$B_d \rightarrow K^{*0} \mu^+ \mu^-$ at LHCb

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$\bar{B}_d \rightarrow \bar{K}^{*0} \mu^+ \mu^-$ is a rare electroweak $b \rightarrow s$ penguin decay that has excellent sensitivity to physics beyond the Standard Model. LHCb should collect $6200^{+1700}_{-1500}$ signal events with a residual background of $1550 \pm 310$ for each nominal year of data taking. This allows for a comprehensive and exciting programme of physics measurements, the details of which will be reviewed in this article.
1. Introduction

As we enter the LHC era, we are confronted with the experimental fact that results from the TeVatron and the $B$-factories are, by and large, in agreement with Standard Model (SM) predictions. The working hypothesis of the LHC project is that there will be new physics (NP) at the TeV scale. However, considerations from flavour physics imply that the NP scale is much larger, assuming its flavour structure is generic. If these two observations are to be reconciled then the study of flavour will be of great interest at the LHC.

LHCb is a high precision experiment for the study of CP-violation and rare decays at the LHC [1]. Of particular interest will be the exclusive $b \rightarrow s$ decay mode $B_d \rightarrow K^{*0} \mu^+ \mu^-$. This is normally treated using the operator product expansion, where the Wilson coefficients $C^7, C^9, C^{10}$ dominate [2]. These have right-handed partners, $C'_{7,9,10}$, that are highly suppressed in the SM and in minimal flavour-violating models [3]. In the presence of NP, the values of these coefficients may change due to new heavy degrees of freedom in the loops. Measuring the Wilson coefficients then allows for entire classes of NP to be observed or excluded.

\begin{figure}[h]
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\includegraphics[width=0.5\textwidth]{kinematics.png}
\caption{Definition of kinematic variables in the decay $B_d \rightarrow K^{*0} \mu^+ \mu^-$ (e.g. Ref. [4]): The $z$-axis is the direction in which the $B$ meson flies in the rest frame of the $\mu^+ \mu^-$. $\theta_l$ is the angle between the $\mu^-$ and the $z$-axis in the $\mu^+ \mu^-$ rest frame, $\theta_K$ is the angle between the $K^-$ and the $z$-axis in the $K^*$ rest frame, and $\phi$ is the angle between the normals to the $\mu^+ \mu^-$ and $K \pi$ decay planes in the $B$ rest frame. In the case of the $B$, the angles are defined relative to the $\mu^+$ and the $K^+$.}
\end{figure}

The kinematics of the decay is described by three angles, $\theta_l$, $\theta_K$, and $\phi$, and the invariant mass squared of the muon pair, $q^2$. The decay angles are shown in Fig. 1 and defined in its caption. A widely-studied observable is the di-lepton forward-backward asymmetry [5],

$$A_{FB}(q^2) = \frac{\int_0^1 \frac{d^2 \Gamma}{dq^2 d \cos \theta_l} d \cos \theta_l - \int_{-1}^0 \frac{d^2 \Gamma}{dq^2 d \cos \theta_l} d \cos \theta_l}{\int_0^1 \frac{d^2 \Gamma}{dq^2 d \cos \theta_l} d \cos \theta_l + \int_{-1}^0 \frac{d^2 \Gamma}{dq^2 d \cos \theta_l} d \cos \theta_l}.$$ (1.1)

The value of $q^2$ for which $A_{FB}(q^2)$ is 0, known as the zero-crossing point, $q^2_{0}(A_{FB})$, has reduced hadronic uncertainties due to leading order form-factor (FF) cancellations [6].

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Figure 2: Recent results from BABAR (red) and BELLE (blue) for $A_{FB}$ (left) and $F_L$ (right) as a function of $q^2$, re-drawn from Refs. [7]. SM theoretical predictions are shown; the orange, light green, and dark green bands show the parametric and estimated 5\% and 10\% contributions from unknown higher order terms in the $1/m_b$ expansion, known as $\Lambda/m_b$ corrections [4]. The light purple bands show the rate-weighted [8] SM average in the region $q^2 \in [1\,\text{GeV}^2, 6\,\text{GeV}^2]$, with all uncertainties. The black points show LHCb 2 fb$^{-1}$ sensitivities using a simultaneous angular projection fit, following Ref. [9] and assuming the SM, where the central value is taken from a single toy experiment.

The SM distributions for $A_{FB}$ and $F_L$, the longitudinal polarization fraction of the $K^*$, can be seen in Fig. 2; theoretical uncertainties are not well controlled outside of the $q^2 \in [1\,\text{GeV}^2, 6\,\text{GeV}^2]$ region [2, 4, 10]. Measurements from both BABAR and BELLE [7] are shown for points that lie inside this theoretically clean region. For comparison, the $A_{FB}$ distributions for a number of NP models are shown in Fig. 3 (left). The current experimental uncertainties for both $A_{FB}$ and $F_L$ are still too large to make any definitive statements about deviations from the SM. The large increase in statistics from LHCb will clarify this situation; the estimated sensitivities for LHCb with 2 fb$^{-1}$ of integrated luminosity are shown as the black points in Fig. 2.

2. Physics Programme

Figure 3: Theoretical $A_{FB}$ and $S_5$ distributions in a number of models, re-drawn from Ref. [10]. The solid lines give the SM prediction. The dashed lines show predictions from a universal extra dimensions (UED) model, a non-minimal flavour violating supersymmetric model (GMSSM) and a flavour blind supersymmetric model (FBMSSM).
Making precision $B$-physics measurements in the LHC environment will be challenging but LHCb has been carefully optimized to make this possible [1]. The detector is expected to collect $6200^{+1700}_{-1500}$ $B_d \to K^{*0} \mu^+ \mu^-$ signal events with $1550 \pm 310$ background events for each nominal year of data taking (2 fb$^{-1}$) [11]. The dominant uncertainty on this yield estimate comes from the branching fraction, which is currently known to an accuracy of $\sim 15\%$ [12]. The large increase in statistics at LHCb relative to the previous generation of experiments allows for an ambitious physics programme to be undertaken [13]. A selection of measurements of the angular distribution are discussed below.

2.1 Measuring $A_{FB}$

The first major analysis target is to map out the $A_{FB}$ distribution and determine $q_0^2(A_{FB})$. This measurement can be made with relatively low integrated luminosity by counting the number of signal events in angular bins of $\theta_l$ to determine $A_{FB}$, following Eq. (1.1). An example of this is shown in Fig. 4 (left). Taking a particular FF model [14], this approach gives a projected uncertainty of $\sigma(q_0^2(A_{FB})) = 0.46\text{GeV}^2$ for $2\text{ fb}^{-1}$ of integrated luminosity [15]. However, the uncertainty is approximately proportional to the gradient of the $A_{FB}$ distribution, which is in turn dependent on the FFs found in nature; the actual experimental uncertainty found may differ from this estimate as the FFs are not well known.

![Figure 4: Left: Counting the number of forward and backwards signal events to determine $A_{FB}$ with 2 fb$^{-1}$ of integrated luminosity [15]. The data-set used was produced with the full LHCb detector simulation and a SM signal simulation following Ref. [14] ($M_{\mu\mu}^2 \equiv q^2$). A straight-line fit is used to extract $q_0^2(A_{FB})$. Right: Estimated sensitivity to $A_{FB}$ in the range $q^2 \in [1\text{GeV}^2, 6\text{GeV}^2]$ as extracted using a full angular analysis to 10 fb$^{-1}$ of toy Monte Carlo LHCb data, with the SM signal simulation following Refs. [2, 4]. The dashed black line shows the input SM distribution, while the solid red line is the median of a thousand toy fits. The 1σ and 2σ confidence levels are marked by the light and dark blue bands. The differing input calculations and FF distributions lead to the variations in gradient and $q_0^2(A_{FB})$ between the two figures.](image-url)

A particular challenge of making the $A_{FB}$ measurement will be dealing with detector and selection acceptance effects, particularly at low $q^2$. While a measurement of $q_0^2(A_{FB})$ should be safe from biases due to these, the rest of the $A_{FB}$ distribution can experience considerable distortion, leading to a systematic effect on $\sigma(q_0^2(A_{FB}))$ from the change in the $A_{FB}$ gradient induced. These effects must be controlled to an accuracy of $\sim 10\%$ if their contribution to the final uncertainty is to be small [13]. Fig. 5 (left) shows the effects on the $\theta_l$ efficiency of the LHCb detector geometry.
and reconstruction in the low $q^2$ region, $q^2 \in [1\text{ GeV}^2, 2\text{ GeV}^2]$. Many more events with $\theta_l$ close to 0 or $\pi$ are lost than those with $\theta_l$ close to $\pi/2$. In the former configuration, $\theta_l[0, \pi]$, events typically feature one high $p_T$ and one low $p_T$ muon; at low $q^2$ the lower momentum $\mu$ often fails to reach the muon chambers and is not reconstructed.

The events with $\theta_l[0, \pi]$ are particularly important for the $A_{FB}$ analysis, as they will show the largest asymmetry in $\theta_l$, and are expected to be produced much less frequently than those with $\theta_l \approx \pi/2$. Fig. 5 (middle) shows the effect of introducing a 300MeV $p_T$ cut on both muons. In this case the effect is disastrous; the vast majority of these significant events are lost. While these effects are less important at higher $q^2$, considerable care must be taken to avoid producing these kind of effects in the theoretically clean region. Fig. 5 (right) shows the effect of the LHCb trigger and offline selections on a sample of fully simulated signal events. It can be seen that there is little distortion introduced; the main source of acceptance effects is expected to be from the detector geometry and reconstruction.

**Figure 5:** Left: The effect of the detector geometry and reconstruction on the signal selection efficiency as a function of $\theta_l$ found from a full detector simulation. Middle: The effect of requiring that both muons have a $p_T > 300$MeV on the signal selection efficiency using a generator-level simulation of the decay kinematics. Right: The total effect on the signal selection efficiency from the complete trigger chain, relative to the detector geometry and reconstruction, found from a full detector and trigger simulation.

### 2.2 Beyond $A_{FB}$

Measurements involving the counting of signal events as a function of $\theta_l$ are attractive as they require a relatively modest understanding of the detector and backgrounds. However, there is much more information available in the decay which can be extracted at the price of a more challenging analysis. These measurements are important as they are complementary to $A_{FB}$ and provide sensitivity to Wilson coefficients beyond $C_7$ and $C_9$. Projections of the full angular distribution can be used to perform a simultaneous fit to the decay angles [9]. This gives additional sensitivity to $A_{FB}$ and $F_L$, as shown in Fig. 2, and to non-SM values of $C_7$ via a new observable, $A_T^{(2)}$ [3, 16]. Sensitivity to $C_7$ may also be gained via the observable $S_5$ [10], theoretical distributions for which are shown in Fig. 3 (right). $S_5$ features a zero-crossing point, the measurement of which may be attractive experimentally due to the steep gradient at this point for SM-like scenarios [17]. $S_5$ may
be extracted by counting the number of signal events as function of the decay angles $\theta_K$ and $\phi$, using the expression

$$S_5 = \frac{4}{3} \left[ \int_0^{\pi/2} + \int_{\pi/2}^{2\pi} - \int^{3\pi/2} \right] \partial \phi \left[ \int_0^1 - \int_{-1}^0 \right] \partial \cos \theta_K \frac{\partial^3 \Gamma}{\partial q^2 \partial \phi \partial \cos \theta_K} / \partial \Gamma;$$

however, controlling acceptance effects in the two angles will be a challenge.

Finally, it is possible to perform a full angular analysis [4, 18]. In this case all four experimental measurables are utilized to extract the underlying decay amplitudes. This allows for the measurement of additional observables which cannot be accessed in other ways. Fig. 6 shows the estimated LHCb sensitivity to the theoretically clean observables $A^{(3)}$ and $A^{(4)}$ for a simulated 10 fb$^{-1}$ data-set. In addition, significant improvement can be gained on $A_{FB}$ and $q_0^2(A_{FB})$. Fig. 4 (right) shows the expected sensitivity to $A_{FB}$ with 10 fb$^{-1}$ of integrated luminosity, giving $\sigma(q_0^2) = +0.18^{-0.16}$ GeV$^2$. A further factor of two improvement might be expected if the FF model from [14] had been used instead of that from [2], due the increase in the gradient at $q_0^2(A_{FB})$. This gradient change may be seen by comparing the central values of the right- and left-hand plots in Fig. 4 and is further explored in Ref. [18].

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{Experimental sensitivity bands (1$\sigma$ and 2$\sigma$ uncertainties are marked light and dark blue) for the theoretically clean observables $A^{(3)}$ and $A^{(4)}$ for 10 fb$^{-1}$ of LHCb data assuming a supersymmetric model [4, 16]. The dashed blue line shows the distribution taken as input, while the solid red line is the median of a thousand toy fits. The SM theoretical distributions are also shown with the same colour scheme as in Fig. 2. These two distributions must be statistically distinguishable if an observation of NP is to be made.}
\end{figure}

2.3 CP-Violation

LHCb reconstructs $B_d \rightarrow K^{*0} \mu^+ \mu^-$ through the charged decays of the $K^{*0}$ so the sign of the $K$ tags the $B$ flavour. Both model dependent [10] and model independent [19] considerations indicate that significant non-SM-like CP-violation is possible in the decay though the addition of complex phases in the Wilson coefficients. These remain poorly constrained and could in principle provide a source of CP-violation beyond the CKM mechanism. The SM contribution of CP-violation is very small, coming from doubly-Cabibbo suppressed diagrams; any CP-violation seen in this decay would be a robust sign of NP. Furthermore, the transformation of some parts of the angular distribution under $T$-parity means that $q^2$-integrated measurements can be made without separating the data into $B$ and $\bar{B}$ samples [19]. Fig. 7 shows some model-independent predictions for the
angular asymmetry $A_8$, defined in Ref. [10], and the estimated sensitivity using a full angular analysis. It is clear that more work is required before these sort of measurements become competitive for anything other than very large NP phases. In the case of a full angular analysis, other more theoretically clean observables may also be considered [20].

Figure 7: Left: Model-independent distributions for the CP asymmetry in the angular component $I_8$ ($A_8$) for the SM (green) and NP with $\phi_{\text{NP}} = 2e^{i3\pi/4}$ (red), $\phi_{\text{10NP}} = 2e^{i3\pi/4}$ (grey), and $\phi_{10} = 3e^{i3\pi/4}$ (blue). The error bands show estimated $\Lambda/m_b$ corrections [20] however other theoretical uncertainties are not shown. Right: Estimated experimental sensitivity to $A_8$ after performing a full angular fit to the $B$ and $\bar{B}$ samples assuming the SM for 10 fb$^{-1}$ of LHCb data [20]. The bands have the same meaning as in Fig. 6.

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[8] i.e. \( \langle \theta' \rangle_{q_{\min}^{\max},q_{\max}^{\max}} = \int_{q_{\min}}^{q_{\max}} dq^2 \left( \theta' (q^2) \frac{d\Gamma}{dq^2} \right) / \int_{q_{\min}}^{q_{\max}} dq^2 \frac{d\Gamma}{dq^2} \) for an observable \( \theta'(q^2) \).

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