

Numerical study of chiral magnetic effect in quenched SU(2) lattice gauge theory *

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A possible experimental observation of the chiral magnetic effect in heavy ion collisions at RHIC was recently reported by the STAR Collaboration. We study signatures of this effect in SU(2) lattice gluodynamics with the chirally invariant Dirac operator. We find that at zero temperature the local fluctuations of an electric current of quarks and chirality fluctuations increase with a growth of an abelian external magnetic field. The external magnetic field leads to a spatial separation of the quark's electric charges. The separation increases with the strength of the magnetic field. As temperature gets higher the dependence of these quantities on the strength of the magnetic field becomes weaker. In the deconfinement phase the local fluctuations of the chiral density and of the spatial components of the quarks electric current are large and are almost independent on the external magnetic field. The local fluctuations of the electric charge density decrease with the strength of the magnetic field in this phase.

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1. Introduction

Very strong magnetic fields of the hadronic scale can significantly modify the properties of strongly interacting matter: they change the order of the phase transition from the confinement phase to the quark-gluon plasma, shift the position of the transition line [1], etc. At the Relativistic Heavy Ion Collider (RHIC) in noncentral heavy-ion collisions the strong magnetic field arises due to the relative motion of the ions and products of the collision [2]. The induced magnetic field is perpendicular to the reaction plane. At first moments ($\tau \sim 1$ fm/c) of the collision the value of magnetic field at RHIC may reach the hadronic scale, $\sqrt{qB} \sim (10 - 100$ MeV) [2, 3]. Such strong magnetic fields can also be created in future at the ALICE experiment at LHC, at the Facility for Antiproton and Ion Research (FAIR) at GSI, and at the Nuclotron Ion Collider fAcility (NICA) in Dubna.

The strong magnetic fields can also enhance the chiral symmetry breaking. The chiral perturbation theory predicts a linear rise of the chiral condensate $\langle \bar{\psi}\psi \rangle$ with the strength of the field [4], that is in an accordance with our lattice results [5]. In the AdS/QCD approach [6], in Nambu-Jona-Lasinio model [7] and in sigma models [8] the value of $\langle \bar{\psi}\psi \rangle$ rises quadratically with the external magnetic field.

The strong magnetic fields lead also to the chiral magnetization of QCD vacuum at zero and finite temperature. This effect has a paramagnetic nature because the quarks are 1/2-spin particles and the external magnetic field leads to a polarization of magnetic moments of quarks in the direction of the magnetic field. The magnetization of the QCD vacuum is essential for the properties of the nucleon magnetic moments [10]. Our lattice simulations [9] demonstrate at the relatively weak magnetic fields the chiral magnetization rises linearly in accordance with Ref. [10]. At the large values of the magnetic fields a behavior of the magnetization becomes a nonlinear function of the magnetic field [11].

We also observed an anomalous quark magnetization in intense magnetic fields which appears due to the topological fluctuations of the QCD vacuum [14]. The essence of this effect is that quark gets a local electric dipole moment in addition to the magnetic moment. Both magnetic and anomalous electric moments are parallel to the axis of the external magnetic field. The generation of the anomalous electric dipole moment is a spin analogue of the chiral magnetic effect (CME).

The CME is the generation of a local electric current in the direction of the external magnetic field in the topologically nontrivial background of gluons [2, 15]. If we consider the u and d -quarks as massless particles then the right-handed quarks should move in the direction of the magnetic field and the left-handed quarks should move in the opposite direction because in the external field magnetic moments of quarks are parallel to the direction of the field. A topological charge of a gluon configuration is non-zero leads to a local imbalance between left-handed and right-handed quarks arises which, in turns, gives rise to a nonzero net electric current long the axis of the magnetic field. An evidence of the CME was observed by the STAR Collaboration at RHIC as a non-statistical asymmetry of negatively and positively charged particles emitted at different sides of the reaction plane of noncentral heavy-ion collisions [16, 17]. The observed effect indicates the presence of the chirality fluctuations in heavy-ion collisions. Below we report the evidence of the CME in the lattice gauge theory following our original study [21].

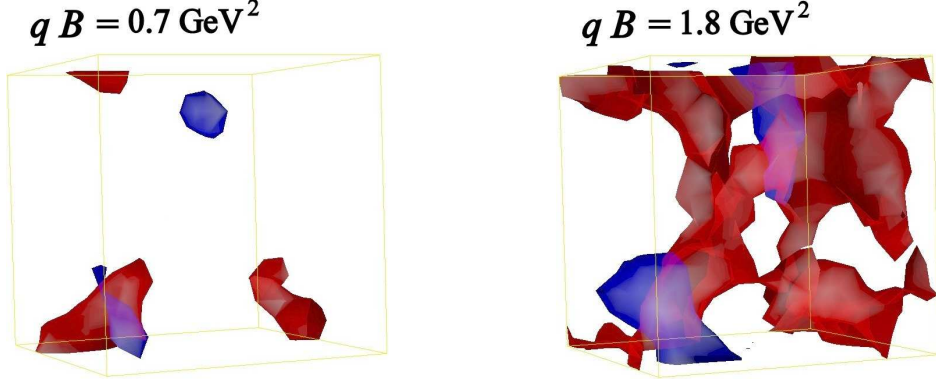


Figure 1: The excess of the electric charge density due to the external magnetic field $qB = 0.7 \text{ GeV}^2$ (left) and $qB = 1.81 \text{ GeV}^2$ (right) on 14^4 lattice corresponding to the physical volume of $(1.44 \text{ fm})^4$. The red regions corresponds to the excess of the positive charge density, while the blue color marks the excess of the negative charge density.

2. Details of calculations

We generate statistically independent $SU(2)$ gauge configurations with the tadpole improved Symanzik action [18]. Using the chirally invariant Neuberger operator [19], we solve the Dirac equation $D\psi_k = \lambda_k \psi_k$ numerically and determine the corresponding eigenfunctions ψ_k and eigenvalues λ_k . Here $D = \gamma^\mu (\partial_\mu - iA_\mu)$ is the massless Dirac operator in the gauge field background A_μ . The uniform magnetic field is introduced into this operator as described in Ref. [5].

We performed the zero-temperature simulations on 14^4 lattice with the lattice spacing $a = 0.103 \text{ fm}$ and on 16^4 lattice with the lattice spacings $a = 0.103 \text{ fm}$ and $a = 0.089 \text{ fm}$. At finite temperature we used $16^3 \times 6$ lattices with the spacings $a = 0.128 \text{ fm}$ ($T = 256 \text{ MeV} = 0.82 T_c$) and $a = 0.095 \text{ fm}$ ($T = 350 \text{ MeV} = 1.12 T_c$). The critical temperature of the phase transition in $SU(2)$ gauge theory is $T_c = 313.(3) \text{ MeV}$ [20]. The values of the magnetic field are quantized due to the periodic boundary conditions imposed in finite lattice volume. In our simulations a minimal nonzero value of magnetic field is $qB_{min} = (348 \text{ MeV})^2$.

3. Fluctuations of electric charge density

In order to study the simplest signatures of the chiral magnetic effect we first explore the fluctuations of the electromagnetic current

$$j_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x). \quad (3.1)$$

The zeroth component of (3.1) corresponds to a local charge density. We remove the diverging ultraviolet contributions by subtracting the charge density at zero magnetic field B from the charge density calculated at nonzero B . The result of the subtraction is a nonperturbative infrared quantity which is plotted in Fig. 1. Here we visualize the level surfaces of two typical charge density distributions in a fixed timeslice of the zero-temperature gauge field configuration for two values of the external magnetic field B . The magnetic field is directed from the bottom to the top.

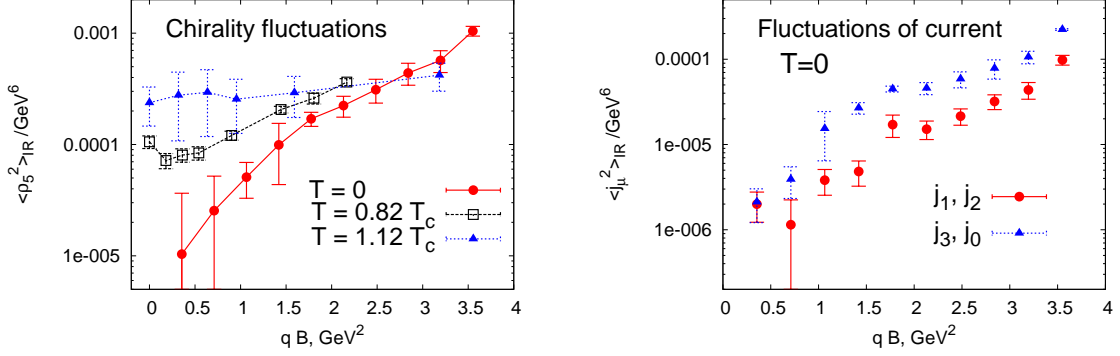


Figure 2: The expectation values of the squared chirality (left) and of the squares of the longitudinal (j_0 and j_3) and the transverse (j_1 and j_2) electric currents (right) vs the external magnetic field at three temperatures.

We see in Fig. 1 that the local charge density grows with the external magnetic field. The regions with the excess of a charge are extended along the direction of the magnetic field because at a sufficiently strong field the quarks occupy the lowest Landau level. In the absence of the gluon background the trajectories of the quarks should be elongated along the direction of the magnetic field. The gluon field distorts the motion of quarks, so that the trajectories of quarks diffuse.

4. Fluctuations of chirality

The other quantity which characterizes the chiral magnetic effect is the local chirality:

$$\rho_5(x) = \bar{\psi}(x)\gamma_5\psi(x). \quad (4.1)$$

The average value of (4.1) equals to zero, so that we measure the excess of the squared chirality:

$$\langle \rho_5^2 \rangle_{IR}(B, T) = \frac{1}{V} \int_V d^4x \langle \rho_5(x)^2 \rangle_{B, T} - \frac{1}{V} \int_V d^4x \langle \rho_5(x)^2 \rangle_{B=0, T=0}, \quad (4.2)$$

where V is the lattice volume. The subtraction of the $B = 0$ and $T = 0$ quantity is performed in order to regularize the total result in the ultraviolet region.

Fig. 2 (left) shows the squared chirality at different temperatures T . At zero temperature the chirality fluctuations increase quickly with the value of B . In the confinement phase at nonzero temperature ($T = 0.82 T_c$) the grows rate becomes smaller and in the deconfinement phase ($T = 1.12 T_c$) the squared chirality is almost independent on the value of the magnetic field.

The value of the chirality fluctuations at zero temperature and sufficiently large magnetic fields ($qB \sim 1\text{GeV}^2$) is close to the value of $\langle \rho_5^2 \rangle$ in the deconfinement phase ($T = 1.12 T_c$) at zero magnetic field. Thus, we can conclude that the chiral magnetic effect – if the confinement of quarks is absent – could be also observed in a cold nuclear matter exposed to the strong magnetic fields.

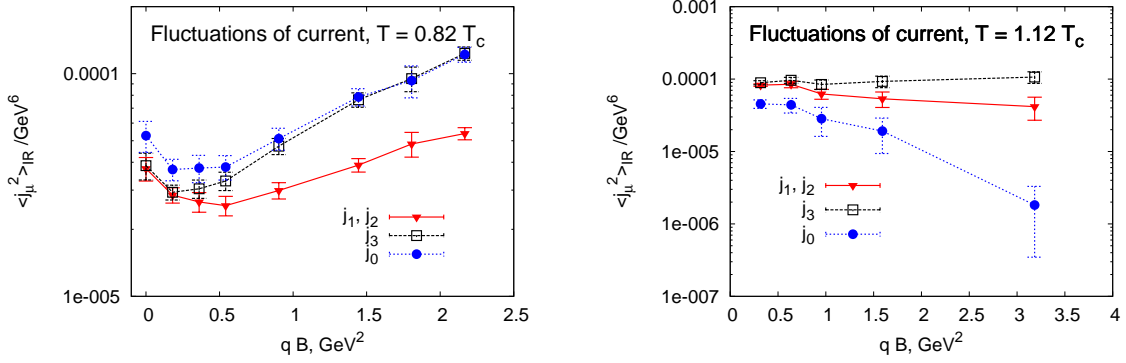


Figure 3: The squares of the longitudinal and transverse components of the electric currents vs the external magnetic field in the confinement phase (left) and in the deconfinement phase (right).

5. Fluctuations of electromagnetic current

We study the fluctuations of the electric current:

$$\langle j_\mu^2 \rangle_{IR}(B, T) = \frac{1}{V} \int_V d^4x \langle j_\mu^2(x) \rangle_{B, T} - \frac{1}{V} \int_V d^4x \langle j_\mu^2(x) \rangle_{B=0, T=0}. \quad (5.1)$$

We checked on the lattice, that all components of the electric current (3.1) are on average equal to zero within statistical error bars. The component of the current along the direction of the magnetic field is called longitudinal. Transverse components are perpendicular to the direction of the field. At $T = 0$ the magnetic field in the $\mu = 3$ direction breaks the rotational symmetry, so that we have: $\langle j_1^2 \rangle = \langle j_2^2 \rangle$ and $\langle j_0^2 \rangle = \langle j_3^2 \rangle$.

In Fig. 2 (right) we show all components of the electric field. All of them grow with the strength of the field. The longitudinal component of the electric current and the electric charge fluctuate stronger than the transverse ones. The fluctuations of the spatially transverse components grow with the field because the transverse momentum of a quark occupying the lowest Landau level increases with the strength of the magnetic field.

At $T \neq 0$ the rotational symmetry is broken by the temperature and by the external magnetic field, so that the longitudinal and zero components are no more equivalent, $\langle j_0^2 \rangle \neq \langle j_3^2 \rangle$, while the spatially transverse components are still degenerate, $\langle j_1^2 \rangle = \langle j_2^2 \rangle$.

In Fig. 3 (left) we show the fluctuations of the electric current in the confinement phase. At weak fields all components decrease slightly (the decrease is, however, within the statistical errors). Then the fluctuations start to increase with the value of the magnetic field. The components $\langle j_0^2 \rangle$ and $\langle j_3^2 \rangle$ are larger than $\langle j_1^2 \rangle$ and $\langle j_2^2 \rangle$. From Fig. 3 (right) we see, that in the deconfinement phase the components $\langle j_1^2 \rangle$, $\langle j_2^2 \rangle$ and $\langle j_3^2 \rangle$ are almost independent on the external magnetic field B , while the charge fluctuation $\langle j_0^2 \rangle$ is a decreasing function of B . This observation can be explained by the Debye screening in the deconfinement phase.

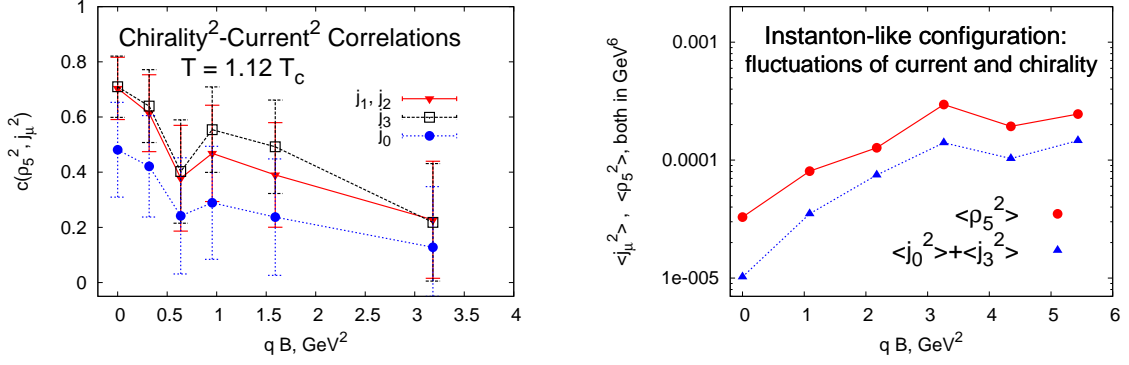


Figure 4: Left: the correlation function of $\rho_5^2(x)$ and $j_\mu^2(x)$, Eq. (6.1). Right: The values of $\langle \rho_5^2 \rangle$ and $\langle j_\mu^2 \rangle$ at an instanton-like configuration with $Q = 1$.

6. Chirality-current correlations

We observed that the $\langle \rho_5 j_\mu \rangle = 0$. This fact is natural due to the CP -oddness of this quantity. Next, we study the correlation between the squares of the chirality and the induced current:

$$c(\rho_5^2, j_\mu^2) = \frac{\langle \rho_5^2 j_\mu^2 \rangle - \langle \rho_5^2 \rangle \langle j_\mu^2 \rangle}{\langle \rho_5^2 \rangle \langle j_\mu^2 \rangle}. \quad (6.1)$$

We found that in the confinement phase the value of (6.1) vanishes for all components of the electric currents. In the deconfinement phase the correlator is decreasing function of the field [Fig. 4 (left)]. Therefore, the enhancement of the fluctuations of the electric current in the direction of the field is not locally correlated with the chirality fluctuations.

7. Instanton-like configuration

We have also studied signatures of the chiral magnetic effect at an instanton-like gauge field configuration with unit topological charge, $Q = 1$. In Fig. 4 (right) we show the fluctuations of the local chirality and the fluctuations of the squared longitudinal electric current in the instanton background. We observed the growth of the chiral density with the strength of the applied external magnetic field. The growth is caused by a difference between the particles with positive and negative chiralities in the $Q = 1$ background. The net longitudinal electric current, $j_\parallel^2 = j_0^2 + j_3^2$, increases with the strength of the field as well. This is the signature of the chiral magnetic effect.

8. Conclusion

We found the signatures of the chiral magnetic effect in lattice SU(2) gluodynamics in a background of the strong external magnetic field. This effect arises due to imbalance between left- and right-handed quarks (the local chirality). The average chirality in a large enough volume is equal

to zero, but the local fluctuations of the chiral density can be sufficiently strong. We found that the fluctuations of the chirality at zero temperature increase with the strength of the magnetic field. As the temperature increases the fluctuations become weaker dependent on the strength of the field B . In the deconfinement phase the local chirality fluctuations are almost independent on B .

We found that in the confinement phase all components of the electric current of quarks are growing in intense magnetic fields. The longitudinal (with respect to the axis of the magnetic field) currents fluctuate stronger than the transverse ones. In the deconfinement phase the thermal fluctuations lead to the increase of the current fluctuations while all spatial components of the current are insensitive to the value of the magnetic field. The charge fluctuations decrease as the field gets stronger. The fluctuations of the electric current and chirality are almost independent.

Finally, we observed the chiral magnetic effect at the instanton-like configuration.

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