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Spectroscopy and Decay Constants from Nonperturbative HQET at Order $1/m$

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We carry out a thorough analysis with the GEVP method to obtain ground-state and first-excited-state masses and decay constants of bottom-strange (pseudo-scalar and vector) mesons. This computation is done for quenched, nonperturbatively renormalized HQET, including order $1/m_b$ terms. The continuum limit is obtained using three lattice spacings and two static actions.

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1. Introduction

It is not yet feasible to perform simulations on lattices that can simultaneously represent the two relevant scales of B-physics: the low energy scale Λ_{QCD}, requiring large physical lattice size, and the high energy scale of the b-quark mass m_b, requiring very small lattice spacing a.

A promising alternative is to consider (lattice) heavy-quark effective theory (HQET), which allows for an elegant theoretical treatment, with the possibility of fully nonperturbative renormalization [1] (see [2] for a review). The approach is briefly described as follows. HQET provides a valid low-momentum description for systems with one heavy quark, with manifest heavy-quark symmetry in the limit $m_b \to \infty$. The heavy-quark flavor and spin symmetries are broken at finite values of m_b respectively by kinetic and spin terms, with first-order corrections to the static Lagrangian parametrized by ω_{kin} and ω_{spin}

$$\mathscr{L}^{\mathrm{HQET}} = \overline{\psi}_h(x)\, D_0\, \psi_h(x) - \omega_{\mathrm{kin}}\, \mathscr{O}_{\mathrm{kin}} - \omega_{\mathrm{spin}}\, \mathscr{O}_{\mathrm{spin}}\,, \tag{1.1}$$

where

$$\mathscr{O}_{\mathrm{kin}} = \overline{\psi}_h(x)\, \mathbf{D}^2\, \psi_h(x)\,, \qquad \mathscr{O}_{\mathrm{spin}} = \overline{\psi}_h(x)\, \sigma\cdot\mathbf{B}\, \psi_h(x)\,. \tag{1.2}$$

These $\mathrm{O}(1/m_b)$ corrections are incorporated by an expansion of the statistical weight in $1/m_b$ such that $\mathscr{O}_{\mathrm{kin}}$, $\mathscr{O}_{\mathrm{spin}}$ are treated as insertions into static correlation functions. This guarantees the existence of a continuum limit, with results that are independent of the regularization, provided that the renormalization be done nonperturbatively.

As a consequence, expansions for masses and decay constants are given respectively by

$$m_B = m_{\mathrm{bare}} + E^{\mathrm{stat}} + \omega_{\mathrm{kin}}\, E^{\mathrm{kin}} + \omega_{\mathrm{spin}}\, E^{\mathrm{spin}} \tag{1.3}$$

and

$$f_B \sqrt{\frac{m_B}{2}} = Z_A^{\mathrm{HQET}}\, p^{\mathrm{stat}}\, (1 + c_A^{\mathrm{HQET}}\, p^{\delta A} + \omega_{\mathrm{kin}}\, p^{\mathrm{kin}} + \omega_{\mathrm{spin}}\, p^{\mathrm{spin}})\,, \tag{1.4}$$

where the parameters m_{bare} and Z_A^{HQET} are written as sums of a static and an $\mathrm{O}(1/m_b)$ term (denoted respectively with the superscripts "stat" and "$1/m_b$" below), and c_A^{HQET} is of order $1/m_b$ (see e.g. [3] for unexplained notation). Bare energies (E^{stat}, etc.) and matrix elements (p^{stat}, etc.) are computed in the numerical simulation.

The divergences (with inverse powers of a) in the above parameters are cancelled through the nonperturbative renormalization, which is based on a matching of HQET parameters to QCD on lattices of small physical volume — where fine lattice spacings can be considered — and extrapolation to a large volume by the step-scaling method. Such an analysis has been recently completed for the quenched case [4]. In particular, there are nonperturbative (quenched) determinations of the static coefficients $m_{\mathrm{bare}}^{\mathrm{stat}}$ and Z_A^{stat} for HYP1 and HYP2 static-quark actions [5] at the physical b-quark mass, and similarly for the $\mathrm{O}(1/m_b)$ parameters ω_{kin}, ω_{spin}, $m_{\mathrm{bare}}^{1/m_b}$, Z_A^{1/m_b} and c_A^{HQET}.

The newly determined HQET parameters are very precise (with errors of a couple of a percent in the static case) and show the expected behavior with a. They are used in our calculations reported here, to perform the nonperturbative renormalization of the (bare) observables computed in the simulation. Of course, in order to keep a high precision, also these bare quantities have to be accurately determined. This is accomplished by an efficient use of the generalized eigenvalue problem (GEVP) for extracting energy levels E_n and matrix elements, as described below.

A significant source of systematic errors in the determination of energy levels in lattice simulations is the contamination from excited states in the time correlators

$$C(t) \;=\; \langle O(t)\, O(0)\rangle \;=\; \sum_{n=1}^{\infty} |\langle n|\, \hat{O}\, |0\rangle|^2 \, e^{-E_n t} \tag{1.5}$$

of fields $O(t)$ with the quantum numbers of a given bound state.

Instead of starting from simple local fields O and getting the (ground-state) energy from an effective-mass plateau in $C(t)$ as defined above, it is then advantageous to consider all-to-all propagators [6] and to solve, instead, the GEVP

$$C(t)\, v_n(t,t_0) \;=\; \lambda_n(t,t_0)\, C(t_0)\, v_n(t,t_0)\,, \tag{1.6}$$

where $t > t_0$ and $C(t)$ is now a matrix of correlators, given by

$$C_{ij}(t) \;=\; \langle O_i(t) O_j(0)\rangle \;=\; \sum_{n=1}^{\infty} \mathrm{e}^{-E_n t}\, \Psi_{ni}\Psi_{nj}\,, \quad i,j = 1,\ldots,N\,. \tag{1.7}$$

The chosen interpolators O_i are taken (hopefully) linearly independent, e.g. they may be built from the smeared quark fields using N different smearing levels. The matrix elements Ψ_{ni} are defined by

$$\Psi_{ni} \;\equiv\; (\Psi_n)_i \;=\; \langle n|\hat{O}_i|0\rangle\;, \quad \langle m|n\rangle \;=\; \delta_{mn}\,. \tag{1.8}$$

One thus computes C_{ij} for the interpolator basis O_i from the numerical simulation, then gets effective energy levels E_n^{eff} and estimates for the matrix elements Ψ_{ni} from the solution $\lambda_n(t,t_0)$ of the GEVP at large t. For the energies

$$E_n^{\mathrm{eff}}(t,t_0) \;\equiv\; \frac{1}{a}\, \log \frac{\lambda_n(t,t_0)}{\lambda_n(t+a,t_0)} \tag{1.9}$$

it is shown [7] that $E_n^{\mathrm{eff}}(t,t_0)$ converges exponentially as $t \longrightarrow \infty$ (and fixed t_0) to the true energy E_n. However, since the exponential falloff of higher contributions may be slow, it is also essential to study the convergence as a function of t_0 in order to achieve the required efficiency for the method. This has been done in [3], by explicit application of (ordinary) perturbation theory to a hypothetical truncated problem where only N levels contribute. The solution in this case is exactly given by the true energies, and corrections due to the higher states are treated perturbatively. We get

$$E_n^{\mathrm{eff}}(t,t_0) \;=\; E_n + \varepsilon_n(t,t_0) \tag{1.10}$$

for the energies and

$$\mathrm{e}^{-\hat{H}t}\,(\hat{\mathscr{Q}}_n^{\mathrm{eff}}(t,t_0))^{\dagger}|0\rangle \;=\; |n\rangle + \sum_{n'=1}^{\infty} \pi_{nn'}(t,t_0)\,|n'\rangle \tag{1.11}$$

for the eigenstates of the Hamiltonian, which may be estimated through[1]

$$\hat{\mathscr{Q}}_n^{\mathrm{eff}}(t,t_0) \;=\; R_n\,(\hat{O}\,,\, v_n(t,t_0))\,, \tag{1.12}$$

$$R_n \;=\; (v_n(t,t_0)\,,\, C(t)\, v_n(t,t_0))^{-1/2}\, \left[\frac{\lambda_n(t_0+a,t_0)}{\lambda_n(t_0+2a,t_0)}\right]^{t/2}\,. \tag{1.13}$$

[1]The choice of R_n we make here leads to smaller statistical errors than the one in [3], while the form of the corrections π remains unchanged.

In our analysis we see that, due to cancellations of t-independent terms in the effective energy, the first-order corrections in $\varepsilon_n(t,t_0)$ are independent of t_0 and very strongly suppressed at large t. We identify two regimes: 1) for $t_0 < t/2$, the 2nd-order corrections dominate and their exponential suppression is given by the smallest energy gap $|E_m - E_n| \equiv \Delta E_{m,n}$ between level n and its neighboring levels m; and 2) for $t_0 \geq t/2$, the 1st-order corrections dominate and the suppression is given by the large gap $\Delta E_{N+1,n}$. Amplitudes $\pi_{nn'}(t,t_0)$ get main contributions from the first-order corrections. For fixed $t - t_0$ these are also suppressed with $\Delta E_{N+1,n}$. Clearly, the appearance of large energy gaps in the second regime improves convergence significantly. We therefore work with t, t_0 combinations in this regime.

A very important step of our approach is to realize that the same perturbative analysis may be applied to get the $1/m_b$ corrections in the HQET correlation functions mentioned previously

$$C_{ij}(t) \;=\; C_{ij}^{\rm stat}(t) + \omega C_{ij}^{1/m_b}(t) + \mathrm{O}(\omega^2)\,, \tag{1.14}$$

where the combined $\mathrm{O}(1/m_b)$ corrections are symbolized by the expansion parameter ω. Following the same procedure as above, we get similar exponential suppressions (with the static energy gaps) for static and $\mathrm{O}(1/m_b)$ terms in the effective theory. We arrive at

$$E_n^{\rm eff}(t,t_0) \;=\; E_n^{\rm eff,stat}(t,t_0) + \omega E_n^{{\rm eff},1/m_b}(t,t_0) + \mathrm{O}(\omega^2) \tag{1.15}$$

with

$$E_n^{\rm eff,stat}(t,t_0) \;=\; E_n^{\rm stat} + \beta_n^{\rm stat}\,\mathrm{e}^{-\Delta E_{N+1,n}^{\rm stat}\,t} + \ldots\,, \tag{1.16}$$

$$E_n^{{\rm eff},1/{\rm m_b}}(t,t_0) \;=\; E_n^{1/m_b} + [\beta_n^{1/m_b} - \beta_n^{\rm stat}\,t\;\Delta E_{N+1,n}^{1/m_b}]\mathrm{e}^{-\Delta E_{N+1,n}^{\rm stat}\,t} + \ldots\,. \tag{1.17}$$

and similarly for matrix elements.

Preliminary results of our application of the methods described in this section are presented next. A more detailed version of this study will be presented elsewhere [8].

2. Results

We carried out a study of static-light $\mathrm{B_s}$-mesons in quenched HQET with the nonperturbative parameters described in the previous section, employing the HYP1 and HYP2 lattice actions for the static quark and an $\mathrm{O}(a)$-improved Wilson action for the strange quark in the simulations. The lattices considered were of the form $L^3 \times 2L$ with periodic boundary conditions. We took $L \approx 1.5$ fm and lattice spacings 0.1 fm, 0.07 fm and 0.05 fm, corresponding respectively to $\beta = 6.0219$, 6.2885 and 6.4956. We used all-to-all strange-quark propagators constructed from approximate low modes, with 100 configurations. Gauge links in interpolating fields were smeared with 3 iterations of (spatial) APE smearing, whereas Gaussian smearing (8 levels) was used for the strange-quark field. A simple $\gamma_0\gamma_5$ structure in Dirac space was taken for all 8 interpolating fields. Also, the local field (no smearing) was included in order to compute the decay constant.

The resulting (8×8) correlation matrix may be conveniently truncated to an $N \times N$ one and the GEVP solved for each N, so that results can be studied as a function of N. We have considered

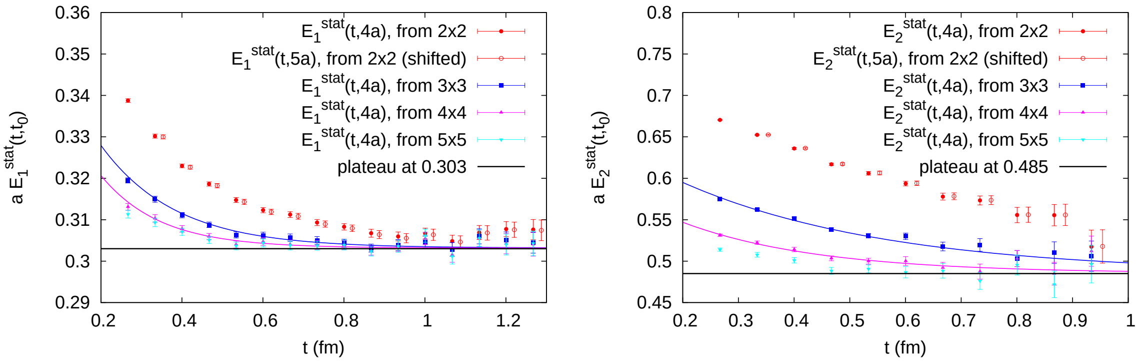

Figure 1: Static energy spectrum obtained from the **first basis** of interpolators for the case of HYP2 action and $\beta \approx 6.3$: ground state (left) and first excited state (right).

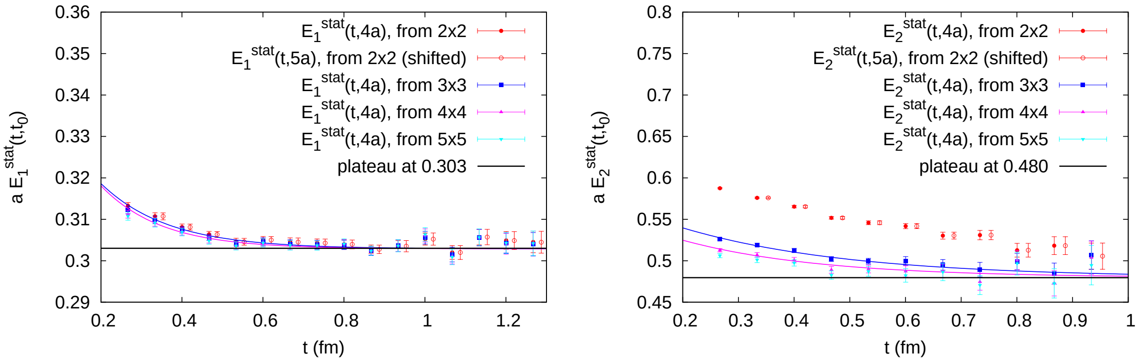

Figure 2: Static energy spectrum obtained from the **second basis** of interpolators for the case of HYP2 action and $\beta \approx 6.3$: ground state (left) and first excited state (right).

two bases of interpolators. The **first basis** was obtained by projecting (at $t_i \approx 0.2$ fm) with the N eigenvectors of $C(t_i)$ with the largest eigenvalues

$$C(t_i)\, b_n = \lambda_n\, b_n \quad \Rightarrow \quad C^{(N\times N)}_{nm}(t) \;=\; b_n^\dagger C(t)\, b_m\,, \quad n,m \le N\,. \tag{2.1}$$

For N not too large, this avoids numerical instabilities and large statistical errors in the GEVP. This basis was used in [3], where also the normalization of the different smeared fields is specified. Note that the (relative) normalization does matter in Eq. (2.1). The **second basis** was picked from unprojected interpolators, sampling the different smearing levels (from 1 to 7) as $\{1,7\}$,$\{1,4,7\}$, etc. Our results for the effective energies using the two bases are shown in Figs. 1 and 2, where the solid lines correspond to a simultaneous fit of the various energy levels and values of N to the behavior[2] in Eq. (1.16). In both cases, we observe the predicted t_0 independence in the GEVP solution for the energies. We see a much stronger dependence on N for the ground state in the first case. Nevertheless, our final results remain unchanged within their errors, which are determined as described in [3].

We then take the continuum limit, as shown in Figs. 3 and 4. We see that the correction to the ground-state energy due to terms of order $1/m_b$, which is positive for finite a, is quite small

[2] For $N = 2$ care must be taken, since the subleading corrections may not be negligible in the available t interval. This happens for our first basis above. In any case, we do not include data from $N = 2$ in our analysis.

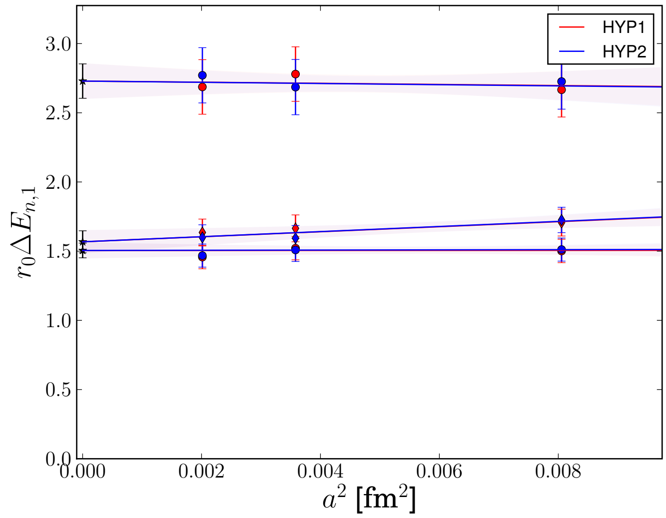

Figure 3: Continuum extrapolation (joint limit for HYP1, HYP2) of the energy splittings (in MeV). Shown are ΔE_{21} (lower two curves, respectively the static and the full values) and ΔE_{31} (static, uppermost curve).

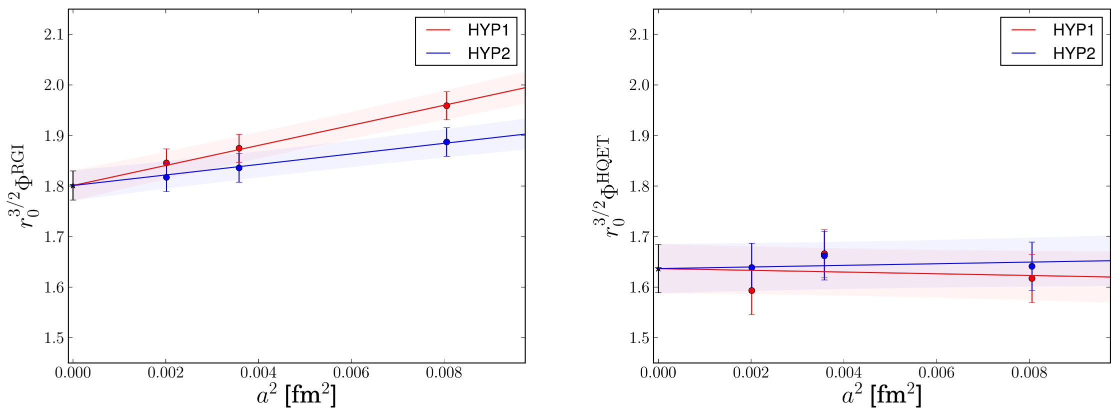

Figure 4: Continuum extrapolation (joint limit for HYP1, HYP2) of the pseudoscalar meson decay constant in the static limit (left) and to order $1/m_b$ (right).

(consistent with zero) in the continuum limit. Our results for the pseudoscalar meson decay constant, both in the static limit and including $\mathrm{O}(1/m_b)$ corrections, are shown in terms of the combination $\Phi^{\rm HQET} \equiv F_{\rm PS}\sqrt{m_{\rm PS}}/C_{\rm PS}$, where $C_{\rm PS}(M/\Lambda_{\rm QCD})$ is a known matching function and $\Phi^{\rm RGI}$ denotes the renormalization-group-invariant matrix element of the static axial current [9]. These two continuum extrapolations are shown in comparison with fully relativistic heavy-light (around charm-strange) data from [9] in Fig. 5 below. Note that, up to perturbative corrections of order α^3 in $C_{\rm PS}$, HQET predicts a behavior $const. + \mathrm{O}(1/r_0 m_{\rm PS})$ in this graph. Surprisingly no $1/(r_0 m_{\rm PS})^2$ terms are visible, even with our rather small errors.

3. Conclusions

The combined use of nonperturbatively determined HQET parameters (in action and currents) and efficient GEVP allows us to reach precisions of a few percent in matrix elements and of a

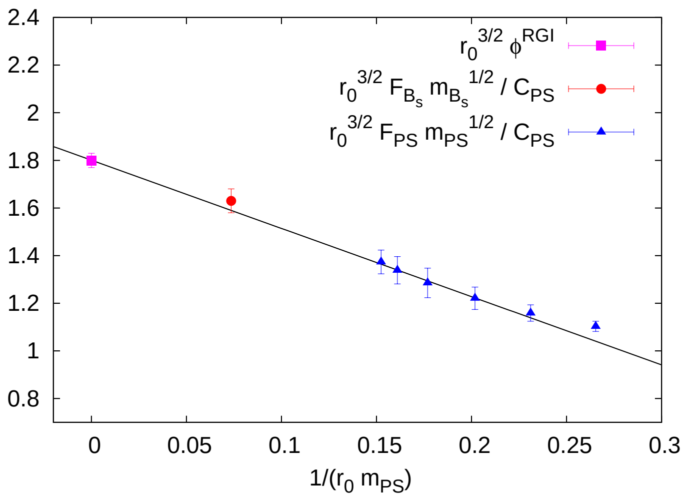

Figure 5: Comparison of the continuum values for the pseudoscalar meson decay constant from Fig. 4 to fully relativistic data in the charm region. The solid line is a linear interpolation between the static limit and the points around the charm-quark mass, which corresponds to $1/r_0 m_{PS} \approx 0.2$.

few MeV in energy levels, even with only a moderate number of configurations. The method is robust with respect to the choice of interpolator basis. All parameters have been determined non-perturbatively and in particular power divergences are completely subtracted. We see that HQET plus $O(1/m_b)$ corrections at the b-quark mass agrees well with an interpolation between the static point and the charm region, indicating that linearity in $1/m$ extends even to the charm point. A corresponding study for $N_f = 2$ is in progress.

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