

Search for the IR fixed point in the twisted Polyakov loop scheme

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We present a non-perturbative study of the running coupling constant in the Twisted Polyakov Loop (TPL) scheme. We investigate how the systematic and statistical errors can be controlled via a feasibility study in SU(3) pure Yang-Mills theory. We show that our method reproduces the perturbative determination of the running coupling in the UV. In addition, our numerical result agrees with the theoretical prediction of this coupling constant in the IR. We also present our preliminary results for $N_f = 12$ QCD, where an IR fixed point may be present.

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1. Introduction

The existence of non-trivial fixed points is one of most intriguing subjects in quantum field theory. Field theories with fixed points exhibit scale-invariant behaviour and are often exactly solvable. In addition, renormalization group (RG) flows around fixed points contain information of the universality class of field theories.

Non-trivial fixed points have been identified non-perturbatively in two-dimensional scalar field theories, *via* the techniques of algebraic method or Wilsonian RG. In the case of three-dimensional scalar field theories, they can be found using the large- N expansion or Wilsonian RG. In four dimensions, there is no non-trivial fixed point for scalar field theories. On the other hand, for gauge theories in four dimensions, there are Gaussian fixed points. Furthermore, the perturbative β function indicates the existence of non-trivial infrared (IR) fixed points for a certain region of large-flavor (N_f) SU(N) gauge theories. Possible appearance of these IR fixed points has stimulated phenomenological studies of topics such as dynamical electro-weak symmetry breaking and unparticle physics. The existence of these IR fixed points depends on the gauge group, the number of flavours, and the representation of fermion fields. For SU(3) gauge theory with fermions in the fundamental representation, such a fixed point has been predicted in the range $8 < N_f \leq 16$ using perturbation theory [1]. However, the value of the renormalized coupling at the point depends on N_f , and it may be in the regime where perturbation theory is not applicable. Therefore it is important to investigate the existence of this IR fixed point non-perturbatively.

First such lattice study for SU(3) gauge theory was carried out in Ref. [2], where the authors investigated the phase structure of the case of $N_f = 16$. Recently, Appelquist *et al.* performed lattice calculation of the running coupling constant in the Schrödinger functional (SF) scheme and discovered evidence of an IR fixed point in the case of $N_f = 12$ [3]. In their work, no such evidence was found for $N_f = 8$. Furthermore, two groups have studied the phase structure of the $N_f = 12$ theory [4, 5]. However, the existence of IR fixed point is not firmly established yet [5]. The difficulty is mainly due to scheme dependence of the running coupling constant and the presence of significant lattice artifacts in the strong-coupling regime. Therefore it is important to measure the running coupling in different renormalisation schemes.

In this work, we perform lattice simulation of the running coupling constant for the fundamental-representation, $N_f = 12$, SU(3) gauge theory. Similar to the approach of Appelquist *et al.*, we measure the step scaling function $\sigma(s, g^2(L)) = g^2(sL)$ keeping the values of β that give constant $g^2(L)$ for each small lattice size. We work in the Twisted Polyakov Loop (TPL) scheme which does not contain $O(a/L)$ discretization errors. These errors are present in the SF scheme due to the boundary counterterm. This TPL scheme was first proposed by de Divitiis *et al.* [6, 7] for SU(2) gauge theory, and we extend the definition of the scheme to the SU(3) case.

In this paper, we give a short review of TPL scheme in §.2. In §.3 we present a validity study of this scheme by calculating the running coupling constant in SU(3) pure Yang-Mills theory. Our preliminary results for $N_f = 12$ SU(3) gauge theory is reported in §.4.

2. Twisted Polyakov Loop scheme

In this section, we present the definition of the Twisted Polyakov Loop scheme in SU(3) gauge

theory. This is an extension of the SU(2) case as discussed in Ref. [6]. To define the TPL scheme, we introduce twisted boundary condition for the link variables in x and y directions on the lattice:

$$U_\mu(x + \hat{\nu}L/a) = \Omega_\nu U_\mu(x) \Omega_\nu^\dagger. \quad (\nu = 1, 2) \quad (2.1)$$

Here, Ω_ν are the twist matrices which have the following properties:

$$\Omega_1 \Omega_2 = e^{i2\pi/3} \Omega_2 \Omega_1, \Omega_\mu \Omega_\mu^\dagger = 1, (\Omega_\mu)^3 = 1, \text{Tr}[\Omega_\mu] = 0. \quad (2.2)$$

The gauge transformation $U_\mu(r) \rightarrow \Lambda(r) U_\mu(r) \Lambda^\dagger(r + \hat{\mu})$ and eq.(2.1) imply

$$\Lambda(r + \hat{\nu}L/a) = \Omega_\nu \Lambda(r) \Omega_\nu^\dagger. \quad (2.3)$$

Because of this twisted boundary condition, the definition of Polyakov loops in the twisted directions are modified,

$$P_1(y, z, t) = \text{Tr} \left(\left[\prod_j U_1(x = j, y, z, t) \right] \Omega_1 e^{i2\pi y/3L} \right), \quad (2.4)$$

in order to satisfy gauge invariance and translation invariance. The renormalized coupling in TPL scheme is defined by taking the ratio of Polyakov loop correlators in the twisted (P_1) and the un-twisted (P_3) directions:

$$g_{TP}^2 = \frac{1}{k} \frac{\langle \sum_{y,z} P_1(y, z, L/2a) P_1(0, 0, 0)^\dagger \rangle}{\langle \sum_{x,y} P_3(x, y, L/2a) P_3(0, 0, 0)^\dagger \rangle}. \quad (2.5)$$

At tree level, this ratio Polyakov loops is proportional to the bare coupling. The proportionality factor k is obtained by analytically calculating the one-gluon-exchange diagram. To perform this analytic calculation, we choose the explicit form of the twist matrices [8],

$$\Omega_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \Omega_2 = \begin{pmatrix} e^{-i2\pi/3} & 0 & 0 \\ 0 & e^{i2\pi/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.6)$$

In the case of SU(3) gauge group,

$$\begin{aligned} k &= \frac{1}{24\pi^2} \sum \frac{(-1)^n}{n^2 + (1/3)^2} \\ &= \frac{1}{24\pi^2} \left[\frac{9}{2} - \frac{3\pi}{2} \text{cosech} \left(\frac{\pi}{3} \right) \right] \\ &= 0.03184 \dots \end{aligned} \quad (2.7)$$

The naive twisted boundary condition for lattice fermions can be written by

$$\psi(x + \hat{\nu}L/a) = \Omega_\nu \psi(x). \quad (2.8)$$

However, this results in an inconsistency when changing the order of translations, namely,

$$\begin{aligned} \psi(x + \hat{\nu}L/a + \hat{\rho}L/a) &= \Omega_\rho \Omega_\nu \psi(x), \\ &\neq \Omega_\nu \Omega_\rho \psi(x). \end{aligned} \quad (2.9)$$

To avoid this difficulty, we introduce "smell" symmetry [9], which is a copy of color symmetry. We identify the fermion field as a $N_c \times N_s$ matrix ($\psi_\alpha^a(x)$). Then we impose the twisted boundary condition for fermion fields to be

$$\psi_\alpha^a(x + \hat{\nu}L/a) = e^{i\pi/3} \Omega_\nu^{ab} \psi_\beta^b(\Omega_\nu)^\dagger_{\beta\alpha} \quad (2.10)$$

for $\nu = 1, 2$ directions. Here, the smell index can be considered as a "flavor" index, then the number of flavors should be a multiple of $N_s (= N_c = 3)$. We use staggered fermion in our simulation. This contains four tastes for each flavour. This enables us to perform simulations with $N_f \geq 12$ in this SU(3) gauge theory with twisted boundary condition.

3. Quenched QCD case

Before carrying out the simulation for $N_f = 12$, we first measure the TPL running coupling in quenched QCD. The gauge configurations are generated by the pseudo-heatbath algorithm and overrelaxation algorithm mixed in the ratio 1:5. One such a combination is called a "sweep" in the following. In order to generate the configurations with the twisted boundary condition we use the trick [10] proposed by Lüscher and Weisz. To reduce large statistical fluctuation of the TPL coupling, as reported in Ref. [11], we measure Polyakov loops at every Monte Carlo sweep and perform a jackknife analysis with large bin size, typically of $O(10^3)$. This enables us to evaluate the statistical error correctly. The simulations are carried out with several lattice sizes ($L/a = 4, 6, 8, 10, 12, 14, 16$) at more than twenty β values in the range $6.2 \leq \beta \leq 16$. We generate 200,000-400,000 configurations for each $(\beta, L/a)$ combination. Figure 1 shows the β dependence of the

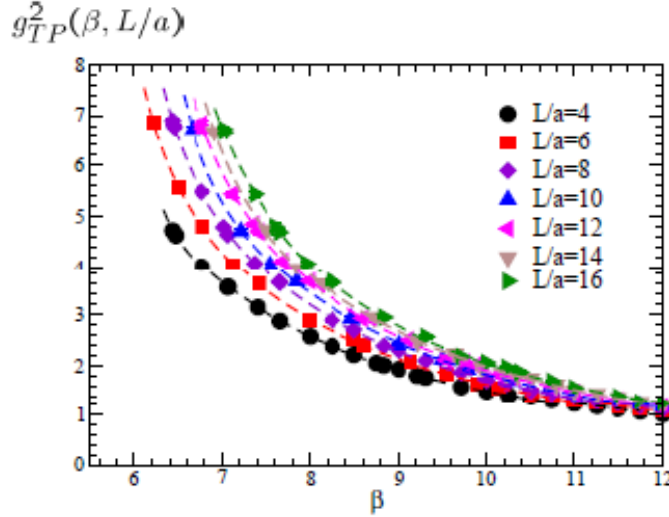


Figure 1: TPL renormalized coupling in the each β and L/a in quenched QCD.

renormalized coupling in TPL scheme at various lattice sizes. The results are fitted at each fixed lattice size to the interpolating function which is similar to the one used in Ref. [3],

$$g_{\text{TP}}^2(\beta) = \sum_{i=1}^n \frac{A_i}{(\beta - B)^i}, \quad (3.1)$$

where A_i are the fit parameters, and $4 \leq B \leq 5$, $n = 3, 4$ are employed. As a small lattice size of the step scaling, we use $L/a = 4, 6, 8, 10$. The step scaling parameter is $s = 1.5$, and we estimate the coupling constant for $L/a = 9, 15$ from interpolations at the fixed β using the above fit results of all the lattice sizes.

We take the continuum limit using a linear function in $(a/L)^2$, because the TPL scheme involves no $O(a/L)$ error. We found that the coupling constant of the TPL scheme exhibits scaling behaviour even at the small lattice sizes, as shown in Fig.2.

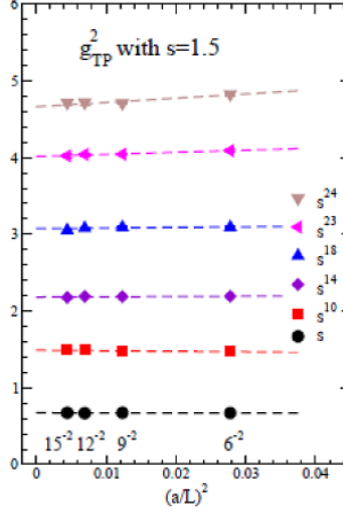


Figure 2: The continuum limit of g_{TP}^2 with $s = 1.5$. The fit function is a linear function of $(a/L)^2$. The statistical error bars are of the same size of the symbols.

The TPL running coupling constant in quenched QCD with 24 steps is shown in Fig.3 together with one- and two-loop perturbative results. The horizontal axis corresponds to the energy scale. All the results are normalized at $L = L_0$ with $g^2(L_0/L) = 0.65$. The nonperturbative running coupling constant is consistent with one- and two-loop perturbative results in the high energy region ($L_0/L \geq 0.1$). On the other hand, in the low energy region, the running is slower than one-loop. This shows the feature of TPL scheme. The TPL running coupling constant in $\mu = 1/L \rightarrow 0$ limit goes to $1/k \sim 32$, since the boundary effects becomes negligible in this limit. Thus the definition of eq.(2.5) goes to the constant. This is the reason why the nonperturbative running coupling constant in this scheme runs slower than the one-loop perturbative result in the low energy region. From this quenched test, we conclude that we can control both the the statistical and systematic errors of the TPL coupling constant, and can obtain reasonable result with this scheme. Furthermore we found the TPL coupling constant in quenched QCD has a robust scaling behaviour even in a small lattice size, which was also observed in the previous quenched SU(2) calculations [6, 11].

4. $N_f = 12$ case

In this section, we present preliminary results for our nonperturbative running coupling constants. It is consistent with the perturbative result at high energy.

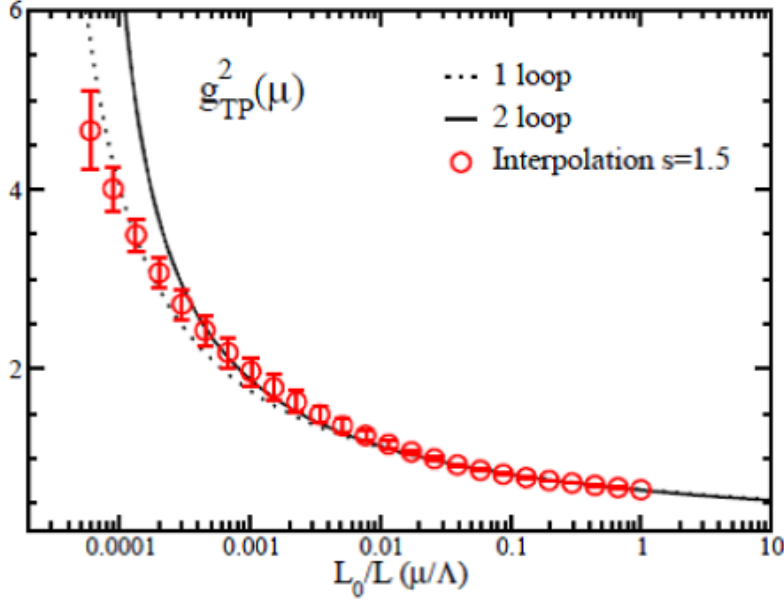


Figure 3: The running coupling constants in TPL scheme, one-loop and two-loop.

The simulation parameters are $4.0 \leq \beta \leq 25.0$ with lattice sizes $L/a = 4, 6, 8, 10, 12$ ¹. Figure 4 shows the global behavior of the ratio of Polyakov loops in eq.(2.5) for each lattice size. This is also the global behaviour of the TPL coupling constant. Note that the behavior in low- β region is different from the SF scheme [3], and the TPL coupling shows the trend of reaching a plateau for each lattice size. The effect of taste breaking for staggered fermions results in significant scheme dependence in this region.

It is difficult to find a good interpolating function to fit all the data. This is due to the plateau behaviour in the IR. In this proceedings, we use the 38 data points in the high- β regime, where the ratios of Polyakov loops are smaller than 0.04, to perform the global fit to the interpolating function $f(x, y) = f(\beta, a/L)$:

$$f(x, y) = \frac{6k}{x + c_1 \log(y)} + \frac{c_2 + c_3 \log(y)}{(x + c_1 \log(y))^2}. \quad (4.1)$$

Here, we fix the coefficient of the first term to be $6k$. This is because the renormalised coupling constant should be equal to the bare one in the UV (high β). From the perturbative analysis at high energy, we can fix a/L dependence.

We carry out step scaling procedure similar to that in the quenched case. Figure 5 shows the running of our 99 steps starting at the UV point $g^2 = 0.298$, which corresponds to the ratio of Polyakov loop being 0.009487. Perturbative results are also shown in this plot. We find good agreements with perturbative running in this UV regime. Presently, we are investigating the issue of finding a good interpolating function to describe the β -dependence in the IR. This is important

¹At the lattice conference, we took the UV starting point of step scaling at $g^2 = 0.542$, a large value. Therefore the running behavior was not consistent with the perturbative results even at this UV starting point. In this proceedings, we will report the modified result which has been obtained using a starting point much deeper into the UV regime.

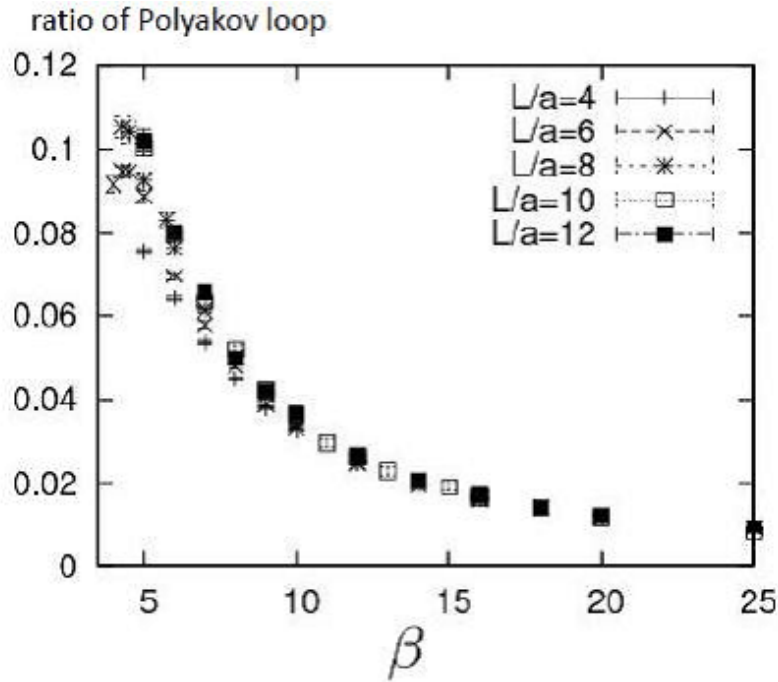


Figure 4: The ratio of Polyakov loop in each β and L/a .

for a detailed study of the IR fixed point. We find that as a function of β the value of the running coupling for fixed $L/a = 4, 6$ stops growing towards smaller β at around $\beta = 4.5$ and deviate from the larger volume data. This is in contrast to the case of SF scheme where the running couplings for each L/a continue to grow towards smaller β and cross with each other around $\beta = 4.5$.

The step scaling analysis in the strong coupling region is in progress.

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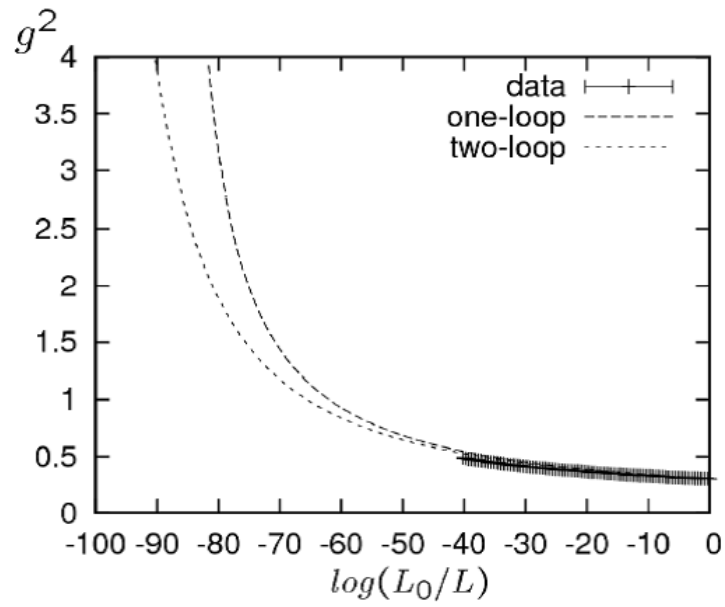


Figure 5: The TPL running coupling constant in high energy region.

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