

Lattice Investigations of Nucleon Structure at Light Quark Masses

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Lattice simulations of hadronic structure are now reaching a level where they are able to not only complement, but also provide guidance to current and forthcoming experimental programmes at, e.g. Jefferson Lab, COMPASS/CERN and FAIR/GSI. By considering new simulations at low quark masses and on large volumes, we review the recent progress that has been made in this exciting area by the QCDSF/UKQCD collaboration. In particular, results obtained close to the physical point for several quantities, including electromagnetic form factors and moments of ordinary parton distribution functions, show some indication of approaching their phenomenological values.

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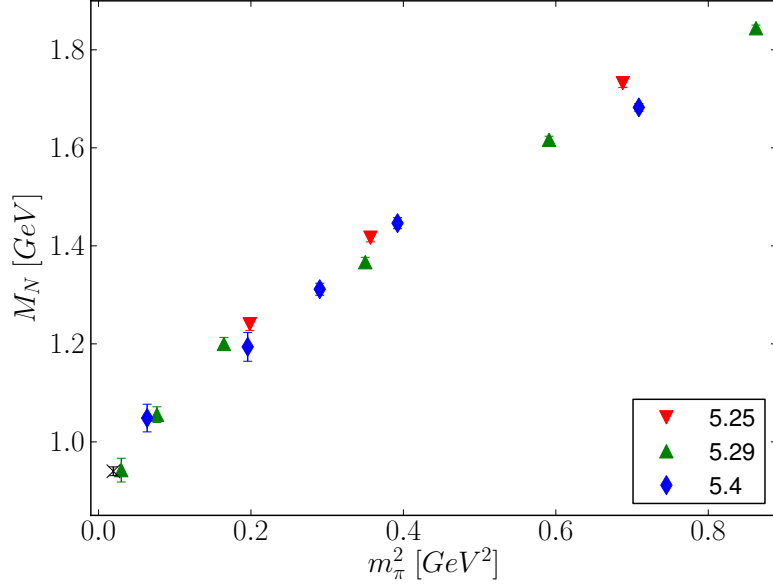


Figure 1: Nucleon mass as a function of m_π^2 for three different values of β . The scale is set using $r_0 = 0.467$ fm and the physical mass is indicated by the cross.

1. Introduction

Much of our knowledge about hadronic structure in terms of quark and gluon degrees of freedom has been obtained from high energy scattering experiments. However, there are still many unresolved issues in hadronic physics that need to be addressed, from both an experimental and theoretical perspective. This is one of the main motivations of the 12 GeV Jefferson Lab upgrade which aims to [1]: search for exotic mesons; study the role of hidden flavours in the nucleon; map out the spin and flavour dependence of the valence parton distribution functions; explore nuclear medium effects; and measure the generalised parton distribution functions of the nucleon. It is imperative that these and other exciting experimental efforts, such as those at COMPASS/CERN and FAIR/GSI, are matched by modern lattice simulations. See recent lattice reviews [2, 3, 4, 5] for latest developments.

In this talk we present the latest results from the QCDSF/UKQCD Collaboration for electromagnetic (EM) form factors, $\langle x \rangle_q$, $\langle x \rangle_{\Delta q}$, g_A and g_T . Simulations are performed with the Wilson gauge action and two flavours of $\mathcal{O}(a)$ -improved Wilson fermions. Four values of $\beta = 5.20, 5.25, 5.29, 5.40$, corresponding to lattice spacings in the range $0.1 < a < 0.07$ fm, allow for the approach to the continuum limit to be assessed, while a range of lattice volumes ($1.1 < L < 3.2$ fm) enable us to search for finite size effects in our simulations. A key feature of the new results presented here is the control we are now able to achieve over the approach to the chiral limit. This is made possible by several new simulations with pion masses reaching as low as 170 MeV, which are summarised in Fig. 1.

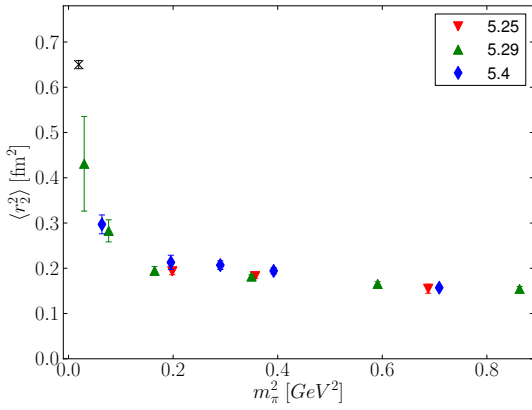


Figure 2: Results for the isovector Pauli radius, r_2 .

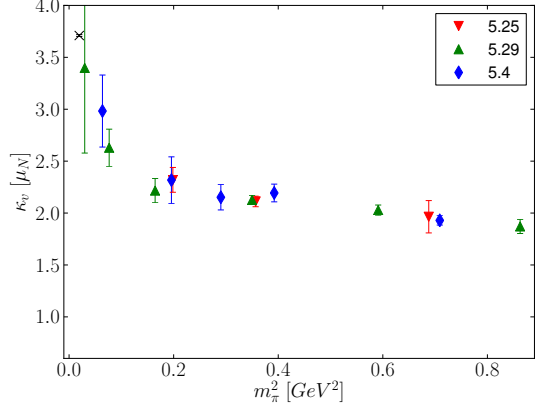


Figure 3: Isovector anomalous magnetic moment, κ_v in nuclear magnetons.

2. Electromagnetic Form Factors

The study of the electromagnetic properties of hadrons provides important insights into the non-perturbative structure of QCD. The EM form factors reveal important information on the internal structure of hadrons including their size, charge distribution and magnetisation.

A lattice calculation of the q^2 -dependence of hadronic electromagnetic form factors can not only allow for a comparison with experiment, but also help in the understanding of the asymptotic behaviour of these form factors, which is predicted from perturbative QCD. Such a lattice calculation would also allow for the extraction of other phenomenologically interesting quantities such as charge radii and magnetic moments. For a recent review see [6].

On the lattice, we determine the form factors $F_1(q^2)$ and $F_2(q^2)$ by calculating the following matrix element of the electromagnetic current

$$\langle p', s' | j^\mu(\vec{q}) | p, s \rangle = \bar{u}(p', s') \left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2M_N} F_2(q^2) \right] u(p, s), \quad (2.1)$$

where $u(p, s)$ is a Dirac spinor with momentum, p , and spin polarisation, s , $q = p' - p$ is the momentum transfer, M_N is the nucleon mass and j_μ is the electromagnetic current. The Dirac (F_1) and Pauli (F_2) form factors of the proton are obtained by using $j_\mu^{(p)} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$, while for isovector form factors $j_\mu^v = \bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d$. It is common to rewrite the form factors F_1 and F_2 in terms of the electric and magnetic Sachs form factors, $G_e = F_1 + q^2/(2M_N)^2 F_2$ and $G_m = F_1 + F_2$.

If one is using a conserved current, then (e.g. for the proton) $F_1^{(p)}(0) = G_e^{(p)}(0) = 1$ gives the electric charge, while $G_m^{(p)}(0) = \mu^{(p)} = 1 + \kappa^{(p)}$ gives the magnetic moment, where $F_2^{(p)}(0) = \kappa^{(p)}$ is the anomalous magnetic moment. From Eq. (2.1) we see that F_2 always appears with a factor of q , so it is not possible to extract a value for F_2 at $q^2 = 0$ directly from our lattice simulations. Hence we are required to extrapolate the results we obtain at finite q^2 to $q^2 = 0$. Form factor radii, $r_i = \sqrt{\langle r_i^2 \rangle}$, are defined from the slope of the form factor at $q^2 = 0$.

In Fig. 2 we see results for the isovector Pauli radius plotted as a function of m_π^2 . Here we observe a common trend of lattice determinations of this quantity, namely that the results are a factor of ~ 3 smaller than experiment and show little variation as a function of m_π^2 for pion masses

larger than $m_\pi > 300$ MeV. While the new results below $m_\pi < 300$ MeV are showing signs of upward curvature, as predicted from chiral perturbation theory [7, 8, 9], it is not yet clear whether this is enough to reach the experimental value. Of course, at such light quark masses, finite volume effects are expected to suppress the nucleon charge radii [9], and the results from three different volumes at $m_\pi^2 \approx 0.09$ GeV² indicate that this is indeed the case, although the effect appears to be rather small at this pion mass.

Results for the isovector anomalous magnetic moment, κ_v , are shown in Fig. 3 and the results are seen to be slowly increasing towards the experimental value as the pion mass is decreased. Closer to the chiral limit, the results are starting to exhibit stronger chiral behaviour, once again as predicted by χ PT [7, 8, 9]. Here finite volume effects are also expected to suppress the magnetic moment [8], so the fact that our results on a finite volume are not increasing fast enough to agree perfectly with experiment is not totally unexpected.

3. Moments of Structure Functions

In this section we discuss our latest findings for the nucleon's axial charge, g_A , tensor charge, g_T , and spin-independent and spin-dependent quark momentum fractions, $\langle x \rangle_q$ and $\langle x \rangle_{\Delta q}$. All operators in this section have been renormalised nonperturbatively using the Rome-Southampton method [10].

3.1 Axial Charge, g_A

The axial coupling constant of the nucleon is important as it governs neutron β -decay and also provides a quantitative measure of spontaneous chiral symmetry breaking. It is also related to the first moment of the helicity dependent quark distribution functions, $g_A = \Delta u - \Delta d$. It has been studied theoretically as well as experimentally for many years and its value, $g_A = 1.2695(29)$, is known to very high accuracy. Hence it is an important quantity to study on the lattice, and since it is relatively clean to calculate (zero momentum, isovector), it serves as useful yardstick for lattice simulations of nucleon structure.

The axial charge is defined as the value of the isovector axial form factor at zero momentum transfer and is determined by the forward matrix element

$$\langle p, s | A_\mu^{u-d} | p, s \rangle = 2g_A s_\mu, \quad (3.1)$$

where $A_\mu = \bar{q} \gamma_\mu \gamma_5 q$, p is the nucleon momentum, and s_μ is a spin vector with $s^2 = -M_N^2$.

g_A has been studied in-depth for many years by the QCDSF/UKQCD [11], LHP [12] and RBC/UKQCD [13] collaborations and has been shown to suffer from large finite size effects. Such effects are also seen in our new results at lighter pion masses as seen in Fig. 4 where we show those results for where we have more than one volume available at a fixed pion mass. Here we normalise the finite volume results for the axial charge, $g_A(L)$, with the result obtained from the largest available volume for that choice of (β, κ) , $g_A(\infty)$.

Another way to visualise our results is to consider the ratio g_A/f_π , as shown in Fig. 5. A good reason for considering this ratio is that the common renormalisation constant, Z_A , cancels in the ratio. It is also a natural ratio to consider in the context of the Goldberger-Treiman relation,

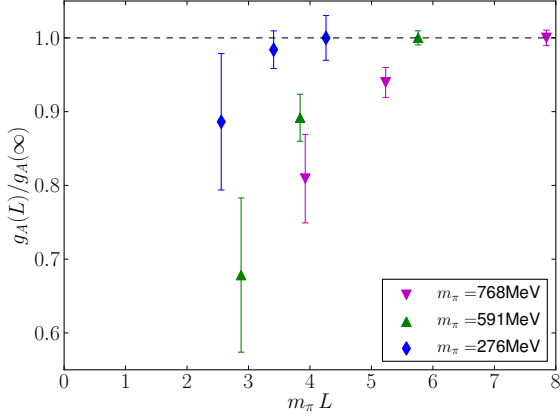


Figure 4: Finite volume results for the nucleon axial charge, $g_A(L)$, normalised with the largest available volume, $g_A(\infty)$, plotted as a function of $m_\pi L$ for $\beta = 5.29$.

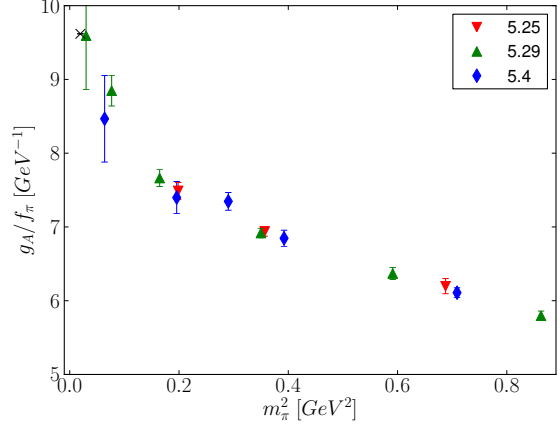


Figure 5: Results for the ratio of the nucleon axial charge and the light pseudoscalar decay constant, g_A/f_π

$g_A = f_\pi g_{\pi NN}/M_N$, which is valid in the chiral limit and approximately valid at larger quark masses. As can be seen in Fig. 5, the results for g_A/f_π show a strong dependence on m_π^2 and increase towards the experimental value as they approach the physical point. This may be an indication that the leading finite size effects cancel in this ratio [11, 14]. It would be nice to see if such cancellations occur in χ PT expressions of these quantities. This is currently under investigation.

3.2 Tensor Charge, g_T

Although the tensor charge has not received as much attention as g_A , there have been recently some new attempts to calculate the quantity on the lattice [15, 16, 17]. Since g_T is not known experimentally, it provides an opportunity for the lattice to make a prediction, although given the difficulties with g_A as discussed above, care must be taken, not only with the chiral extrapolation, but also in the assessment of finite size effects.

In Fig. 6 we show the latest QCDSF/UKQCD results, including the new simulations at low pion masses. Here we observe a similar trend to that observed in the axial charge, g_A , above, namely that increasing the volume at a fixed pion mass increases the result, although the effects are not as strong.

Considering the behaviour of the results at fixed $m_\pi L$, we observe that the results are reasonably flat as a function of m_π^2 . This is to be compared with the above result for g_A where we observed a definite increase in the results.

Given that we still don't have the chiral and finite volume systematics under control, it is not yet possible to make a strong prediction for the isovector tensor charge of the nucleon, however our current results indicate that it is likely to be in the range $0.9 < g_T < 1.1$.

3.3 Nucleon Momentum Fraction, $\langle x \rangle$

Lattice studies of $\langle x \rangle_q$ are notorious in that all lattice results to date at heavy quark masses exhibit an almost constant behaviour in quark mass towards the chiral limit and are almost a factor of 1.5 larger than phenomenologically accepted results, e.g. $\langle x \rangle_{u-d}^{\text{MRST}} = 0.157(9)$ [18]. Despite

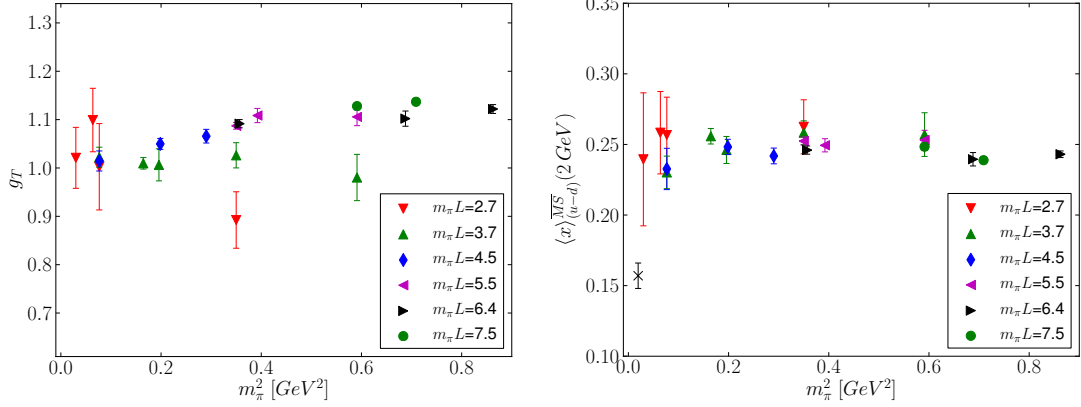


Figure 6: Results for the isovector tensor charge, g_T , **Figure 7:** Results for the isovector quark momentum fraction, $\langle x \rangle_{u-d}$, in the $\overline{\text{MS}}$ scheme at $\mu = 2 \text{ GeV}$.

predictions that the results should drop dramatically in the light quark mass region [19], we are still yet to see conclusive evidence for this from a lattice calculation.

To date, only connected contributions have been simulated to high precision, and although progress is being made towards the evaluation of disconnected contributions [20], here we only quote results for isovector quantities where disconnected contributions cancel.

Figure 7 shows the latest QCDSF/UKQCD results including new results below $m_\pi < 300 \text{ MeV}$. While the results above $m_\pi^2 > 0.1 \text{ GeV}^2$ show the same constant behaviour seen in many lattice simulations, the new results in the region below $m_\pi^2 < 0.1 \text{ GeV}^2$ are beginning to show some interesting behaviour, albeit with larger statistical errors. What is particularly interesting is the results at a fixed pion mass ($\approx 270 \text{ MeV}$), but with three different lattice volumes ($L = 24, 32, 40$). Here we observe that the result from the smallest volume continues the trend of the results at heavier pion masses, while the results from the larger volumes sit slightly lower. This is not totally unexpected, as the dramatic downward curvature has been shown to be only achievable on larger volumes [9, 21, 22]. For this reason, we expect that even after a large increase in statistics, the current point at $m_\pi \approx 170 \text{ MeV}$ with $L = 40$ could well still lie somewhat higher than phenomenological determinations. A further simulation at this pion mass but with spatial volume length of $L = 64$ is currently underway to further test these ideas.

Similar behaviour is seen in the spin-dependent momentum fraction, $\langle x \rangle_{\Delta q}$.

4. Conclusion & Outlook

We have presented some of the latest results on nucleon structure from the QCDSF/UKQCD collaboration, including EM form factors, $\langle x \rangle_q$, $\langle x \rangle_{\Delta q}$, g_A and g_T . These calculations are now becoming available at pion masses as low as $m_\pi \approx 170 \text{ MeV}$, so direct comparison with experimental determinations will soon be possible. However, as we have seen in, e.g. g_A , finite size effects are starting to become a serious issue. As a result, we are now planning a new simulation on a volume of $(5 \text{ fm})^3$, in order to minimise these effects, although corrections from ChPT will still probably need to be taken into account.

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