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Results from the MILC collaboration's SU(3) chiral perturbation theory analysis

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The MILC Collaboration

We present the status of the MILC collaboration's analysis of the light pseudoscalar meson sector with SU(3) chiral fits. The analysis includes data from new ensembles with smaller lattice spacing, smaller light quark masses and lighter than physical strange quark masses. Our fits include the NNLO chiral logarithms. We present results for decay constants, quark masses, Gasser-Leutwyler low energy constants, and condensates in the two- and three-flavor chiral limits.

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a (fm)	$a\hat{m}'$ / am'_s	$10/g^2$	size	# lats.	u_0	r_1/a	$m_\pi L$
$\approx$0.09	0.0124 / 0.031	7.11	$28^3\times96$	531	0.8788	3.712(4)	5.78
$\approx$0.09	0.0093 / 0.031	7.10	$28^3\times96$	1124	0.8785	3.705(3)	5.04
$\approx$0.09	0.0062 / 0.031	7.09	$28^3\times96$	591	0.8782	3.699(3)	4.14
$\approx$0.09	0.00465 / 0.031	7.085	$32^3\times96$	480	0.8781	3.697(3)	4.11
$\approx$0.09	0.0031 / 0.031	7.08	$40^3\times96$	945	0.8779	3.695(4)	4.21
$\approx$0.09	0.00155 / 0.031	7.075	$64^3\times96$	491	0.877805	3.691(4)	4.80
$\approx$0.09	0.0062 / 0.0186	7.10	$28^3\times96$	985	0.8785	3.801(4)	4.09
$\approx$0.09	0.0031 / 0.0186	7.06	$40^3\times96$	580	0.8774	3.697(4)	4.22
$\approx$0.09	0.0031 / 0.0031	7.045	$40^3\times96$	380	0.8770	3.742(8)	4.20
$\approx$0.06	0.0072 / 0.018	7.48	$48^3\times144$	625	0.8881	5.283(8)	6.33
$\approx$0.06	0.0054 / 0.018	7.475	$48^3\times144$	465	0.88800	5.289(7)	5.48
$\approx$0.06	0.0036 / 0.018	7.47	$48^3\times144$	751	0.88788	5.296(7)	4.49
$\approx$0.06	0.0025 / 0.018	7.465	$56^3\times144$	768	0.88776	5.292(7)	4.39
$\approx$0.06	0.0018 / 0.018	7.46	$64^3\times144$	826	0.88764	5.281(8)	4.27
$\approx$0.06	0.0036 / 0.0108	7.46	$64^3\times144$	601	0.88765	5.321(9)	5.96
$\approx$0.045	0.0028 / 0.014	7.81	$64^3\times192$	801	0.89511	7.115(20)	4.56

Table 1: List of ensembles used in this study, with u_0 the tadpole factor and r_1/a the scale from the heavy quark potential. The r_1/a values shown come from a smooth interpolation.

1. Introduction

The MILC collaboration has been carrying out simulations of 2+1 flavor lattice QCD with an improved staggered quark action for about 10 years. The physics program has recently been reviewed in Ref. [1]. An important aspect of the MILC collaboration's research program has been the study of the light pseudoscalar meson sector. Here we give the latest update of this program. Compared to the last status report in Ref. [2] lattice ensembles with smaller lattice spacings, smaller light quark masses and lighter-than-physical strange quark masses are analyzed. Furthermore, we do fits based on both SU(2) and SU(3) chiral perturbation theory (χPT), rather than just SU(3) as before, and we now include NNLO chiral logarithms. The SU(2) chiral fits are described in Ref. [3].

2. The ensembles and the fitting procedures

The MILC collaboration has generated lattice configuration ensembles at six different lattice spacings, ranging from $a \approx 0.18$ fm down to $a \approx 0.045$ fm. In the present analysis, only the $a \approx 0.09$ fm ("fine"), $a \approx 0.06$ fm ("superfine") and $a \approx 0.045$ fm ("ultrafine") ensembles are considered. With our very precise numerical data, adding in coarser lattice spacings would require inclusion of higher order discretization effects in the fits, which is currently not feasible.

The ensembles considered in this study are listed in Table 1. In our notation, $a\hat{m}'$ is the simulation light quark mass, with up and down quark masses being equal, and am'_s is the simulation strange quark mass. Notice that several ensembles have an unphysically light am'_s, about 60%

a (fm)	Goldstone	RMS	singlet
0.15	241	542	673
0.12	265	460	558
0.09 ($a\hat{m}' = 0.00155$, $am'_s = 0.031$ ensemble only)	177	281	346
0.09 (all other fine ensembles)	246	329	386
0.06	224	258	280
0.045 (some valence pions are lighter)	324	334	341

Table 2: Masses (in MeV, using $r_1 = 0.3117$ fm for the scale) for the lightest sea-quark pions of various tastes at each lattice spacing. The Goldstone pion is the taste pseudoscalar and has the lightest mass of all tastes, while the taste singlet has the heaviest mass. The root-mean-squared (RMS) mass is the average that is used in the NNLO chiral logarithms. Unless otherwise indicated, the masses given are also the lightest valence-quark pions on each ensemble at that lattice spacing. We drop the $a \approx 0.15$ fm and $a \approx 0.12$ fm ensembles from the current analysis because of the large splittings and heavy singlet pions.

of the physical strange quark mass, and one ensemble has three degenerate (light) quarks. These ensembles were created specifically to have good control over the SU(3) χPT fits.

We determine the scale r_1 on every ensemble from the static quark potential (see Ref. [1]). The values listed in Table 1 come from a smooth interpolation. For the analysis presented here, however, we use a mass independent scheme, where r_1 is taken from the smooth interpolation with the quark masses set to their physical values. This procedure avoids spurious dependence on the quark masses in the χPT fits.

Even with the use of the improved staggered (asqtad) fermions and the fairly small lattice spacings considered, the taste-violation lattice artifacts are significant, and need to be accounted for in the analysis. We do this, as in our previous studies, by using rooted staggered χPT forms (rSχPT) at NLO in our chiral fits [4, 5]. The "rooting procedure," taking the fourth root of the fermion determinant when generating the lattices, is used to eliminate the unwanted tastes present with the use of staggered fermions. As reviewed in Ref. [1], recent work suggests strongly that the procedure does indeed produce the desired theory in the continuum limit.

As a new feature in the present analysis, our χPT fits now include the NNLO chiral logarithms derived by Bijnens, Danielsson and Lahde [6, 7, 8]. In contrast to the NLO chiral logs, however, lattice artifacts are not included in the NNLO chiral logs. Instead, we use the root mean square average (over tastes) pion mass for the argument of the NNLO chiral logs. This is systematic at this order in χPT only if chiral symmetry violations from taste-violating lattice effects are significantly smaller than the usual chiral violations from mass terms. That begins to be true for the $a \approx 0.09$ fm points, and is better satisfied for the $a \approx 0.06$ and 0.045 fm ensembles. It is not true for ensembles with $a \geq 0.12$ fm, which is why that data is omitted from the analysis. Table 2 gives some representative pion masses for our ensembles.

The SU(3) chiral fits are done in two stages. The first consists of "low-mass" fits used to

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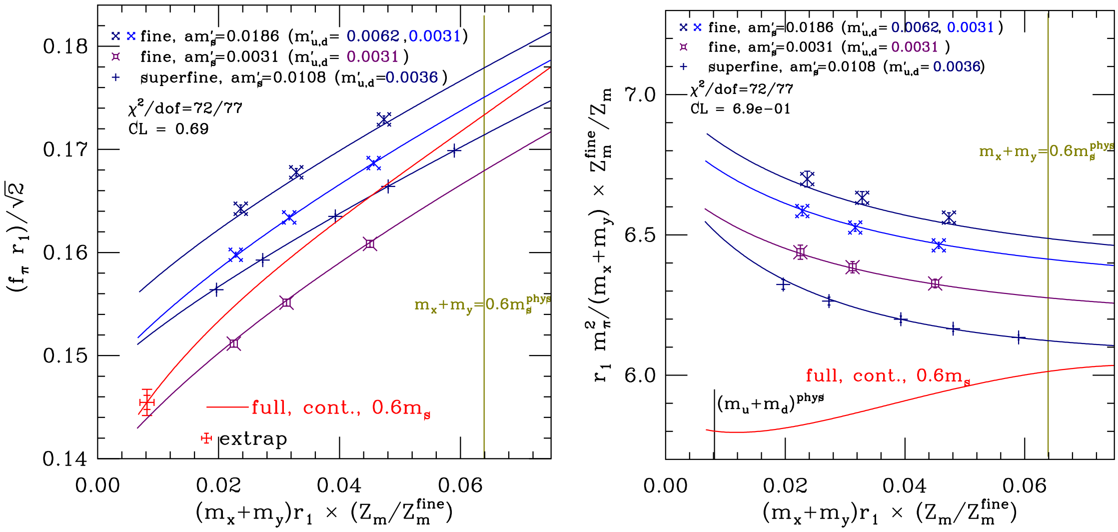

Figure 1: Low-mass SU(3) chiral fits. The red line is the continuum limit with (light) valence and sea quark masses set equal and the strange quark mass fixed at $0.6m_s^{phys}$.

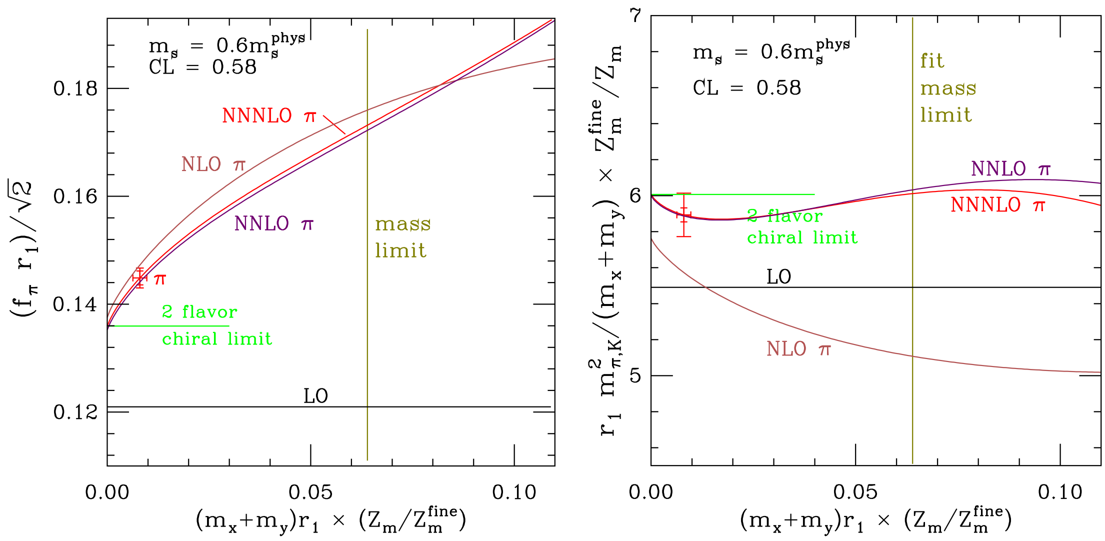

Figure 2: Test of convergence of SU(3) χPT fits in the continuum, with the strange quark mass fixed at $0.6m_s^{phys}$. For this test we also included NNNLO analytic terms in the fit.

determine the LO and NLO low energy constants (LECs), namely what we call f_3 and B_3 (at LO) and the Gasser-Leutwyler parameters L_i (at NLO). Here the goal is to keep only those ensembles and valence points where meson masses (including kaons, which have a quark of mass m_s') are sufficiently light that SU(3) χPT may be expected to be rapidly convergent. In addition, taste splitting as a fraction of the Goldstone pion mass should be small enough that omission of taste-violations from the NNLO terms (but inclusion at NLO) is systematic; this, for example, is another reason to drop the $a \approx 0.09$ fm ensemble with $a\hat{m}' = 0.00155$, $am_s' = 0.031$. After these cuts, only the three fine and one superfine ensembles with $m_s' \lesssim 0.6m_s^{phys}$ are included, and the valence masses are limited by $m_x + m_y \leq 0.6m_s^{phys}$. The fits are illustrated in Fig. 1. To test convergence, the full set of NNNLO analytic terms may also be added; as shown in Fig. 2, the convergence is satisfactory.

Addition of such terms does not improve the goodness of fit, as can be seen by comparing the confidence levels (CL) of the two fits in Figs. 1 and 2. The fits include all partially quenched data for pion and "kaon" (with lighter than physical strange quark mass) decay constants and masses.

In the second stage, the "high-mass" SU(3) χPT fits, all ensembles listed in Table 1 are included with the valence masses restricted to $m_x + m_y \leq 1.2 m_s^{phys}$. The LO and NLO LECs are fixed at the values from the low-mass fits. NNNLO and NNNNLO analytic terms are included, but not the corresponding logs. These terms are needed to obtain good confidence levels, and they allow us to interpolate around the (physical) strange quark mass. The fact that they are required indicates that SU(3) χPT is not converging rapidly at these mass values, unlike the situation in the low-mass case. Since the LO and NLO LECs dominate the chiral extrapolation to the physical point, the results for decay constants and masses are insensitive to the form of these NNNLO and NNNNLO interpolating terms, as long as the fits are good. The high-mass fits are used to give the central values of the physical decay constants and other quantities involving the strange quark mass, such as f_2, B_2 and chiral condensate $\langle \bar{u}u \rangle_2$, which are defined in the two-flavor chiral limit ($\hat{m} \to 0$, m_s fixed at m_s^{phys}). The high-mass fits are illustrated in Fig. 3.

3. Preliminary results

In a first analysis we use, as before, a lattice scale determined from Υ-splittings [9] which leads to $r_1^{phys} = 0.318(7)$ fm [10]. With this, we obtain

$$\begin{aligned} f_\pi &= 128.0 \pm 0.3 \pm 2.9 \text{ MeV} , \\ f_K &= 153.8 \pm 0.3 \pm 3.9 \text{ MeV} , \\ f_K/f_\pi &= 1.201(2)(9) . \end{aligned} \tag{3.1}$$

Here, and in the following results, the first error is statistical and the second is systematic.

Our result for f_π agrees nicely with the latest PDG 2008 value, $f_\pi = 130.4 \pm 0.2$ MeV [11]. Since f_π is our most accurately determined dimensionful quantity, we can use it to determine the scale. This gives $r_1^{phys} = 0.3117(6)(^{+12}_{-31})$ fm. Redoing our analysis with this more accurate scale, we obtain

$$\begin{aligned} f_K &= 156.2 \pm 0.3 \pm 1.1 \text{ MeV} , & f_K/f_\pi &= 1.198(2)(^{+6}_{-8}) , \\ f_2 &= 122.8 \pm 0.3 \pm 0.5 \text{ MeV} , & B_2 &= 2.87(1)(4)(14) \text{ GeV} , \\ f_3 &= 110.8 \pm 2.0 \pm 4.1 \text{ MeV} , & B_3 &= 2.39(8)(10)(12) \text{ GeV} , \\ f_\pi/f_2 &= 1.062(1)(3) , & f_\pi/f_3 &= 1.172(3)(43) , \\ \langle \bar{u}u \rangle_2 &= -(279(1)(2)(4) \text{ MeV})^3 , & \langle \bar{u}u \rangle_3 &= -(245(5)(4)(4) \text{ MeV})^3 , \\ 2L_6 - L_4 &= 0.16(12)(2) , & 2L_8 - L_5 &= -0.48(8)(21) , \\ L_4 &= 0.31(13)(4) , & L_5 &= 1.65(12)(36) , \\ L_6 &= 0.23(10)(3) , & L_8 &= 0.58(5)(7) , \\ m_s &= 89.0(0.2)(1.6)(4.5)(0.1) \text{ MeV} , & \hat{m} &= 3.25(1)(7)(16)(0) \text{ MeV} , \\ m_u &= 1.96(0)(6)(10)(12) \text{ MeV} , & m_d &= 4.53(1)(8)(23)(12) \text{ MeV} , \\ m_s/\hat{m} &= 27.41(5)(22)(0)(4) , & m_u/m_d &= 0.432(1)(9)(0)(39) . \end{aligned} \tag{3.2}$$

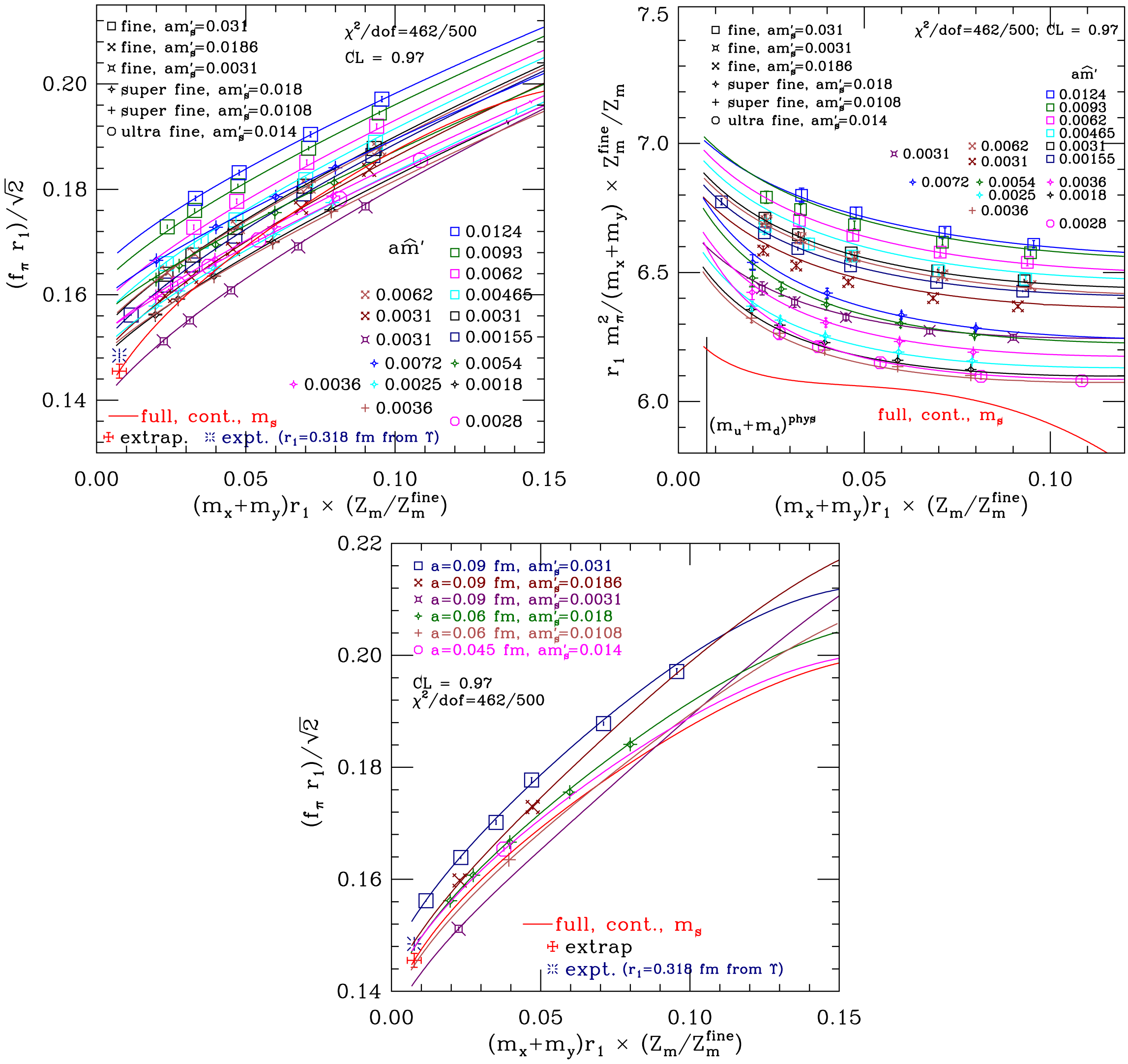

Figure 3: High-mass SU(3) chiral fits: in the top plots selected partially quenched data points are shown, while in the bottom plot only full QCD points, *i.e.,* points with sea and valence quark masses set equal, are shown.

Here the NLO LECs L_i are in units of 10^{-3}, evaluated at chiral scale m_η, and the LO LECs B_j, quark masses and chiral condensates are in the $\overline{\text{MS}}$ scheme at 2 GeV. For the conversion from the bare quantities we use the two-loop renormalization factor of Ref. [12]. The resulting perturbative error is listed as the third error in these quantities. The subscripts "2" and "3" refer to the two-flavor (with m_s at its physical value) and three-flavor chiral limits, respectively. The quark condensates are related to the LO LECs by $\langle \bar{u}u \rangle_j = -f_j^2 B_j/2$. Quark masses, finally, have a fourth error, accounting for our limited knowledge of electromagnetic effects on pion and kaon masses (see Ref. [13] for how we address this).

We note that our new results for the decay constants, quark masses, and condensates agree, well within errors, with our previous analysis using NLO SU(3) χPT supplemented by higher-order analytic terms [2]. Most also have smaller errors. Not surprisingly, however, some of the NLO

LECs changed considerably with the inclusion of NNLO chiral logs. Similar changes have been observed in continuum extractions of these NLO LECs; see for example Ref. [14]. The comparison with our previous results suggests that NLO SU(3) χPT plus analytic terms, when implemented in a careful manner, can be used to reliably extrapolate physical quantities such as light pseudoscalar meson decay constants, B_K, and heavy-light meson decay constants and form factors to the physical light quark masses and continuum [1].

From the ratio of f_K/f_π in Eq. (3.2) we can obtain

$$|V_{us}| = 0.2247(^{+16}_{-13}) \, , \tag{3.3}$$

which is a significant improvement over our previous result, $|V_{us}| = 0.2246(^{+25}_{-13})$ [2].

Using one-loop conversion formulae [15] we obtain from the SU(3) NLO LECs in Eq. (3.2) the scale invariant SU(2) NLO LECs [16]

$$\bar{l}_3 = 3.32(64)(45) \, , \qquad \bar{l}_4 = 4.03(16)(17) \, . \tag{3.4}$$

We observe nice agreement between the SU(3) chiral fit results described here and the results of the SU(2) chiral fits given in Ref. [3] for all quantities that can be directly compared, namely f_π, f_2, B_2, $\hat{m}$, $\langle \bar{u}u \rangle_2$ and $\bar{l}_{3,4}$.

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