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Mixing of S-Wave Charmonia with $\mathrm{D\overline{D}}$ Molecule States

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Charmonium states can decay into pairs of D and $\overline{D}$ mesons if their masses are above the allowed decay thresholds. In general $c\bar{c}$ states near threshold will also undergo mixing with $D\overline{D}$ molecular (or tetraquark) states, by creation and annihilation of light quark-antiquark pairs. The investigation of such effects sheds light on the higher Fock state contributions to charmonium wavefunctions and on mass shifts, relative to a scenario where such mixing effects are neglected. A variational approach is applied to a mixing matrix between operators of both sectors, of $c\bar{c}$ and of $D\overline{D}$ molecular type. The efficient calculation of several diagrams appearing in this matrix requires all-to-all propagators, which are realized by sophisticated stochastic estimator techniques. The runs are performed on $n_F = 2$ $24^3 \times 48$ lattice volumes with $m_\pi \approx 280$ MeV, using the non-perturbatively improved clover Wilson action, both for valence and for sea quarks.

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1. Introduction

Nonperturbative simulations have demonstrated that quantum chromodynamics (QCD) predicts the confinement of objects that carry a color charge. The hadronic color singlet states with baryon number $B = 0$ are called mesons. Within the nonrelativistic quark model these contain exactly one quark and one antiquark. However, at least in principle, QCD offers the possibility to construct such states entirely out of gluons (glueballs), out of quark, antiquark and gluons (hybrids), out of two quarks and two antiquarks (molecules or tetraquarks) and even higher Fock states. In general these states will undergo mixing and not all these states will be stable against strong decays.

The charmonium sector provides us with a particularly rich laboratory for the study of these effects. Several experimentally discovered charmonium states are suspected to contain large contributions from non-$c\bar{c}$ configurations [1, 2, 3]. Here we wish to address the contribution of $c\bar{q}q\bar{c}$ molecules[1] to S-wave charmonia and of $c\bar{c}$ states to $\mathrm{D\overline{D}}$ bound states.

QCD predicts the spectrum within a sector of fixed J^{PC}, isospin I, strangeness S and charm C. Classifying the resulting states according to their (presumed) quark content will be model dependent. However, there are examples where clearly one Fock state dominates over another and where mixing is small. For instance the ($J^{PC} = 0^{-+}$, $I = S = C = 0$) η_c state is clearly different from the η' state with exactly the same quantum numbers and in this case dynamic mixing effects turn out to be negligible [4], such that we can identify the leading Fock contribution to the former state as $c\bar{c}$ while the η' is composed of strange and light quark-antiquark pairs.

With such caveats in mind, we will somewhat carelessly use the term "η_c-$D_1\overline{D}^*$-mixing" when we mean mixing between states that couple to $c\gamma_5\bar{c}$-type operators and states created by, $(c\gamma_5\gamma_i\bar{q})(\bar{c}\gamma_i q)$. These "unperturbed" states can be accessed on the lattice by appropriate interpolating fields. Their mixing is studied by applying a variational approach. One of the problems is that, unless the two sectors completely decouple from each other, in the limit of large Euclidean times the states created by all these operators will decay into the same ground state: the ground state created by the $c\bar{c}$ type operators, that we call the unmixed η_c, will intrinsically already contain a $D_1\overline{D}^*$ contribution. However, if the perturbative approach that we outline below is justified, then this implicit mixing [5] will first appear at second order in an expansion parameter λ, while the transition matrix element between the two sectors (explicit mixing) will be of order λ. Consequently, at intermediate Euclidean times implicit mixing might be negligible, reducing the model dependence of our ansatz.

Our perturbative set-up is as follows. We expect the physical η_c wavefunction at first order in a parameter λ to read,

$$|\eta_c\rangle = \frac{1}{\mathcal{N}}\left(|c\bar{c}\rangle + \lambda\frac{\langle c\bar{q}q\bar{c}|H_1|c\bar{c}\rangle}{E(c\bar{c}) - E(c\bar{q}q\bar{c})}|c\bar{q}q\bar{c}\rangle\right), \tag{1.1}$$

with a normalisation factor $\mathcal{N}$ and a (hopefully) small[2] coupling constant λ appearing in the mixing vertex of the Hamiltonian H_1. While we do not know the functional form of H_1 or of the unperturbed wavefunctions, we can evaluate all the relevant matrix elements on the lattice.

[1] We use the term "molecule" synonymous also for tetraquarks.

[2] In some cases λ might be large but we should be able to detect this through implicit mixing, i.e. decay of the state created by the $c\bar{q}q\bar{c}$ operators into the $c\bar{c}$ states at small Euclidean times.

One important thing to note here is the dependence of the mixing on the light quark mass. With decreasing m_q, the denominator obviously becomes smaller, but the mixing matrix element in the numerator is expected to increase, since the probability for creating a light quark-antiquark pair should be inversely dependent on the light quark mass. Therefore we expect mixing effects to increase at smaller light quark masses.

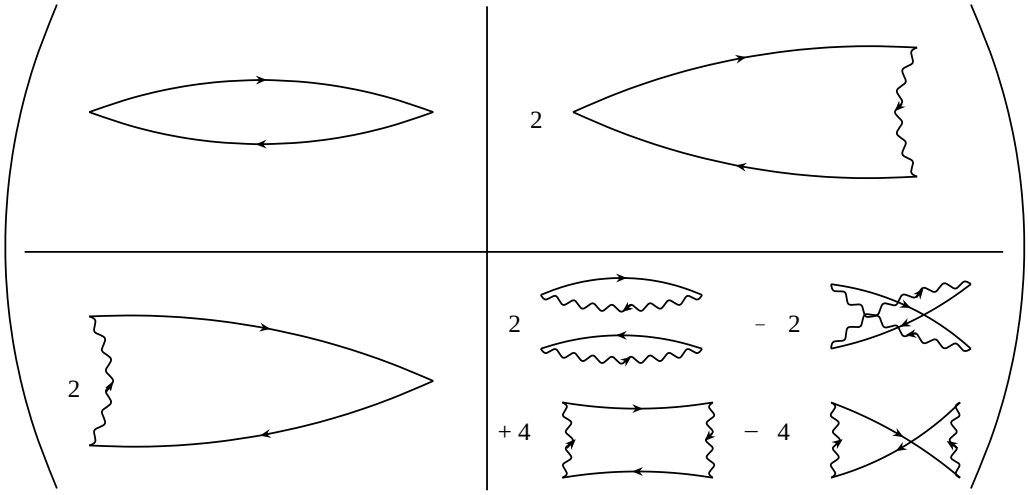

Figure 1: The mixing matrix. Solid lines represent charm quarks, wiggled lines light quarks.

2. Simulation

We aim to calculate the coefficients in the expansion of the QCD eigenstates into the trial interpolating fields nonperturbatively, by diagonalizing a matrix of cross correlators including both $c\bar{c}$ and molecular operators. For each type of operator we apply three different types of smearing: local, narrow and wide, where these terms indicate the number of Wuppertal smearing steps with $\delta = 0.3$, employing spatial APE smeared ($n_{\mathrm{APE}} = 15$, $\alpha = 0.3$) parallel transporters to smooth the trial wavefunctions. The number of fermion field smearing iterations is determined by optimizing the effective masses separately for each of the two sectors.

In figure 1 we sketch the structure of the mixing matrix. The different smearing levels are omitted for the sake of clarity. Solid lines represent charm quark propagators and wiggly lines light quark propagators. The prefactors are due to the two mass degenerate light sea quark flavors. The upper left corner contains the $c\bar{c}$, the lower right corner the molecular sector. Nonvanishing off-diagonal elements indicate mixing. The spatial separation within the molecular operators was tuned by maximizing the magnitude of the off-diagonal element. The optimal value was $r = 4a \approx 0.3\,\mathrm{fm}$.

The charm-anticharm annihilation diagrams were omitted in the present study where we focus on $c\bar{c}$-molecule mixing near threshold. We studied these previously in the context of η_c-η' mixing where their effect turned out to be negligible [4].

For the evaluation of the last two diagrams of the molecular sector, light all-to-all propagators are necessary. $O(\sim 100)$ complex $\mathbb{Z}_2$ stochastic estimates per configuration were calculated for this purpose, with the application of sophisticated noise reduction methods like staggered-spin-color dilution and hopping parameter acceleration as already used in previous studies [6, 4, 7].

The generic form of our meson interpolators, centred around a position x, reads,

$$M(x) = (\bar{c}\Gamma_M c)_x\,, \tag{2.1}$$

while the molecular interpolators with separation r look like,

$$Y(x,r) = \frac{1}{\sqrt{2}}\left((\bar{q}\Gamma_Y^1 c)_x(\bar{c}\Gamma_Y^2 q)_{x+r} + (-)^s(\bar{c}\Gamma_Y^1 q)_x(\bar{q}\Gamma_Y^2 c)_{x+r}\right)\,. \tag{2.2}$$

J^{PC}	Γ_M	Γ_Y^1	Γ_Y^2	s
0^{-+}	γ_5	γ_i	$\gamma_i\gamma_5$	0
1^{--}	γ_i	γ_5	$\gamma_i\gamma_5$	1
1^{++}	$\gamma_i\gamma_5$	γ_5	γ_i	1

Table 1: Γ structures of meson and molecular interpolating fields.

In table 1 we display the explicit Γ structures for the $J^{PC} = 0^{-+}, 1^{--}$ and 1^{++} channels.

The variational method consists of solving a generalized eigenvalue problem,

$$C(t_0)^{-1/2}C(t)C(t_0)^{-1/2}\psi^\alpha = \lambda^\alpha(t,t_0)\psi^\alpha\,, \tag{2.3}$$

see refs. [8, 9, 10] for details. For sufficiently large times the eigenvalues and -vectors will approach their asymptotic values. The components of a given eigenvector can be interpreted as the coupling strengths of the corresponding operators to the state under consideration.

Our strategy differs from that of many previous studies that utilized the variational method, in as far as our primary interest lies in the couplings and not only in the resulting spectrum. We first determine the eigenvalues of the three by three submatrices within each of the Fock sectors, separately, in order to obtain an "unperturbed" spectrum. This is then used to identify the affiliation of the eigenvectors to the eigenstates on the jacknifes samples. Finally, we will compare spectrum and eigenvector components with the mixing elements switched on to this unmixed reference point.

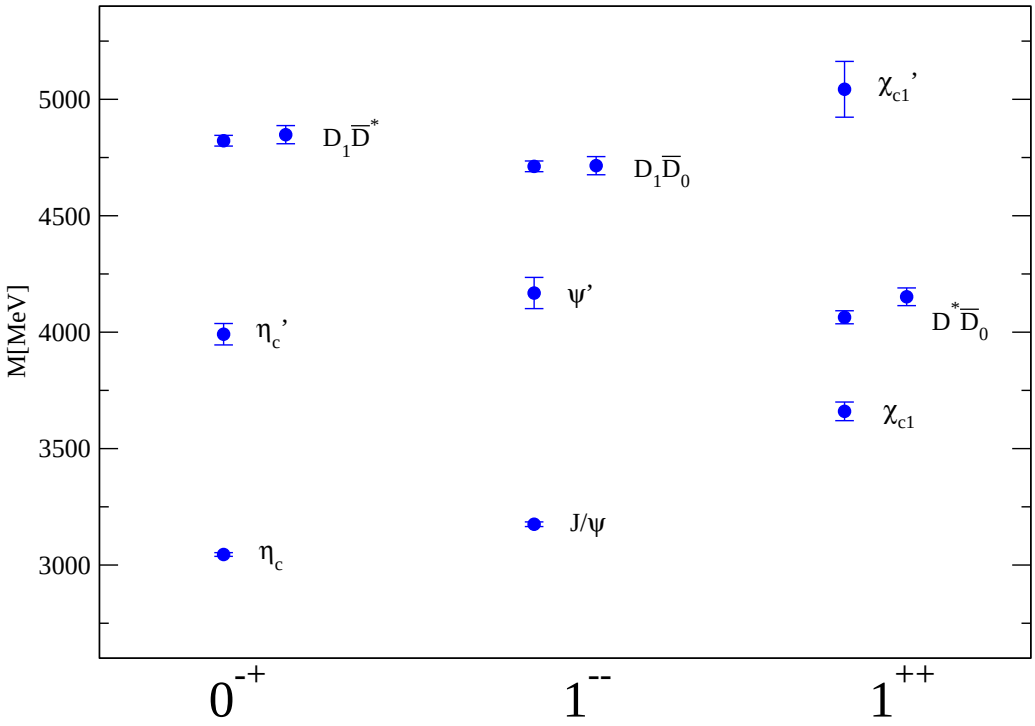

Figure 2: Mass spectra from the separate diagonalization of the submatrices within each sector.

Our runs are performed on 100 effectively de-correlated $n_F = 2\,24^3 \times 48$ configurations generated by the QCDSF collaboration [11], with a lattice spacing $a \approx 0.076\,\mathrm{fm}$, obtained from the

chirally extrapolated nucleon mass. We employ the same non-perturbatively order a improved clover action for the valence quarks. The charm quark mass was set by tuning the spin-averaged charmonium mass $\frac{1}{4}(m_{\eta_c}+3m_{J/\Psi})$ to its experimental value. The pion on these lattices is reasonably light, $m_\pi \approx 280\,\mathrm{MeV}$, to embrace the above mentioned dependence of the mixing on the light quark mass.

Computations were performed using the Chroma software library [12] partly on the local HPC cluster and partly on the BlueGene/P of the Jülich Supercomputer Center.

3. Spectrum

An extra benefit bucked off by our analysis is the mass spectrum in the investigated channels. The separate diagonalization of the three by three submatrices provides us with at least four reliable eigenvalues, two for each subsector. However, since the molecular channels typically are rather noisy, we are only able to extract the ground states there, within reasonable errors. So we are left with three states in each channel, plotted in figure 2. For the molecular masses we give two data points: the left ones are from the diagonalisation procedure, the right ones represent the sums of the masses of the corresponding pairs of non-interacting D mesons. Note that in the 1^{++} channel, the radially excited χ_{c1} is heavier than the molecular state, in contrast to the other channels. If we consider the fact that our pion is about 130-140 MeV too heavy, the mass of the molecular state is indeed consistent with the $X(3872)$, which most likely has $J^{PC}=1^{++}$ [13].

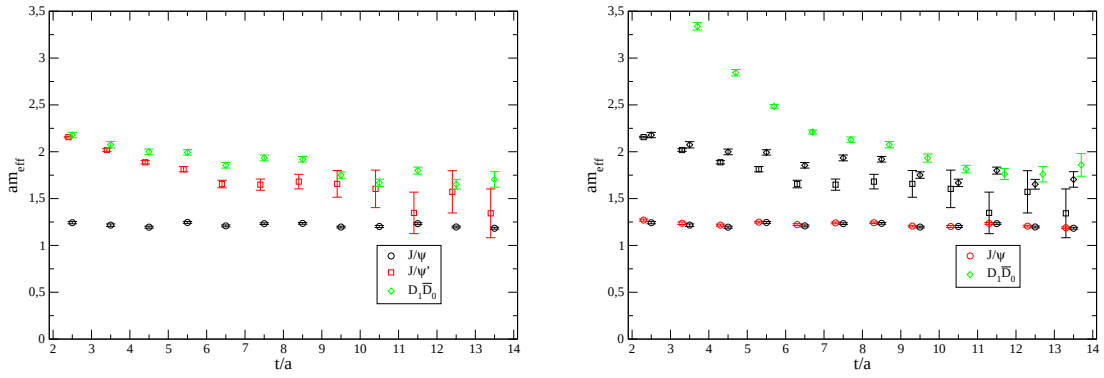

Figure 3: Effective masses of the eigenvalues of the submatrices (left hand side) and of the full matrix (right hand side) in the 1^{--} channel.

4. Mixing

Equipped with the reference eigenvalues from the submatrices, we go for the diagonalization of the full six by six matrix. However, due to limited statistics, we find this to be numerically unstable and restrict ourselves to the sub-basis $M_{local}, M_{narrow}, Y_{local}, Y_{narrow}$. We discuss the vector state as one example. In the left of figure 3 we display the effective masses from the diagonalization of the two submatrices. The data points for J/Ψ and its radial excitation are from the $c\bar{c}$ submatrix, the ones for the $D_1\overline{D}_0$ from the molecular sector.

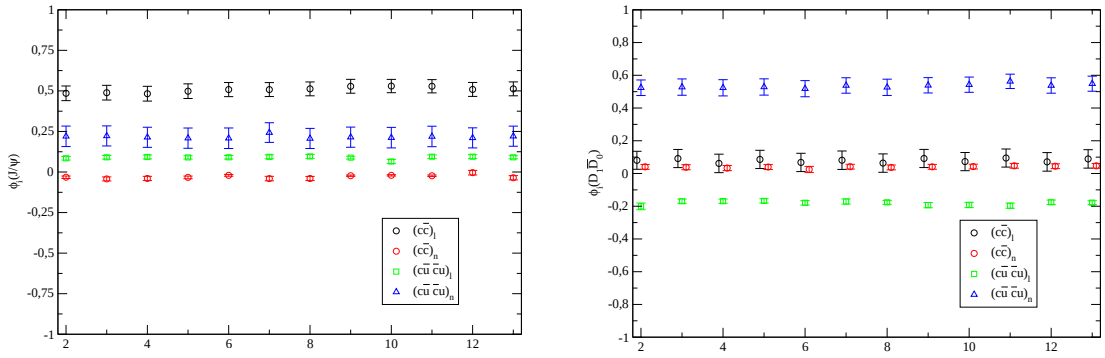

Figure 4: Eigenvector components from the diagonalization of the full matrix in the 1^{--} channel.

The unmixed reference points can also be found in black color in the right of figure 3. In addition, the two lowest lying effective masses from the diagonalization of the full matrix are shown there. We are able to identify these two states with J/Ψ and $D_1\bar{D}_0$, respectively. Interestingly, the J/Ψ' state is not found in the diagonalisation of the four by four system.

state	$(c\bar{c})_l$	$(c\bar{c})_n$	$(c\bar{c}u\bar{c})_l$	$(c\bar{c}u\bar{c})_n$
η_c	0.54(3)	-0.02(1)	-0.1(1)	-0.31(5)
$D_1\bar{D}^*$	0.07(1)	0.01(1)	-0.46(8)	0.14(2)
J/ψ	0.51(4)	-0.03(1)	0.09(1)	0.21(6)
$D_1\bar{D}$	0.08(6)	0.04(1)	-0.18(1)	0.53(4)
χ_{c1}	0.39(5)	0.69(3)	-0.22(3)	-0.49(4)
$D\bar{D}^*$	0.63(4)	-0.23(3)	-0.73(4)	0.12(3)

Table 2: Eigenvector components in the full basis.

The corresponding eigenvector components are plotted in figure 4 for the J/ψ (left) and for the $D_1\bar{D}_0$ molecule. The J/Ψ receives the dominant contribution from the local $c\bar{c}$ operator. However, the molecular configurations seem to contribute significantly too. The $D_1\bar{D}_0$ state in contrast only contains small (but non-vanishing) $c\bar{c}$ admixtures.

In table 2 we summarize the results for all channels that we investigated. In each of them we see significant mixing effects. The large molecular contribution to the χ_{c1} is particularly noteworthy to mention.

5. Conclusion & Outlook

In all of the investigated channels (0^{-+}, 1^{--}, and 1^{++}) we detect significant mixing effects between $c\bar{c}$ and four-quark states. Although the precise values of the eigenvector components should not be taken too seriously since the operator basis is rather small and thus may miss non-negligible parts of the physical wavefunction, our analysis clearly substantiates the assumption of charmonium states having a rich Fock structure.

In the near future this study will be extended to other channels including additional interpolating fields, e.g. a molecular operator containing two mesons in a relative P-Wave.

Acknowledgments

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