Heavy-light mesons in 2+1 flavor lattice QCD

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Heavy-light meson system is investigated using the relativistic heavy quark action on the 2+1 dynamical flavor PACS-CS configurations at the lattice spacing $a^{-1} = 2.2$ GeV and the spatial extent $L = 3$ fm. Dynamical up-down and strange quark masses as well as the valence charm quark mass are set around their physical values. We measure the charm-*ud* and charm-strange meson masses and decay constants. Our results are consistent with the experimental values except the hyperfine splitting of the charm-strange meson. We also estimate the CKM matrix elements in the second row.

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1. Introduction

Precise determination of the CKM matrix is an important step to establish the validity range of the Standard Model, thereby setting the basis for exploring physics at smaller space-time scales. In this respect the CKM matrix elements in the second row, especially $|V_{cd}|$ and $|V_{cs}|$, are still poorly determined with errors in the 10% range [1]. This situation contrasts sharply with those of the first row where $|V_{ud}|$ and $|V_{us}|$ are known at sub percent level.

Lattice QCD provides a variety of means for precise determination of the second row of the CKM matrix. With $|V_{cs}|$, for example, the leptonic decay width of the D_s meson $\Gamma(D_s \to l\nu)$ is given by

$$\Gamma(D_s \to l\nu) = \frac{G_F^2}{8\pi} m_l^2 m_{D_s} f_{D_s}^2 \left(1 - \frac{m_l^2}{m_{D_s}^2}\right)^2 |V_{cs}|^2. \tag{1.1}$$

so that a lattice determination of the decay constant f_{D_s} combined with the experimental value of $\Gamma(D_s \to l\nu)$ will allow us to extract $|V_{cs}|$. The other matrix element $|V_{cd}|$ can be obtained in the same way.

An intriguing topic in the charm quark sector is a disagreement of the value of the D_s meson decay constant from lattice QCD and experiment. In 2007 HPQCD and UKQCD Collaboration reported a calculation of this quantity using the HISQ action for heavy quark on the 2+1 dynamical flavor MILC configurations [2]. Their value $f_{D_s} = 241 \pm 3$ MeV exhibited significant deviation from those of the CLEO and Belle experiments at the time. The latest CLEO value [3] $f_{D_s} = 259.5 \pm 7.3$ MeV, though smaller than 2007, is still two standard deviations away.

FNAL Collaboration also calculated the decay constant on the MILC configurations using the clover heavy quark formalism. Their original value $f_{D_s} = 249(3)(16)$ MeV [4] was also smaller than experiment, but an update this year [5] including a 2% change in the scale setting of the MILC ensemble moved the value up to $f_{D_s} = 260(10)$ MeV.

In this paper we present a status report of our work on the charmed mesons using the relativistic heavy quark formalism on the 2+1 dynamical flavor PACS-CS configurations.

2. Set up

The charm quark system is simulated with a relativistic heavy quark action on the 2+1 flavor lattice QCD configurations. The configurations are generated by the PACS-CS Collaboration [6] using the nonperturbatively $O(a)$-improved Wilson quark action with $c_{SW}^{NP} = 1.715$ [7] and the Iwasaki gauge action. The lattice size is $32^3 \times 64$ whose spatial extent is $L = 3$ fm with the lattice spacing of $a = 0.09$ fm. The dynamical up-down quark masses range from 8 MeV down to 3 MeV, which is close to the the physical value. We utilize the same quark action for the light quark propagators in our calculation.

For the heavy quarks, we employ a relativistic heavy quark action proposed in Ref. [8]. The cutoff errors are reduced from $O((m_Q a)^n)$ to $O(f(m_Q a)(a\Lambda_{QCD})^2)$ where $f(m_Q a)$ is an analytic function around $m_Q a = 0$. We use one-loop perturbative values for parameters in the heavy quark action [9]. In addition, nonperturbative contributions at the massless limit is included for the clover

Table 1: Simulation parameters. Quark masses are perturbatively renormalized in the $\overline{\text{MS}}$ scheme. The renormalization scale is $\mu = 1/a$ for each simulation point and $\mu = 2$ GeV for the physical point.

κ_{ud}	κ_{s}	$m_{\text{ud}}^{\overline{\text{MS}}}(\mu)$ [MeV]	$m_{\text{s}}^{\overline{\text{MS}}}(\mu)$ [MeV]	#conf	
				measured	MD time/total
0.13781	0.13640	3.5(2)	86.4(1)	65	1625/1625
0.13770	0.13660	8.3(5)	74.1(4)	60	1500/1500
0.137785	0.13660	3.5(1)	72.8(2)	200	1000/2000
physical point		2.53(5)	72.7(8)		
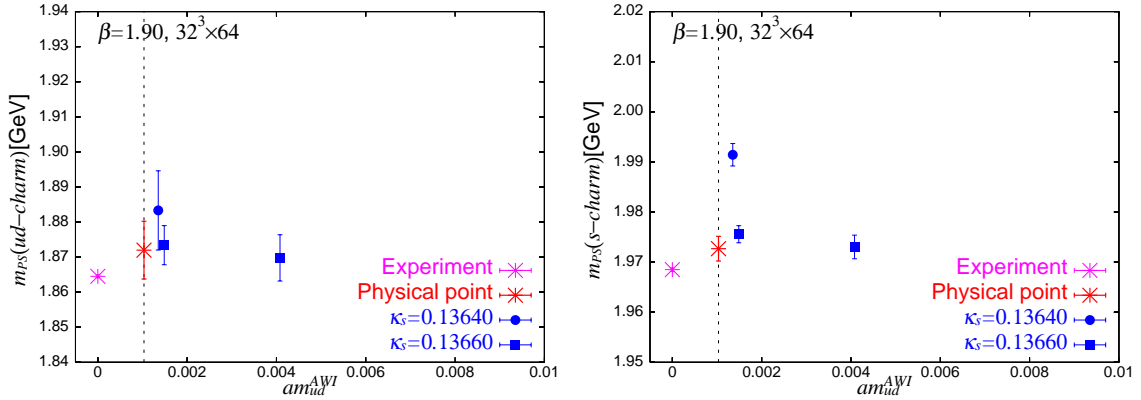
Figure 1: m_{ud}^{sea} quark mass dependence of m_D (left panel) and m_{D_s} (right panel).

coefficients, and the parameter ν is adjusted nonperturbatively to reproduce the relativistic dispersion relation of the charmonium. Our valence heavy quark masses are always tuned to the physical charm quark mass point, which is determined by the spin-averaged $1S$ state of the charmonium, $M(1S) = (M_{\eta_c} + 3M_{J/\psi})/4 = 3.0677(3)$ GeV. Our results for charmonium spectrum have been reported in Ref. [10].

Table 1 summarizes our simulation parameters and the statistics of the configuration sets we have used for the heavy quark measurements. The number of the source points is quadrupled to suppress statistical fluctuations.

3. Results for D and D_s meson masses and decay constants

We present our results for D and D_s meson masses and their decay constants. Since the physical charm quark mass has already been fixed with the heavy-heavy spectrum, all heavy-light meson quantities can be predicted.

Figure 1 compares our results for the D and D_s meson masses with the experimental values [3]. Since our sea quark masses are close to the physical values, we extrapolate our results with a linear function of the up-down and the strange quark masses to the physical point,

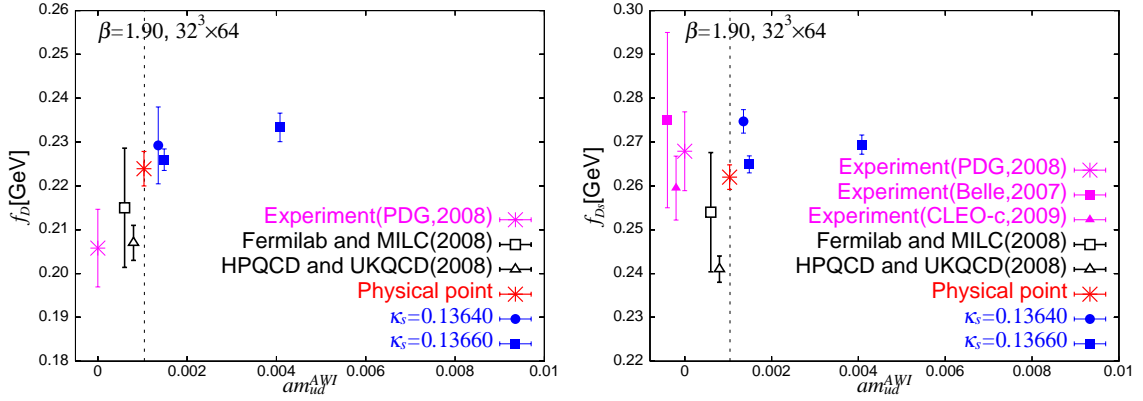$$M = A + B(m_{\text{ud}} - m_{\text{ud}}^{\text{phys}}) + C(m_{\text{s}} - m_{\text{s}}^{\text{phys}}). \tag{3.1}$$

Figure 2: m_{ud}^{sea} quark mass dependence of f_D(left panel) and f_{D_s}(right panel).
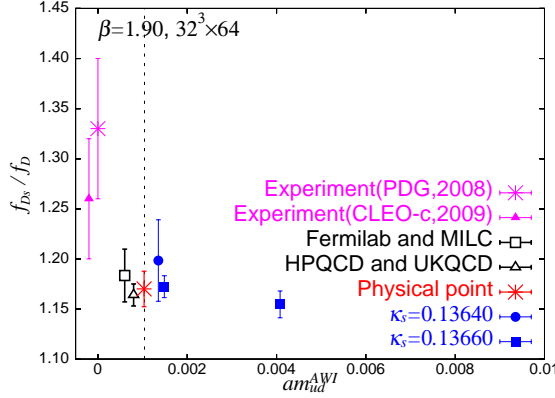

Figure 3: m_{ud}^{sea} quark mass dependence of f_{D_s} over f_D.

Our results at the physical point (red bursts) reproduce the experimental spectrum well.

We also calculate the decay constants of heavy-light pseudoscalar mesons,

$$< 0|A_\mu^{imp}|PS(p) > = i f_{PS} p_\mu, \tag{3.2}$$

$$A_\mu^{imp} = \sqrt{2\kappa_q}\sqrt{2\kappa_Q} Z_{A_\mu} \left\{ \bar{q}(x)\gamma_\mu\gamma_5 Q(x) \right. \tag{3.3}$$

$$\left. -c_{A_\mu}^+ \partial_\mu^+ \left(\bar{q}(x)\gamma_\mu\gamma_5 Q(x) \right) - c_{A_\mu}^- \partial_\mu^- \left(\bar{q}(x)\gamma_\mu\gamma_5 Q(x) \right) \right\}. \tag{3.4}$$

For the renormalization factor and the improvement coefficients of the axial current, we employ one-loop perturbative values [11]. Furthermore, the nonperturbative contribution at the massless limit is incorporated to the improvement coefficient $c_{A_4}^+$ by writing

$$c_{A_4}^+ = (c_{A_4}^+(m_Q a) - c_{A_4}^+(0))^{\mathrm{PT}} + c_A^{\mathrm{NP}} \tag{3.5}$$

with $c_A^{\mathrm{NP}} = -0.03876106$ [12].

Figure 2 shows the decay constants, where we also plot other 2+1 flavor lattice QCD results from HPQCD and UKQCD Collaboration [2] and FNAL group [5], as well as the experimental values [1, 3, 13], for comparison. Our value for f_{D_s} agrees with the experimental determinations, while
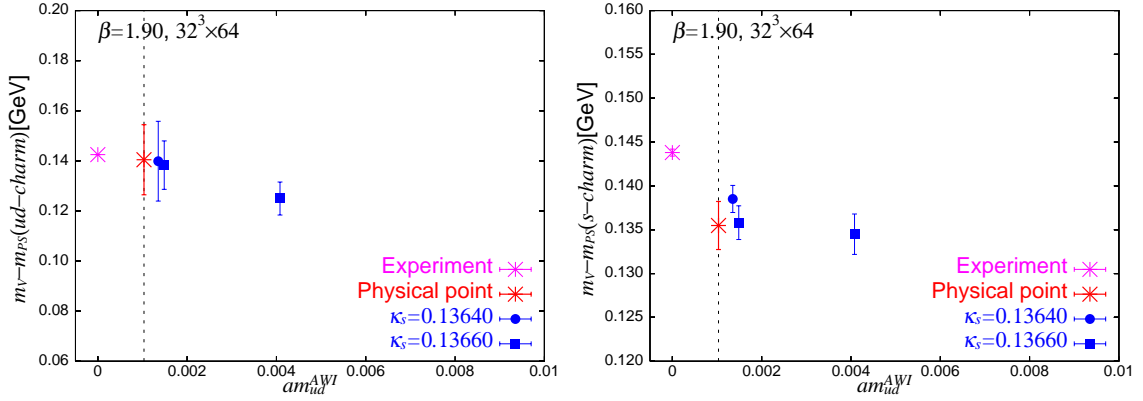
Figure 4: m_{ud}^{sea} quark mass dependence of hyperfine splittings for charm-*ud*(left panel) and and charm-strange mesons (right panel).

that for f_D is somewhat larger. Comparing the three sets of lattice determinations, we observe, both for f_D and f_{D_s}, an agreement between our values and those of the FNAL group, while there seems to be a clear discrepancy between our values and that by the HPQCD and UKQCD Collaboration. Possible reasons are our use of perturbative renormalization factors and the necessity of taking the continuum extrapolation.

In Fig. 3 we plot the ratios of f_{D_s} to f_D in which uncertainties coming from the perturbative renormalization factors cancel out, and perhaps that of the lattice cutoff to some extent. It is interesting that all three lattice results are mutually consistent with small errors of a few percent, but that they seem to lie at the lower edge of the experimental value.

4. Results of hyperfine splittings for charm-*ud* and charm-strange mesons

We evaluate the hyperfine splittings for charm-*ud* and charm-strange mesons. Masses of D^* and D_s^* are obtained from their two-point functions. D^* and D_s^* decays are prohibited energetically on our lattice.

Figure 4 represents our results for the hyperfine splittings with the experimental values [1]. For D^*–D mass difference, our result at the physical point (red bursts) reproduces the experimental spectrum. On the other hand, D_s^*–D_s mass difference shows 10% deviation from the experimental value. A possible origin of this discrepancy is the scaling violation.

5. Estimating the CKM matrix elements

We attempt to estimate the CKM matrix elements from our data of D and D_s meson masses and decay constants combined with experimental values of leptonic decay widths. Using the CLEO value of $\Gamma(D_s \to l\nu)$ [3], we find

$$|V_{cs}|(lattice) = 0.98(2)(3) + O(g^2 a), \tag{5.1}$$

$$|V_{cs}|(PDG) = 1.04(6) \ [1] \tag{5.2}$$

The first error is statistical, and the second is experimental. For comparison, the PDG value is listed. Our preliminary result is consistent with the PDG value. We note that our estimate of $|V_{cs}|$ still has a discretization error of $O(g^2 a)$ due to the use of perturbative renormalization factor. We must take the continuum extrapolation to remove this source of uncertainty.

We can estimate $|V_{cd}|$ from the D meson mass and decay constant and CLEO value of $\Gamma(D \to l\nu)$ [13]:

$$|V_{cd}|(lattice) = 0.207(2)(9) + O(g^2 a), \tag{5.3}$$

$$|V_{cd}|(PDG) = 0.230(11) \ [1] \tag{5.4}$$

Our estimate is smaller than the PDG value by about 10%. For completeness we also record the ratio $|V_{cs}|/|V_{cd}|$ for which $O(g^2 a)$ errors drop out, and is replaced with scaling violation of $O(a^2)$:

$$\frac{|V_{cs}|}{|V_{cd}|}(lattice) = 4.72(13)(26) + O(a^2), \tag{5.5}$$

$$\frac{|V_{cs}|}{|V_{cd}|}(PDG) = 4.52(34) \ [1]. \tag{5.6}$$

6. Conclusion

We measured charm-*ud* and charm-strange meson masses and decay constants with the relativistic heavy quark action on the 2+1 dynamical flavor PACS-CS configurations at $a^{-1} = 2.2$ GeV. Since our sea quark masses are close to the physical point, sea quark mass corrections in our results are small and only short extrapolations to the physical point are needed.

We found our results are consistent with the experimental values in two standard deviations. We do not observe discrepancy from experiments in f_{D_s}.

Combining our data of masses and decay constants with experimental values of leptonic decay widths, CKM matrix elements are evaluated. Our estimates still have discretization errors, which is expected to be 1% by the order counting. Continuum extrapolation is needed to achieve a few percent accuracy.

Acknowledgments

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