Impact of stout-link smearing in lattice fermion actions

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The impact of stout-link smearing in lattice fermion actions is examined through the consideration of the mass and renormalization functions of the overlap quark propagator over a variety of smeared configurations. Up to six sweeps of stout-link smearing are investigated. For heavy quark masses, the quark propagator is strongly affected by the smearing procedure. For moderate masses, the effect appears to be negligible. A small effect is seen for light quark masses, where dynamical mass generation is suppressed through the smearing procedure.

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1. Introduction

The term "fat-link fermions" is used to describe a class of actions in which smeared, or fat, gauge links are incorporated into the standard Wilson or Clover actions. Because the smearing algorithms remove short-distance physics from the gauge field, these actions are also often referred to as UV-filtered actions. There is some flexibility in how these fat links are included in the Dirac operator. They can be used in all terms of the action [1, 2, 3, 4, 5], in just the relevant terms [6], or only in the irrelevant terms [7, 8, 9, 10]. The fat-link irrelevant clover (FLIC) action is an example of the last of these choices.

The FLIC action is $\mathscr{O}(a)$-improved [8, 10], and by only including fat-links in the irrelevant Wilson and Clover terms, the short-distance physics in the relevant operators is preserved. This allows relatively cheap access to the light quark mass regime [9], and excellent scaling with small $\mathscr{O}(a^2)$ errors that provide near continuum results at a finite lattice spacing a [10].

S. Dürr *et al.* advocate [11] a fat-link fermion action in which all gauge links are smeared. Using a tree level, clover improved Wilson action with six sweeps of stout-link smearing, they successfully reproduced the light hadron spectrum. While six sweeps of smearing may seem excessive, there is no clear prescription for determining the correct number of smearing sweeps. Indeed, smearing only introduces extra irrelevant terms into the action, and so as long as the number of sweeps and the smearing parameter are held fixed, the smearing should not alter the continuum limit of the action.

Although not a direct physical observable, the quark propagator is a fundamental component of QCD, with many physical quantities depending on its properties. By studying the momentum-dependent quark mass function in the infrared region, one can gain valuable insights into the mechanisms of dynamical chiral symmetry breaking and the associated dynamical generation of mass.

In the following, we investigate the properties of the momentum space overlap quark propagator over a variety of smeared quenched configurations. Of particular interest is how the mass and renormalization functions of the fermion propagator change as the gauge fields undergo more smearing.

As the configurations are smeared, the short-distance, ultraviolet fluctuations are removed from the gauge field. For heavy quarks one might expect that the Compton wavelength would be short enough to reveal the void in gluon interactions left by the smearing procedure. As more smearing sweeps are applied, this should cause the asymptotic value of the mass function to approach the bare input mass.

The ultraviolet physics that is most affected by the smearing algorithm is linked to the infrared physics through approximate zero modes. In the QCD vacuum, there is a finite density of zero modes of the Dirac operator, $\rho(0)$, due to the presence of topologically nontrivial, instanton-like objects. The Dirac zero modes are intimately related to dynamical chiral symmetry breaking via the Banks-Casher relation,

$$\langle \bar{q}q \rangle = -\pi \rho(0), \text{ as } m_q \to 0. \tag{1.1}$$

One might therefore expect that at light quark masses, the removal of short-distance, nontrivial topological fluctuations through smearing would suppress the generation of approximate zero-modes. This would compromise dynamical mass generation, through dynamical chiral symmetry breaking, for fat-link fermion actions.

2. Overlap quark propagator

The massive overlap operator can be written as [12]

$$D(\mu) = \frac{1}{2}\left[1 + \mu + (1 - \mu)\gamma_5\varepsilon(H_w)\right], \tag{2.1}$$

where $H_w(x,y) = \gamma_5 D_w(x,y)$ is the Hermitian Wilson-Dirac operator, $\varepsilon(H_w) = H_w/\sqrt{H_w^2}$ is the matrix sign function, and the dimensionless quark mass parameter μ is

$$\mu \equiv \frac{m^0}{2m_w}, \tag{2.2}$$

where m^0 is the bare quark mass and m_w is the Wilson quark mass which, in the free case, must lie in the range $0 < m_w < 2$.

In a covariant gauge, the continuum, renormalized quark propagator has the form

$$S_\zeta(p) = \frac{Z_\zeta(p^2)}{i\not{p} + M(p^2)}. \tag{2.3}$$

Whereas on the lattice we have at tree level,

$$S^{(0)}(p) \equiv \frac{1}{i\not{q} + m^0}, \tag{2.4}$$

where we have defined the kinematic lattice momentum q. The lattice mass and renormalization functions are then defined by [13],

$$S_\zeta(p) = \frac{Z_\zeta(p^2)}{i\not{q} + M(p^2)}. \tag{2.5}$$

3. Simulation details

We consider $16^3 \times 32$ lattices, generated with a tadpole-improved Lüscher-Weisz action, with a lattice spacing of $a = 0.093$ fm [14]. To these configurations we apply standard, isotropic stout-link smearing [15] with $\rho = 0.1$, to create ensembles of 1, 3, and 6-sweep smeared configurations. Each level of smearing is then fixed to $\mathcal{O}(a^2)$-improved Landau gauge [16].

We use the mean-field improved Wilson action in the overlap fermion kernel. The value $\kappa = 0.19163$ is used in the Wilson action, which provides $m_w a = 1.391$ for the Wilson regulator mass in the interacting case [17]. The overlap quark propagator is calculated for 15 bare quark masses on each ensemble and we report results for three different bare quark masses; $m_0 = 53$ MeV (light), 177 MeV (moderate), and 531 MeV (heavy). All data is cylinder cut [18]. Statistical uncertainties are estimated via a second-order, single-elimination jackknife.

In a standard lattice simulation, one begins by tuning the value of the input bare quark mass m^0 to give the desired renormalized quark mass. However, smearing a lattice configuration filters out the ultraviolet physics and gives a different renormalized quark mass. The input bare quark mass must then be re-tuned in order to reproduce the same physical behavior as on the unsmeared configuration. We replicate this re-tuning procedure in our results by interpolating between neighboring quark masses, such that the functions agree at a given reference momentum ζ. For full details see Ref. [13].
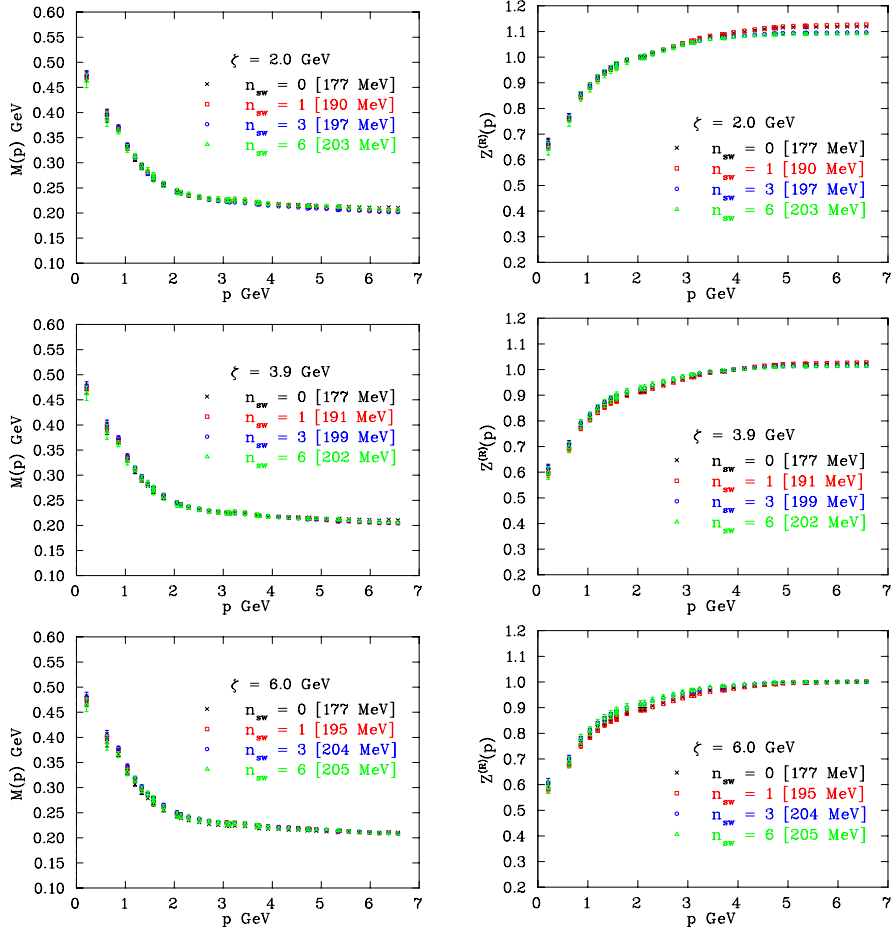
Figure 1: The interpolated mass and renormalization functions for the moderate bare quark mass, $m^0 = 177$ MeV, with the three choices of ζ. The effective bare quark masses are given in square brackets. Apart from a splitting of the renormalization function for $\zeta = 2.0$ GeV, the smearing has little effect on the quark propagator.

4. Results

We begin with an analysis of the moderate bare quark mass, $m_0 = 177$ MeV. The interpolated mass and renormalization functions, for three choices of reference momentum, are shown in Fig. 1. For this choice of bare quark mass the smearing algorithm appears to have little effect on the quark propagator. The most obvious dependence is in the UV region of the renormalization function for $\zeta = 2.0$ GeV, where the smearing introduces a splitting of the curves.

Next we consider the lighter quark mass, $m_0 = 53$ MeV. The mass and renormalization functions are given in Fig. 2. As in the case of the moderate quark mass there appears to be little effect on the quark propagator. However when the quark masses are matched in the UV at $\zeta = 6.0$ GeV there is a significant suppression of dynamical mass generation for $n_{sw} = 6$.

Finally, we consider the heavy quark mass, $m_0 = 531$ MeV, and provide the relevant functions in Fig. 3. The impact of smearing is most apparent for this heavy mass. The most interesting
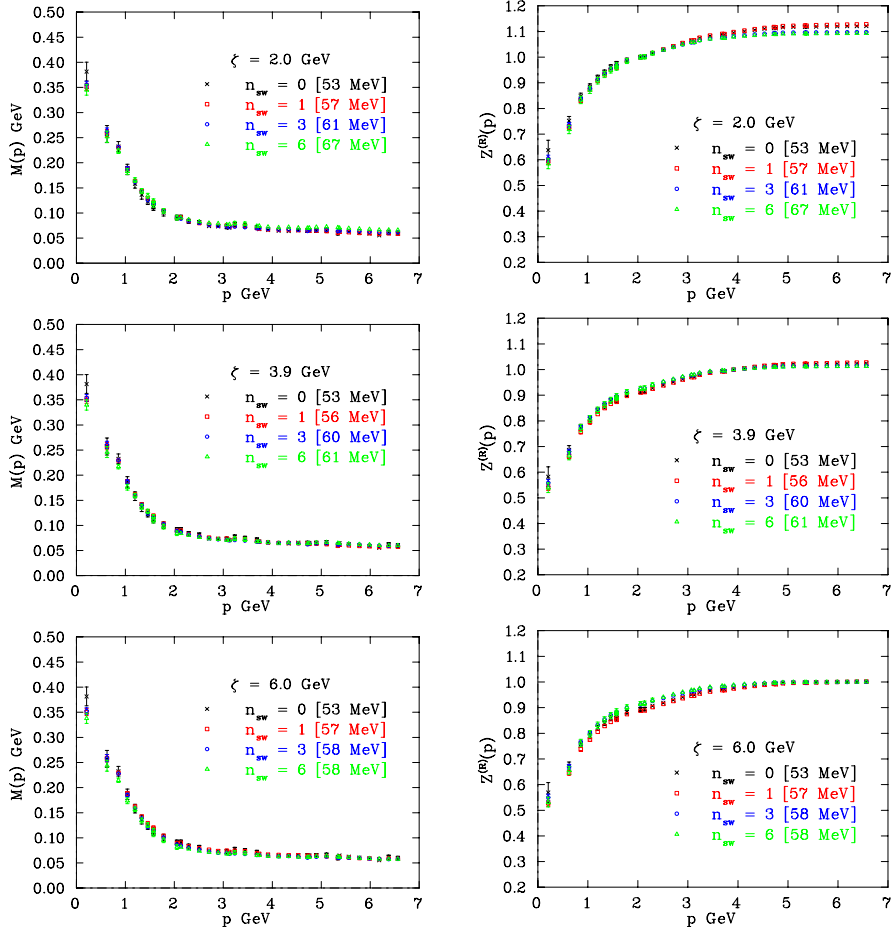
Figure 2: The interpolated mass and renormalization functions for the small bare quark mass, $m^0 = 53$ MeV, with three choices of ζ. As with the moderate quark mass, the smearing has little effect on the propagator. However when the quark masses are matched in the UV at $\zeta = 6.0$ GeV there is a significant suppression of dynamical mass generation for $n_{sw} = 6$.

effect is in the mass function with the choice of $\zeta = 2.0$ GeV, matching the infrared physics. Here the mass function decreases dramatically as progressively more smearing is applied. We see that smearing the gauge field causes the asymptotic value of the mass function to approach the input bare quark mass, clearly illustrating the short-distance void created by 6 sweeps of stout-link smearing.

5. Conclusion

Smearing all links in the gauge field only has a strong effect on the quark propagator for heavy quarks. Here suppression of the short distance physics through smearing spoils the physics of the theory above about 2 to 3 GeV. Renormalization of the bare quark mass is not enough to restore the flattening of the bare quark mass at large momenta. We also saw that the asymptotic value of the mass function approaches the input bare quark mass.
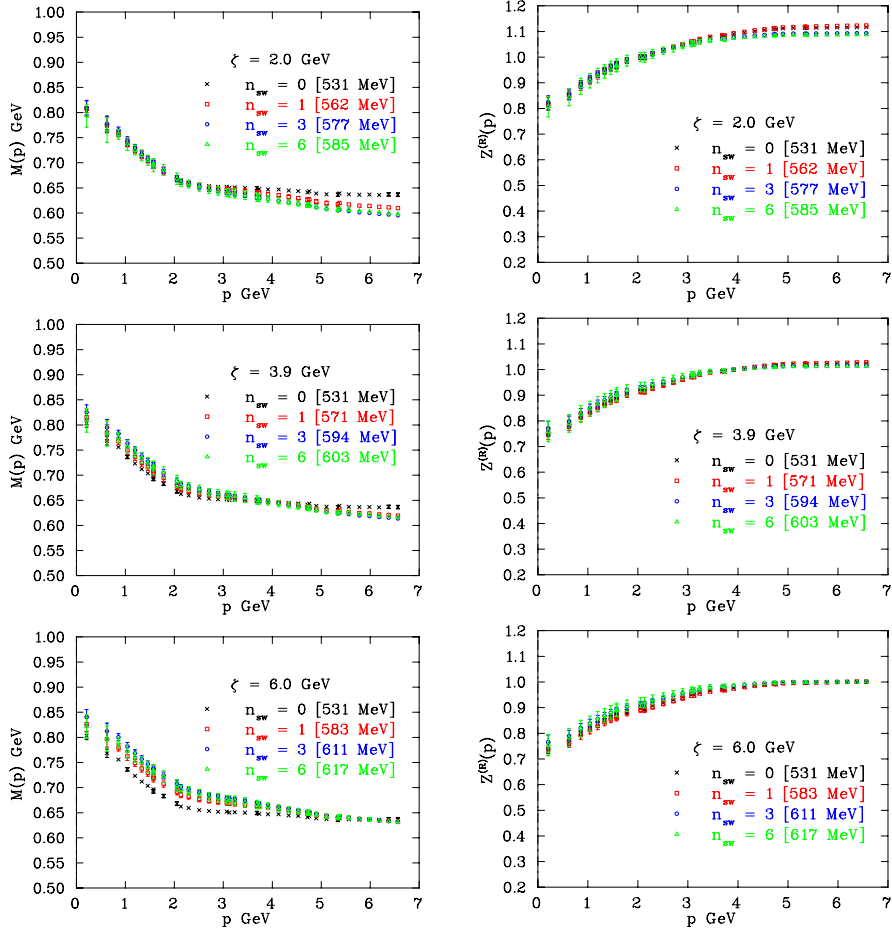
Figure 3: The interpolated mass and renormalization functions for the heavy bare quark mass, $m^0 = 531$ MeV, for the three choices of ζ. The effective bare quark masses are given in square brackets. For this choice of m^0 the smearing has a strong impact on the quark propagator.

For all values of the quark mass considered, the effect of one sweep of smearing on the renormalization function is negligible. The small discrepancies, which are of the order of 2%, are insensitive to the value of the quark mass. We also note a significant reduction in the statistical error, even after a single sweep of smearing.

At the lightest quark mass, the anticipated suppression of dynamical mass generation is apparent. In the case of six smearing sweeps, we find an order 2σ reduction in the infrared dynamical mass generation when the quark masses are matched. In this case, we also see that for $p < 2$ to 3 GeV, the mass function for six sweeps of smearing lies systematically low. This effect is clearly illustrated in the lower left panel of Fig. 2.

Acknowledgments

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