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Low-lying positive-parity excited states of the nucleon

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We present an overview of the correlation-matrix methods developed recently by the CSSM Lattice Collaboration for the isolation of excited states of the nucleon. Of particular interest is the first positive-parity excited-state of the nucleon known as the Roper resonance. Using eigenvectors of the correlation matrix we construct parity and eigenstate projected correlation functions which are analysed using standardized methods. The robust nature of this approach for extracting the eigenstate energies is presented. We report the importance of using a variety of source and sink smearings in achieving this. Ultimately the independence of the eigenstate energies from the interpolator basis is demonstrated. In particular we consider 4×4 correlation matrices built from a variety of interpolators and smearing levels. Using FLIC fermions to access the light quark mass regime, we explore the curvature encountered in the energy of the states as the chiral limit is approached. We report a low-lying Roper state contrasting earlier results using correlation matrices. To the best of our knowledge, this is the first time a low-lying Roper resonance has been found using correlation matrix methods. Finally, we present our results in the context of the Roper results reported by other groups.

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1. Introduction

The first positive-parity excited state of the nucleon, known as the Roper resonance, $N\frac{1}{2}^{+}$(1440 MeV) P_{11}, has been a long-standing puzzle since its discovery in the 1960's due to its lower mass compared to the adjacent negative parity, $N\frac{1}{2}^{-}$ (1535 MeV) S_{11}, state. In constituent quark models with harmonic oscillator potentials, the lowest-lying odd parity state naturally occurs below the P_{11} state (with principal quantum number $N = 2$) [1, 2], whereas, in nature the Roper resonance is almost 100 MeV below the S_{11} state.

Lattice QCD is very successful in computing many properties of hadrons from first principles. In particular, in hadron spectroscopy, the ground states of the hadron spectrum are now well understood. However, the excited states still prove a significant challenge. The first detailed analysis of the positive parity excitation of the nucleon was performed in Ref. [3] using Wilson fermions and an operator product expansion spectral ansatz. Since then several attempts have been made to address these issues in the lattice framework, but in many cases no potential identification of the Roper state has been made. Recently, however, in the analysis of Refs. [4, 5, 6] a low-lying Roper state has been identified using Bayesian techniques.

Another state-of-the-art approach in hadron spectroscopy is the 'variational method' [7, 8], which is based on a correlation matrix analysis. The identification of the Roper state with this method wasn't successful in the past. However, very recently, in Ref. [9] a low-lying Roper state has been identified with this approach employing a diverse range of smeared-smeared correlation functions.

Here we discuss the new correlation matrix construction for isolating the puzzling Roper state [9] and present our Roper results in the context of the previous results reported by other groups in recent times.

2. Variational Method

The two point correlation function matrix for $\vec{p} = 0$ can be written as,

$$G_{ij}(t) = (\sum_{\vec{x}} \mathrm{Tr}_{\mathrm{sp}}\{\Gamma_{\pm}\langle\Omega|\chi_i(x)\bar{\chi}_j(0)|\Omega\rangle\}), \quad (2.1)$$

$$= \sum_{\alpha} \lambda_i^{\alpha}\bar{\lambda}_j^{\alpha} e^{-m_{\alpha}t}, \quad (2.2)$$

where, Dirac indices are implicit. Here, λ_i^{α} and $\bar{\lambda}_j^{\alpha}$ are the couplings of interpolators χ_i and $\bar{\chi}_j$ at the sink and source respectively and α enumerates the energy eigenstates with mass m_{α}. $\Gamma_{\pm}$ projects the parity of the eigenstates.

Since the only t dependence comes from the exponential term, one can seek a linear superposition of interpolators, $\bar{\chi}_j u_j^{\alpha}$, such that,

$$G_{ij}(t+\triangle t)\, u_j^{\alpha} = e^{-m_{\alpha}\triangle t}\, G_{ij}(t)\, u_j^{\alpha}, \quad (2.3)$$

for sufficiently large t and $t+\triangle t$. More detail can be found in Refs. [10, 11, 12]. Multiplying the above equation by $[G_{ij}(t)]^{-1}$ from the left leads to an eigenvalue equation,

$$[(G(t))^{-1}G(t+\triangle t)]_{ij}\, u_j^{\alpha} = c^{\alpha}\, u_i^{\alpha}, \quad (2.4)$$

where $c^{\alpha} = e^{-m_{\alpha}\triangle t}$ is the eigenvalue. Similar to Eq.(2.4), one can also solve the left eigenvalue equation to recover the v^{α} eigenvector,

$$v_i^{\alpha}\,[G(t+\triangle t)(G(t))^{-1}]_{ij} = c^{\alpha} v_j^{\alpha}. \tag{2.5}$$

The vectors u_j^{α} and v_i^{α} diagonalize the correlation matrix at time t and $t+\triangle t$ making the projected correlation matrix,

$$v_i^{\alpha} G_{ij}(t) u_j^{\beta} \propto \delta^{\alpha\beta}. \tag{2.6}$$

The parity projected, eigenstate projected correlator,

$$G_{\pm}^{\alpha} \equiv v_i^{\alpha} G_{ij}^{\pm}(t) u_j^{\alpha}, \tag{2.7}$$

is then analyzed using standard techniques to obtain masses of different states.

3. Lattice Details

Our lattice ensemble consists of 200 quenched configurations with a lattice volume of $16^3 \times 32$. Gauge field configurations are generated using the DBW2 gauge action [13, 14] and an $\mathcal{O}(a)$-improved FLIC fermion action [15] is used to generate quark propagators. The lattice spacing is $a = 0.1273$ fm, as determined by the static quark potential, with the scale set using the Sommer scale, $r_0 = 0.49$ fm [16]. In the irrelevant operators of the fermion action we apply four sweeps of stout-link smearing. Various sweeps (1, 3, 7, 12, 16, 26, 35, 48) of gauge invariant Gaussian smearing [17] are applied at the source (at $t = 4$) and at the sink. The analysis is performed on ten different quark masses corresponding to pion masses $m_{\pi} = \{0.797, 0.729, 0.641, 0.541, 0.430, 0.380, 0.327, 0.295, 0.249, 0.224\}$ GeV. Error analysis is performed using a second-order single elimination jackknife method, where the χ^2/dof is obtained via a covariance matrix analysis method. Our fitting method is discussed extensively in Ref. [11].
The nucleon interpolator we consider is the local scalar-diquark interpolator [3, 18],

$$\chi_1(x) = \varepsilon^{abc}(u^{Ta}(x) C\gamma_5\, d^b(x))\, u^c(x). \tag{3.1}$$

4. Results

We consider several 4×4 correlation matrices. Each matrix is constructed with different sets of correlation functions, each set element corresponding to a different number of sweeps of gauge invariant Gaussian smearing at the source and sink of the $\chi_1\bar{\chi}_1$ correlators. This provides a large basis of operators with a wide range of overlaps among energy states.

We consider seven smearing combinations {1=(1,7,16,35), 2=(3,7,16,35), 3=(1,12,26,48), 4=(3,12,26,35), 5=(3,12,26,48), 6=(12,16,26,35), 7=(7,16,35,48)} of 4×4 matrices. In Ref. [11] it was shown that one cannot isolate a low-lying excited eigenstate using a single fixed-size source smearing. The superposition of states manifested itself as a smearing dependence of the effective mass.

In Fig.1, masses extracted from all the combinations of 4×4 matrices (from 1^{st} to 7^{th}) are shown for the pion mass of 797 MeV. Some dependence of the excited states on smearing sweep

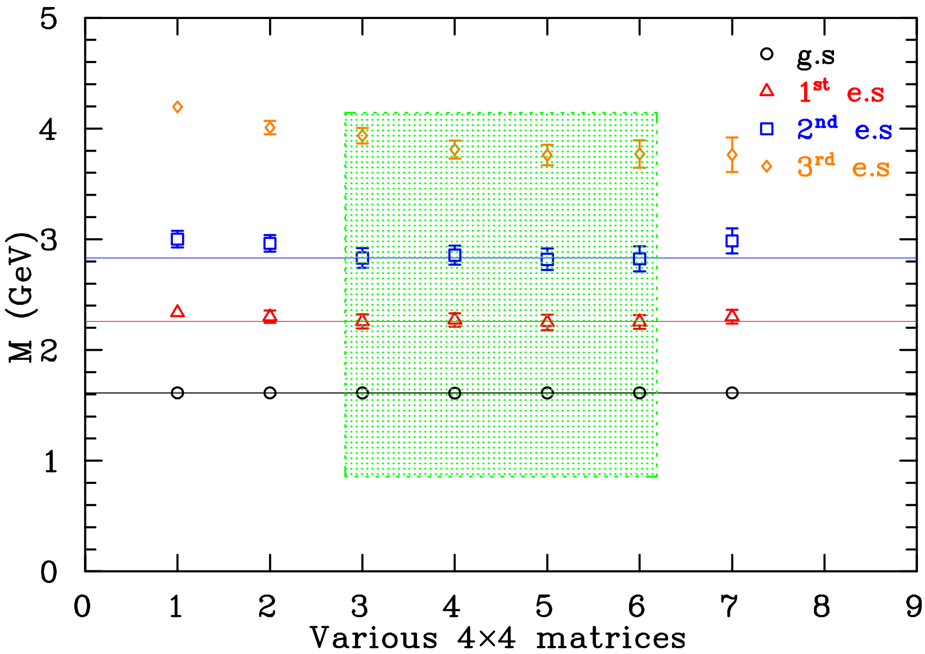

Figure 1: (Color online). Masses of the nucleon, $N\frac{1}{2}^{+}$ - states, from projected correlation functions as shown in Eq.(2.7) for the pion mass of 797 MeV. Numbers in the horizontal scale correspond to each combination of smeared 4×4 correlation matrices. For instance, 1 and 2 correspond to the combinations of (1,7,16,35) and (3,7,16,35) respectively and so on, as discussed in the text following Eq.(3.1). Masses are extracted according to the selection criteria described in Ref. [11]. The horizontal lines are drawn for the average mass over the four combinations (from 3rd to 6th as shown inside the shaded box).

count is observed here as in Ref. [11] for a few of the interpolator basis smearing sets. However the ground and first excited states are robust against changes in the interpolator basis, providing evidence that an energy eigenstate has been isolated. It should be noted that the highest excited state (the third excited state) is influenced more by the level of smearing than the lower excited states. This is to be expected as this state must accommodate all remaining spectral strength and this is dependent on smearing.

The 1st combination in Fig.1 provides heavier excited states as this basis begins with a low number of smearing sweeps (a sweep count of 1) and also contains another low smearing set of 7 sweeps. The first and second excited states sit a little high in comparison with the other bases. This is also evident in Fig.2 for the lighter quark mass. Hence, extracting masses with this basis is not as reliable as other sets containing a great diversity of large numbers of smearing sweeps. The 2nd combination also contains elements with a small smearing sweep count (3 and 7), hence this basis also provides heavier excited states and shows some systematic drift in the second excited state. However, this basis has reduced contamination from the excited states when compared with the first basis.

We can observe at this point that including basis elements with 2 consecutive low smearing sweep counts (for instance, consecutive low numbers of smearing sweeps 1,7 of 1st combination and 3,7 of 2nd combination, respectively), provides a basis which does not span the space well. We

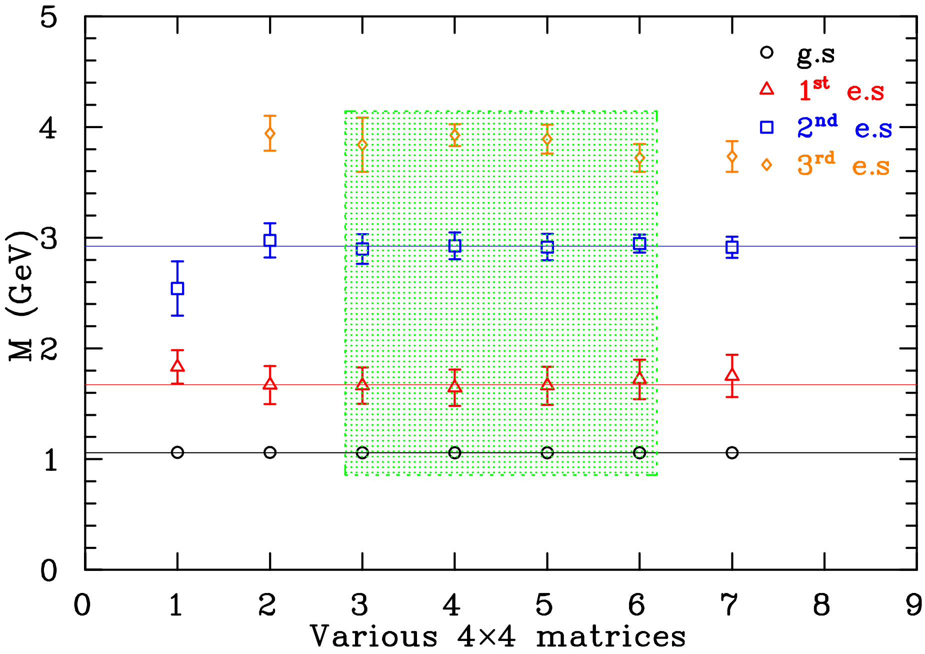

Figure 2: (Color online). As in Fig.1, but for the light pion mass of 249 MeV.

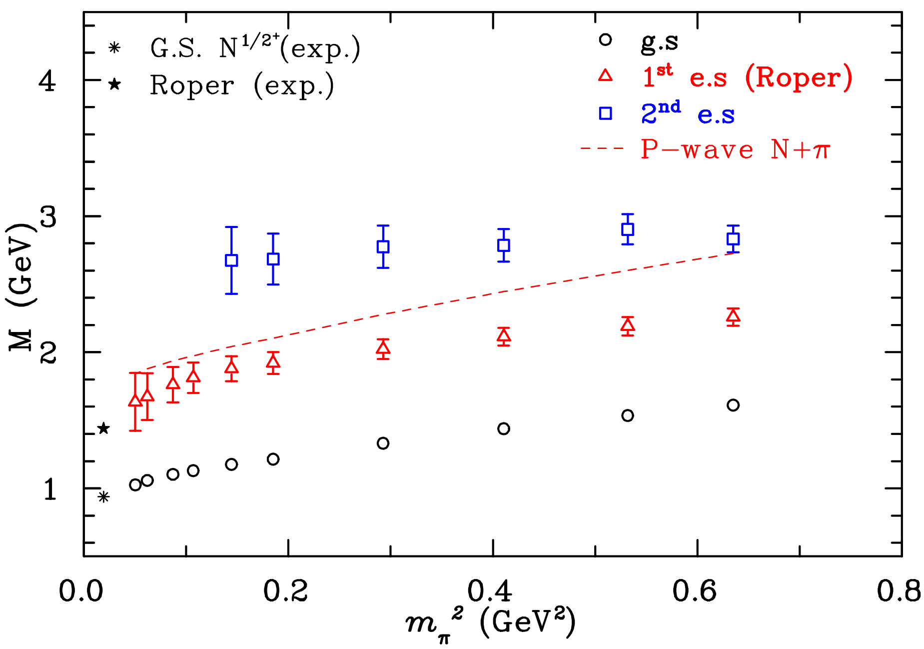

Figure 3: (Color online). The ground, the Roper and second excited states and the non-interacting P-wave $N+\pi$ are illustrated. The black filled symbols are the experimental values of the ground and the Roper states.

also observe that the inclusion of basis elements with 2 consecutive high levels of smearing (for instance, a sweep count of 35, 48 as in the 7$^{\rm th}$ combination) does not span the space well and gives rise to larger uncertainties.

The 3$^{\rm rd}$, 4$^{\rm th}$ and 5$^{\rm th}$ combinations are well spread over the given range of smearing sweeps. They don't include successive lower smearing sweep counts. The 5$^{\rm th}$ combination contains the basis element with a sweep count of 48 but has only slightly larger statistical errors than the 4$^{\rm th}$ basis choice. All these bases provide diversity. It is observed that the 3$^{\rm rd}$ through the 6$^{\rm th}$ combinations provide consistent results for the first and second excited states. An analysis is performed to calculate the systematic errors associated with the choice of basis over the preferred four combinations (from 3$^{\rm rd}$ to 6$^{\rm th}$) with $\sigma_b = \sqrt{\frac{1}{N_b-1}\sum_{i=1}^{N_b}(M_i - \bar{M})^2}$, where, N_b is the number of bases, in this case equal to 4. The statistical and systematic errors due to basis choices are added in quadrature, $\sigma = \sqrt{\bar{\sigma}_s^2 + \sigma_b^2}$, which is shown in Figs. 3 and 4.

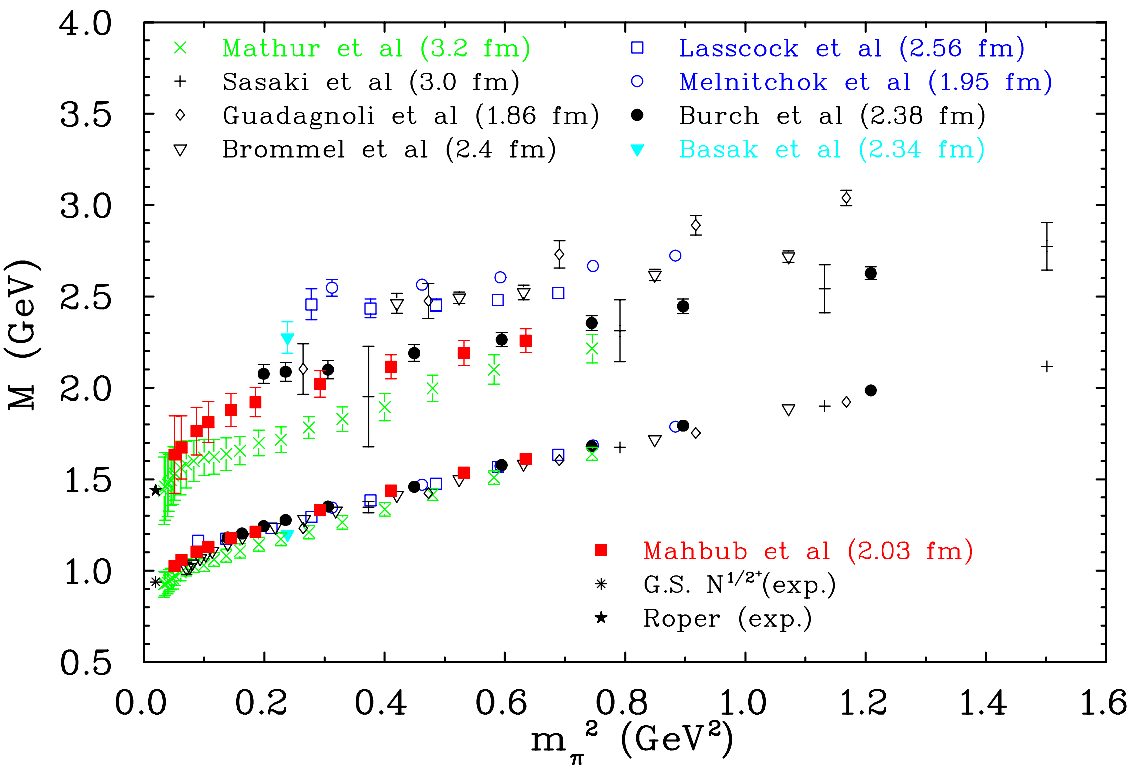

Figure 4: (Color online). Compilation of current lattice results of the nucleon, $N\frac{1}{2}^+$, for the ground and first excited states reported in Refs. [5, 6, 9, 10, 19, 20, 21, 22, 23] as described in the text.

In Fig. 3, it is interesting to note that the observed lattice Roper state sits lower than the P-wave $N+\pi$, indicative of attractive πN interactions producing a resonance at physical quark masses.

In Fig. 4, the ground state results are consistent for all the reported works. However, significant differences are readily observed in the results of the first positive parity excited (Roper) state. In Ref. [5], Mathur *et al*. used a constrained curve fitting method. In Ref. [6], Sasaki *et al*. used the Maximum Entropy Method. In Ref. [19], Guadagnoli *et al*. used a modified correlator technique. Refs. [9, 10, 20, 21, 22, 23] all used the variational approach. Brommel *et al*. [20] considered standard nucleon interpolators χ_1, χ_2 and χ_3 and employ Jacobi smeared sources. They have performed simulations in a range of pion masses of 270-866 MeV and used 3×3 correlation matrices. They reported results that are too high to be interpreted as the Roper state. The early analysis of CSSM Collaboration by Melnitchouk *et al.* [10] used χ_1 and χ_2 interpolators and used a single Gaussian source smearing of 20 sweeps. They found no evidence for the Roper state as the energy states of their 2×2 correlation matrix analysis also proved too high to be interpreted as the Roper resonance. The results from Lasscock *et al.* [21] from 3×3 correlation matrix analysis of standard interpolators also sits as high as those of Brommel and Melnitchouk, and cannot be interpreted as the Roper state. Burch *et al.* [22] considered the alternative approach of using Jacobi smeared sources of two different widths (narrow and wide) to increase the basis of operators and performed 6×6 correlation matrix analysis for a pion mass down to 450 MeV. Though their lightest quark mass results sit slightly above our results labeled Mahbub *et al.* in Fig. 4 [9], in the heavy quark-mass region the significant overlap between the results of Burch *et al.* and ours is apparent. Basak *et al.* [23] used non-local operators to form the basis of their correlation matrix and simulated at a pion mass of 490 MeV. Their results are also high. They reported that they did not find any Roper like positive-parity excitation. Using smeared-smeared correlators to construct a basis of correlation matrices, we identified a low-lying Roper state with significant curvature apparent at lighter quark masses [9].

5. Conclusions

Through the use of a variety of smeared-smeared correlation functions in constructing correlation matrices, the first positive parity excited state of the nucleon $N\frac{1}{2}^+$, the Roper state, has been observed for the first time using the variational analysis [9]. The current status of results for the Roper resonance from a number of groups are reviewed. Our lattice Roper state has a tendency to approach the physical Roper state showing a significant curvature as the chiral limit is approached [9]. This work signifies the importance of using a diverse range of smeared-smeared correlation functions when constructing correlation matrices for the identification of the elusive Roper state. This robust approach should also be applied for larger dimensions of the correlation matrices not only built from the $\chi_1\bar{\chi}_1$ correlators but also using $\chi_1\bar{\chi}_2$ and in the negative parity channel. This will be the subject of future investigations.

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