.g., using the quite accurate value of b-quark $\overline{MS}$ mass extracted from the bottomonium QCD sum rules in [12]. A knowledge of the masses of radially excited B states, contributing to the hadronic sum in (2.2), can also improve the accuracy of the duality approximation. However, the overall uncertainty of this calculation can hardly be decreased below $O(10\%)$ level.

The sum rule predictions are in agreement with the lattice QCD results for f_B presented in Table 1 and taken from the recent review talk [4]. In future, lattice calculation of f_B is expected to become more precise, e.g., according to [13], a $(1.0-1.5\%)$ accuracy can be achieved.

Another possibility is to avoid the duality approximation and use the positivity of the sum on r.h.s. of (2.2) which yields an upper bound

$$f_B < 270 \text{ MeV}. \tag{2.3}$$

I use the same method as in [14] where the upper bound for the $D_{(s)}$- meson decay constants f_{D_s} was obtained from the correlation function with c-quark currents, similar to (2.1). (A more detailed analysis will be presented elsewhere.) In [14] it was mentioned that the bound for f_B is not constraining, simply because the value (2.3) considerably overshoots the QCD predictions presented in Table 1. However, the current central value of f_B extracted from the $B \to \tau\nu_\tau$ width seems to violate this bound. The interval presented in the last line of Table 1 is calculated from the average over BABAR and Belle measurements [1]: $BR(B \to \tau\nu_\tau) = [1.73 \pm 0.35] \times 10^{-4}$ and employing $|V_{ub}| = [3.5^{+.15}_{-.14}] \times 10^{-3}$, a value which is in agreement with the CKM fit [15] and with determination from the $B \to \pi l \nu_l$ decay (see next section).

Summarizing, there is some tension between QCD predictions and the $B \to \tau\nu_\tau$ width. But I believe, one has to be patient, having in mind that a similar tension for f_{D_s} is gradually being resolved (for the current status see, e.g., [16]).

3. $B \to \pi$ form factors

Another well studied $b \to u$ transition is $B \to \pi l \nu_l$. Its hadronic matrix element:

$$\langle \pi^-(p_\pi) \mid \bar{u}\gamma_\mu b \mid \bar{B}^0(p_B)\rangle = f^+_{B\pi}(q^2)\Big(p_{B\mu} + p_{\pi\mu} - \frac{m_B^2 - m_\pi^2}{q^2}q_\mu\Big) + f^0_\pi(q^2)\frac{m_B^2 - m_\pi^2}{q^2}q_\mu\,, \qquad (3.1)$$

where $q = p_B - p_\pi$ is the momentum transfer, contains two form factors and $f^0_{B\pi}(0) = f^+_{B\pi}(0)$. The knowledge of the vector form-factor $f^+_{B\pi}(q^2)$ is sufficient for the extraction of $|V_{ub}|$ from $B \to \pi\ell\nu_\ell$, when $l = \mu, e$, because the contribution of the scalar form factor to the semileptonic width is suppressed by $O(m_\ell^2)$. Before one starts to calculate $f^+_{B\pi}(q^2)$ using any QCD method, the analytical properties of this function and certain bounds [17, 18] obtained from a correlation function similar to (2.1) allow one to express the q^2-dependence in terms of a few parameters (*z-series parameterization*). The form factor shape is also directly measured from the decay distribution in q^2 (see e.g.[2]). Hence, one basically needs a normalization of $f^+_{B\pi}(q^2)$ at some fixed value q^2.

The lattice QCD results for $f^{+,0}_{B\pi}(q^2)$ are available at large $q^2 \geq 15$ GeV^2. At small momentum transfers a "non-lattice" LCSR approach is used [19, 20, 21]. In this method a correlation of two currents $\bar{u}\gamma_\mu b$ and j_5 is taken between the vacuum and on-shell pion state and expressed via hadronic sum:

$$\int d^4x e^{iqx}\langle \pi(p_\pi)|T\{\bar{u}\gamma_\mu b(x) j_5(0)\}|0\rangle = \frac{\langle \pi|\bar{u}\gamma_\mu b|B\rangle\langle B|j_5|0\rangle}{m_B^2 - (p_\pi + q)^2} + \sum_{B_{exc}} \frac{\langle \pi|\bar{u}\gamma_\mu b|B_{exc}\rangle\langle B_{exc}|j_5|0\rangle}{m_{B_{exc}}^2 - (p_\pi + q)^2}. \qquad (3.2)$$

The ground-state B contribution contains the $B \to \pi$ form factors multiplied by f_B.

At $q^2, (p_\pi + q)^2 \ll m_b^2$ the correlation function (3.2) is calculable in terms of light-cone OPE, i.e., as a sum of QCD diagrams where calculable short-distance parts related to the virtual b-quark propagation are convoluted with the vacuum-pion matrix elements of the type $\langle \pi|\bar{u}(x)\Gamma_a d(0)|0\rangle$, $\langle \pi|\bar{u}(x)G^a_{\mu\nu}\frac{\lambda^a}{2}\Gamma_b^{\mu\nu}d(0)|0\rangle$ ($\Gamma_{a,b}$ are combinations of γ-matrices). These matrix elements are cast in a form of universal functions, the pion distribution amplitudes (DA's), which play the role of nonperturbative input in this approach. I skip many important details which can be found in the reviews [8, 9]. The most recently updated LCSR calculation [21] of $B \to \pi$ form factors was used to extract $|V_{ub}|$ from $B \to \pi l \nu_l$ data. This result, together with the lattice QCD and other determinations, is presented in Table 2. The same LCSR method and input was used in our recent calculation [22] of $D \to \pi, K$ form factors (replacing b by c in the correlation function) and the results are in a good agreement with lattice QCD. Another check is provided by the pion e.m. form factor obtained from LCSR for spacelike momentum transfers [23] and compared with the currently available data in [24].

Note that the $B \to \pi$ form factors obtained from LCSR are not only in agreement with the lattice QCD results, but also have comparable uncertainties. This will change, when the lattice calculations achieve their future goal of $\sim 2-3\%$ accuracy for $B \to \pi$ form factors [13].

Having in mind the situation with $B \to \tau\nu_\tau$, described in the previous section, it will be interesting, although difficult, to investigate the $B \to \pi\tau\nu_\tau$ channel, where the scalar $b \to u\tau\nu_\tau$ transition is probed in the (unsuppressed) contribution to the decay width. This contribution is determined by the scalar form factor $f^0_{B\pi}(q^2)$ at $q^2 \geq m_\tau^2$, predicted [21] in the same LCSR approach and with the

ref.	$f^+_{B\pi}(q^2)$ calculation	$f^+_{B\pi}(q^2)$ input	$\lvert V_{ub}\rvert \times 10^3$
[25]	lattice	-	3.38±0.36
[26]	lattice	-	3.55±0.25±0.50
[20]	LCSR	-	$3.5\pm0.4\pm0.1$
[27]	-	lattice $\oplus$ LCSR	$3.47\pm0.29\pm0.03$
[21]	LCSR	-	$3.5\pm0.4\pm0.2\pm0.1$
[18]	-	lattice$\oplus$ LCSR	3.54 ± 0.24

Table 2: $|V_{ub}|$ determination from $B\to\pi\ell\nu_\ell$.

same input as $f^+_{B\pi}(q^2)$. Hence a combined observable

$$\frac{d\Gamma(B\to\pi\tau\nu_\tau)/dq^2}{d\Gamma(B\to\pi\mu\nu_\mu)/dq^2}=\frac{(q^2-m_\tau^2)^2}{(q^2)^2}\Big(1+\frac{m_\tau^2}{2q^2}\Big)\times\Bigg\{1+\frac{3m_\tau^2(m_B^2-m_\pi^2)^2}{4(m_\tau^2+2q^2)m_B^2p_\pi^2}\frac{|f^0_{B\pi}(q^2)|^2}{|f^+_{B\pi}(q^2)|^2}\Bigg\}, \quad (3.3)$$

where m_μ is neglected, and p_π is the pion momentum in B rest frame, can be measured and compared with r.h.s. predicted in SM. Note that this observable is independent of V_{ub} and only depends on the ratio of the two $B\to\pi$ form factors. In any of QCD methods, this ratio has a smaller uncertainty than the individual form factors.

Concluding this brief discussion, I am convinced that in future, the "non-lattice" nonperturbative methods, such as QCD sum rules and LCSR will remain useful practical tools for various exclusive B and D decays and will complement the lattice QCD studies.

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References

[1] T.Iijima, talk at XXIV Int. Lepton-Photon Symposium, Hamburg (2009).

[2] J. Dingfelder, talk at this conference.

[3] V. Lubicz and C. Tarantino, Nuovo Cim. **123B** (2008) 674 [arXiv:0807.4605 [hep-lat]]; R. S. Van de Water, arXiv:0911.3127 [hep-lat].

[4] C. Aubin, arXiv:0909.2686 [hep-lat].

[5] R. Sommer, arXiv:0906.3790 [hep-ph].

[6] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147**, 385, 448 (1979).

[7] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. **B312** (1989) 509; V. M. Braun and I. E. Filyanov, Z. Phys. **C44** (1989) 157; V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. **B345** (1990) 137.

[8] P. Colangelo and A. Khodjamirian, In Shifman, M. (ed.): At the frontier of particle physics/Handbook of QCD, vol. 3, 1495-1576; [hep-ph/0010175].

[9] A. Khodjamirian and R. Ruckl, Adv. Ser. Direct. High Energy Phys. **15** (1998) 345 [arXiv:hep-ph/9801443].

[10] M. Jamin and B. O. Lange, Phys. Rev. D **65**, 056005 (2002) [arXiv:hep-ph/0108135].

[11] A. A. Penin and M. Steinhauser, Phys. Rev. D **65** (2002) 054006 [arXiv:hep-ph/0108110].

[12] K. G. Chetyrkin, J. H. Kühn, A. Maier, P. Maierhofer, P. Marquard, M. Steinhauser and C. Sturm, Phys. Rev. D **80** (2009) 074010 [arXiv:0907.2110 [hep-ph]].

[13] M. Bona *et al.*, arXiv:0709.0451 [hep-ex], Appendix A (Lattice QCD)

[14] A. Khodjamirian, Phys. Rev. D **79** (2009) 031503 [arXiv:0812.3747 [hep-ph]].

[15] J. Charles *et al.* [CKMfitter Group], Eur. Phys. J. C **41** (2005) 1; Moriond 2009 update at http://ckmfitter.in2p3.fr .

[16] The CLEO Collaboration, arXiv:0910.3602 [hep-ex].

[17] C. G. Boyd, B. Grinstein and R. F. Lebed, Phys. Rev. Lett. **74**, 4603 (1995);
L. Lellouch, Nucl. Phys. B **479**, 353 (1996);
M. C. Arnesen, B. Grinstein, I. Z. Rothstein and I. W. Stewart, Phys. Rev. Lett. **95** (2005) 071802;
T. Becher and R. J. Hill, Phys. Lett. B **633**, 61 (2006).

[18] C. Bourrely, I. Caprini and L. Lellouch, Phys. Rev. D **79** (2009) 013008.

[19] V. M. Belyaev, A. Khodjamirian and R. Rückl, Z. Phys. C **60** (1993) 349;
V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Rückl, Phys. Rev. D **51** (1995) 6177;
A. Khodjamirian, R. Rückl, S. Weinzierl and O. I. Yakovlev, Phys. Lett. B **410** (1997) 275;
E. Bagan, P. Ball and V. M. Braun, Phys. Lett. B **417**, 154 (1998).

[20] P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014015.

[21] G. Duplancic, A. Khodjamirian, T. Mannel, B. Melic and N. Offen, JHEP **0804**, 014 (2008).

[22] A. Khodjamirian, C. Klein, T. Mannel, N. Offen, arXiv:0907.2842 [hep-ph].

[23] V. M. Braun, A. Khodjamirian and M. Maul, Phys. Rev. D **61** (2000) 073004 [arXiv:hep-ph/9907495].
J. Bijnens and A. Khodjamirian, Eur. Phys. J. C **26** (2002) 67 [arXiv:hep-ph/0206252].

[24] A. Khodjamirian, arXiv:0909.2154 [hep-ph].

[25] J. A. Bailey *et al.*, Phys. Rev. D **79** (2009) 054507.

[26] E. Dalgic *et al.* , Phys. Rev. D **73**, 074502 (2006) [Erratum-ibid. D **75**, 119906 (2007)].

[27] J. M. Flynn and J. Nieves, Phys. Rev. D **76**, 031302 (2007).

PoS(BEAUTY 2009)045