Flavor-singlet mesons in $N_f = 2 + 1$ QCD with dynamical overlap quarks

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We report on our study of flavor-singlet mesons in three-flavor QCD with dynamical overlap quarks. Gauge ensembles are generated on a $16^3 \times 48$ lattice at a lattice spacing of 0.10 fm with the strange quark masses around its physical value $m_{s,\text{phys}}$ and up and down quark masses down to $m_{s,\text{phys}}/5$. Connected and disconnected meson correlators are calculated using the all-to-all quark propagator. We present our preliminary results on the spectrum of flavor-singlet pseudoscalar and vector mesons.

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1. Introduction

A quantitative understanding of interesting properties of the flavor-singlet mesons is an important subject in lattice QCD. The famous $U(1)$ problem is a long-standing issue albeit past ceaseless efforts. It is also well-known that the flavor-singlet and octet vector mesons mix with each other almost ideally, though this has not yet been confirmed from first principles.

In this article, we report on our study of the flavor-singlet pseudoscalar (PS) and vector mesons. There are two salient features of this work: i) we simulate $N_f = 2+1$ QCD including the effects of dynamical strange quarks, which have been often ignored in previous studies of the flavor-singlet mesons, and ii) we use the all-to-all quark propagator [1] to calculate disconnected meson correlators, which induce the meson mixings and the mass splittings from flavor-non-singlet mesons.

2. Simulation method

Our gauge configurations of $N_f = 2+1$ QCD are generated on a $16^3 \times 48$ lattice using the Iwasaki gauge action and the overlap quark action. We also introduce a topology fixing term [2] into our lattice action to reduce the computational cost, and simulate only the trivial topological sector $Q = 0$ at this stage. The lattice spacing determined from $F_\pi$ is 0.100(5) fm. We take four values of the degenerate up and down quark masses $m_t = 0.015, 0.025, 0.035$ and 0.050, which cover a range of the pion mass from 350 to 610 MeV. Two values $m_s = 0.080$ and 0.100 are chosen for the strange quark mass. The physical quark masses fixed from $M_\pi$ and $M_K$ are $m_{s,\text{phys}} = 0.002$ and $m_{s,\text{phys}} = 0.065$. Statistics are 2,500 HMC trajectories at each combination of $m_t$ and $m_s$. We refer readers to Ref.[3] for further details on our gauge configurations.

We measure PS and vector meson correlators using the all-to-all quark propagator. For each configuration, we prepare 160 low-lying modes $\lambda_m^{(k)} u^{(k)}$ ($k = 1, \ldots, N_e(=160)$) of the overlap-Dirac operator $D(m)$, where $m$ is the valence quark mass. Their contribution to the quark propagator is calculated exactly. The higher modes are taken into account stochastically by the noise method. We prepare a single noise vector $\eta$ for each configuration, and dilute [1] it into our lattice action to reduce the computational cost, and simulate only the trivial topological Iwasaki gauge action and the overlap quark action. We also introduce a topology fixing term [2] and a smearing function $\Gamma$.

We then construct the following meson field at the temporal coordinate $t$ with the Dirac matrix $\Gamma$ and a smearing function $\phi(r)$

$$\phi^{(k, l)}_{\Gamma, \phi}(m; t) = \sum_{x, r} \phi(r) w(x + r, t)^{(k)} \Gamma v_m(x, t)^{(l)}. \quad (2.4)$$
We calculate all possible correlators with the following five different choices of the smearing function (namely $i, j = 0, \ldots, 4$)

\[
\phi_0(r) = \delta_r, 0,\quad \phi_1(r) = \exp[-0.4|r|],\quad \phi_2(r) = \exp[-0.4|w|],
\]

(2.8)

\[
\phi_3(r) = \exp[-1.0|w|],\quad \phi_4(r) = \text{constant}
\]

with the normalization $\sum_r |\phi_i(r)|^2 = 1$. The calculation of all these meson correlators is computationally cheap, once we prepare the $v$ and $w$ vectors of Eq. (2.2).

Since the quark propagator is decomposed into low- and high-mode contributions, the disconnected correlators can be divided into four contributions, i.e., $D = D_{LL} + D_{LH} + D_{HL} + D_{HH}$. We calculate these four contributions separately in our measurement.

### 3. Meson correlators

In Fig. 2, we show an example of the light PS and vector meson correlators. We observe that, at relatively small $\Delta t$, the disconnected piece $D_{P,ij}$ is not a large correction to the full correlator $G_{P,ij}$ and it is dominated by the low-mode contribution $D_{LL}$. Therefore, we may safely ignore the high mode contributions to $D_{P,ij}$, namely $D_{LH}^H$, $D_{HL}^H$, and $D_{HH}^H$, to calculate the full correlator as

\[
G_{P,ab,ij}^{D=LL}(\Delta t) = C_{P,ab,ij}(\Delta t) - D_{P,ab,ij}(\Delta t) \quad (a \neq b, \{1, 2\}).
\]

(3.1)
Figure 2 actually shows that $G_{P,||}$ is well approximated by $G_{P,||}^{D=LL}$ in the whole region of $\Delta t$. As shown in the same figure, the vector meson full correlator $G_{V,||}$ turns out to be noisy at relatively large $\Delta t$. The large uncertainty mainly comes from those of high-mode contributions $D_{V,||}^{[LL,HL,HH]}$ due to the noise method with the small number of noise samples. We observe that $G_{V,||}$ at small $\Delta t$ is well approximated by $G_{V,||}^{D=LL}$ defined as in Eq. (3.1), and expect that the high-mode contributions $D_{V,||}^{[LL,HL,HH]}$ remain to be small at larger $\Delta t$ since they mainly describe short distance physics. Then $G_{V,||}$ is expected to be well approximated by $G_{V,||}^{D=LL}$ also at large $\Delta t$.

From these observations, we use the meson correlators $G_{P,||}^{D=LL}$ ignoring the noisy contributions $D_{P,||}^{[LL,HL,HH]}$ to study the spectrum of the flavor-singlet mesons. The superscript “$D=LL$” is suppressed in the following for simplicity.

4. Vector mesons

We plot the vector meson correlators in Fig. 3. In the light-strange basis, the off-diagonal correlators $G_{V,\{ls,ls\}}$ are about two orders of magnitude smaller than the diagonal ones $G_{V,\{ll,ss\}}$. There is no such large hierarchy in $G_{V,\{88,00,80,08\}}$ in the octet-singlet basis. Since the off-diagonal correlators induce the meson mixing, the above observations indicate that the mixing of the vector mesons is close to the ideal mixing: namely, $V_{1}$ and $V_{0}$ mesons mix significantly with each other to form $\omega$ and $\phi$ mesons, which are well approximated by $V_{1}$ and $V_{0}$.

For a more quantitative examination, we solve the generalized eigenvalue problem (GEVP)

$$C(\Delta t')^{-1/2}C(\Delta t)C(\Delta t')^{-1/2}\tilde{u}_{n} = \tilde{\lambda}_{n} \tilde{u}_{n} \quad (n = 0, 1),$$

where $C(\Delta t)$ is $2 \times 2$ correlator matrix with specified smearing functions for source and sink

$$C(\Delta t) = \begin{pmatrix} G_{V,88}(\Delta t) & G_{V,80}(\Delta t) \\ G_{V,08}(\Delta t) & G_{V,00}(\Delta t) \end{pmatrix}.$$  

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The creation operators of the energy eigenstates, namely, \( \phi \) and \( \omega \) mesons, are determined from the eigenvectors \( \tilde{u}_n \). We obtain the following relation for the local operators

\[
\begin{align*}
\phi &= 0.84(5) V_5 - 0.55(7) V_0 = 1.00(1) V_s - 0.04(9) V_l \\
\omega &= 0.55(7) V_5 + 0.84(5) V_0 = 0.04(9) V_s + 1.00(1) V_l
\end{align*}
\]

(4.3)

which implies the ideal mixing of the vector mesons. We observe that this relation of vector meson operators does not change significantly with other choices of the smearing functions.

In Fig. 4, we plot the effective masses of \( \omega \) and \( \phi \) mesons determined from the eigenvalues \( \tilde{\lambda}_a = \exp[-E_n(\Delta t - \Delta t')] \). By taking a sufficiently large \( \Delta t' \), the effective masses show small dependence on \( \Delta t \) as well as on the smearing functions. The same figure also shows that \( M_\omega \) and \( M_\phi \) are close to those from the connected correlators of the light and strange mesons \( \tilde{C}_{V,\{l,s\}} \). This is because the diagonal disconnected correlators \( D_{V,\{l,s\}} \) are small as seen in Fig. 2, and hence they have small effects to \( M_\omega \) and \( M_\phi \).

In this analysis, we extrapolate \( M_{\omega(\phi)} \) to the physical point using a simple linear form

\[
M_{\omega(\phi)} = a_{\omega(\phi)} + b_{\omega(\phi)} m_l + c_{\omega(\phi)} m_s.
\]

(4.4)

Figure 3: Vector meson correlators in the light-strange (left panel) and octet-singlet bases (right panel). Both panels show results with the exponential smearing function \( \phi_I \) at (\( m_l, m_s \)) = (0.025, 0.080). Filled and open symbols represent diagonal and off-diagonal correlators, respectively.

Figure 4: Left panel: effective masses of \( \omega \) (solid symbols) and \( \phi \) mesons (shaded symbols) at (\( m_{\text{phys}}, m_s \)) = (0.025, 0.080). We also plot effective masses of \( C_{V,l} \) (open circles) and \( C_{V,s} \) (open squares). Right panel: chiral extrapolation of \( M_\phi \). Dotted and dashed lines show fit lines at \( m_s \) and \( m_{s,\text{phys}} \), respectively.
As shown in Fig. 4, this fit describes our data reasonably well with $\chi^2$/d.o.f $\sim 1.3$. We obtain $M_\eta = 909(25)_{\text{stat}}(-98)_{\text{sys}}$ MeV and $M_\phi = 1102(13)_{\text{stat}}(-97)_{\text{sys}}$ MeV, where the systematic error is estimated by including a higher order term $d_{G_\phi} m_l^{3/2}$ [4] and by using a different input $M_\Omega$ to fix the lattice spacing. These results are consistent with the experimental values $M_\eta = 783$ MeV and $M_\phi = 1019$ MeV. Note, however, that our data may suffer from significant finite volume corrections at two smallest quark masses $m_l = 0.015$ and 0.025, where $2.8 \leq M_\pi L \leq 3.2$. We are planning to extend this work to a larger volume $24^3 \times 48$ for a more precise comparison with the experiment.

5. PS mesons

Figure 5 shows correlators of the light and strange PS mesons. In contrast to the vector mesons, the off-diagonal correlators $G_{P,i|l,s,l}$ are not so small compared to the diagonal ones $G_{P,i|l,l}$ in the light-strange basis. This leads to a significant strange (light) quark component in $\eta$ ($\eta'$). For the local operators, we obtain

$$\eta = 0.96(1) P_l - 0.28(3) P_s, \quad \eta' = 0.28(3) P_l - 0.96(1) P_s.$$ (5.1)

As in phenomenological analyses [5], more unambiguous determination of the mixing matrix could be provided by constructing the local $\eta$ and $\eta'$ operators so that their decay constants reproduce the experimental values. We leave this for a future study.

As predicted analytically [6] and as seen in Figs. 2 and 5, disconnected contribution $D_{P,ab}$ induces a constant term in the full correlator $G_{P,ab}$ at fixed topology, e.g.

$$m_l G_{P,l} \xrightarrow{\Delta t \to \infty} \frac{\chi_l}{V} \left(1 - \frac{Q_l^2}{2\chi_l} + \frac{c_4}{2\chi_l^2} \right),$$ (5.2)

which is suppressed by $1/V$. While this term is useful to determine the topological susceptibility $\chi_l$ [7], this forces us to use $G_{P,ab}$ at small $\Delta t$ to extract the PS meson masses $M_\eta$ and $M_{\eta'}$. For the light-strange basis. This leads to a significant strange (light) quark component in $\eta$ ($\eta'$). For the local operators, we obtain

To eliminate excited state contamination at such small $\Delta t$, we solve the GEVP with the $10 \times 10$ correlator matrix including the smearing degrees of freedom

$$C(\Delta t) = \begin{pmatrix}
G_{P,88,00}(\Delta t) & \cdots & G_{P,88,04}(\Delta t) \\
\vdots & \ddots & \vdots \\
G_{P,08,40}(\Delta t) & \cdots & G_{P,00,44}(\Delta t)
\end{pmatrix}.$$ (5.3)

Effective masses of $\eta$ and $\eta'$ mesons are plotted in Fig. 6. Although $\eta$ seems to be lighter than $\eta'$, the existence of the constant term in Eq. (5.2) leads to a large uncertainty of $M_{\eta'}$: 10–15% already at simulated quark masses.

From a linear chiral extrapolation in terms of $m_l$ and $m_s$, we obtain $M_\eta = 639(50)_{\text{stat}}$ MeV and $M_{\eta'} = 840(136)_{\text{stat}}$ MeV at the physical point. These are consistent with the experimental values $M_\eta = 548$ MeV and $M_{\eta'} = 958$ MeV, though the statistical significance of the $\eta' - \eta$ mass splitting (1.4 $\sigma$) is not sufficient.
6. Conclusion

In this article, we report on our study of the flavor-singlet mesons in $N_f = 2 + 1$ QCD using the all-to-all quark propagator to calculate the disconnected correlators. For the vector meson, we observe that the small disconnected contributions in the light-strange basis lead to the almost ideal mixing and the small mass shift. This is consistent with the experimental fact $M_\omega \sim M_\rho$, and explains why the previous calculations of $M_\phi$ ignoring the disconnected contributions show reasonable agreement with experiment [8].

We need to improve the accuracy of $M_{\eta'}$ to establish the $\eta^\prime - \eta$ mass splitting. This could be done by simulating non-trivial topological sectors as well as by suppressing the fixed topology effects in Eq. (5.2) on a larger lattice. The latter is also important to suppress finite volume corrections to the PS and vector meson masses for a more detailed comparison with the experiment. Such simulations on a $24^3 \times 48$ lattice are in progress.

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References

[5] For a recent analysis, see F. Ambrosino et al. (KLOE collaboration), JHEP 0907, 105 (2009).
[8] For a recent review, see E. Scholz in these proceedings.