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The strange quark content of the nucleon in 2+1 flavor lattice QCD

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The strangeness of the nucleon, $\langle N|\bar{s}s|N\rangle - \langle 0|\bar{s}s|0\rangle$, is a quantity of interest for interpreting the results of dark matter detection experiments as well as for exploring the structure of the nucleon itself. We present a calculation of this quantity in 2+1 flavor lattice QCD using a range of lattice spacings and quark masses. The method is based on calculating quark-line disconnected contributions on the MILC lattice configurations, which include the effects of dynamical strange quarks. After continuum and chiral extrapolations, the value is $\langle N|\bar{s}s|N\rangle - \langle 0|\bar{s}s|0\rangle = 0.69 \pm 0.07_{stat} \pm 0.09_{sys}$ in the $\overline{ms}(2GeV)$ regularization.

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1. Introduction and motivations

The strange-quark content of the nucleon is difficult to measure experimentally but is a quantity of wide interest. In particular, the interaction cross section between some proposed dark matter candidates (for instance, neutralinos) and ordinary matter may have a large contribution from interactions with sea strange quarks in the nucleon. Specifically, the interest is in the quantity $\langle N|\int d^3x\bar{s}s|N\rangle - \langle 0|\int d^3x\bar{s}s|0\rangle$: the connected part of the strange quark condensate, integrated over the volume of the nucleon. Knowledge of this matrix element is crucial to design experimental schemes for dark matter detection and to interpret their results. If the matrix element is known, it is possible to determine the constraints on the parameter space of dark matter candidates accessible to a given experiment. [2, 3]

As the quantity in question cannot be measured experimentally or calculated perturbatively, we must turn to lattice QCD to compute it. Previous lattice calculations to answer this question have been done with quenched simulations[4, 5], with a 2-flavor sea using Wilson or overlap quarks[6, 7, 8], and 2+1 flavor stout quarks[9]. Recently this quantity was extracted from baryon mass fits to 2+1 flavor simulations [10]. The results from these studies often have large uncertainties, and some conclude that $\langle N|\bar{s}s|N\rangle$ may be significantly larger than its natural size of unity.

We outline a method for calculating this quantity by evaluating disconnected quark-line diagrams, and present the results of applying this method to the MILC Collaboration's library of gauge configurations using 2+1 flavors of Asqtad-improved staggered quarks. An additional very long ensemble of 4566 lattices from the UKQCD Collaboration with $\beta = 6.75$, $m_l = 0.06a$, $m_s = 0.30a$, $a \sim 0.125$ fm using the same action is included, allowing a measurement of $\langle N|\bar{s}s|N\rangle$ using 25788 gauge configurations using a full 2+1 flavor sea [1].

The method outlined here can be used for light quarks as well, and for hadrons other than the nucleon. While measurements of their quark condensates are not as immediately in demand, they may provide useful information about chiral perturbation theory low-energy constants, and may lead to further progress in understanding the structure of the QCD sea within hadrons.

2. The MILC lattice generation program

The MILC collaboration is engaged in an extensive project of QCD simulations using a Symanzik-improved gauge action and the Asqtad-improved staggered-quark action with a 2+1 flavor sea. This project consists of a number of runs at nominal lattice spacings of $a = .12$ fm, $a = .09$ fm, and $a = .06$ fm, along with other coarser runs not considered here. Details of the action, the ensembles of gauge configurations, and the method of extracting nucleon correlators can be found in Ref. [11]. An additional very long ensemble of 4566 lattices from UKQCD with $\beta = 6.75$, $m_l = 0.06a$, $m_s = 0.30a$ using the same action is included in the analysis.

3. The method

In a chiral fermion formulation, the matrix element in question is equal to $\frac{\partial M_N}{\partial m_s}$ by the Feynman-Hellman theorem. We emphasize that the derivative should be taken holding all other parameters in the action fixed. A sizeable value for $\frac{\partial M_N}{\partial m_q}$ does not imply a large dependence of the *physical*

nucleon mass on m_s. Changing the value of m_s changes the value of *all* physical quantities by a similar amount, and this change is interpreted as an overall rescaling of the lattice rather than a shift in the physical values. Likewise, a large result does not imply the presence of many $\bar{s}s$ loops in the nucleon. Rather, it comes from the suppression of the vacuum $\bar{s}s$ condensate near the nucleon.

One needs large-scale simulations with a true 2+1 flavor sea to determine $\langle N|s\bar{s}|N\rangle$ with reasonable accuracy. Previous methods used to determine this quantity require particular choices for the lattice parameters, namely ensembles with different m_s but the same β. On the other hand, in a general program of lattice QCD simulations such as MILC's, m_s is typically held fixed at its physical value while β is changed with the m_q's to keep the physical lattice spacing fixed. Thus, computing $\langle N|s\bar{s}|N\rangle$ using the standard method requires special-purpose simulations which must be limited in scope due to economics. We have developed a method to determine $\frac{\partial M_N}{\partial m_s}$ from any *single* lattice ensemble, allowing its use on the MILC lattice configurations. The many ensembles are simply used to conduct an extrapolation to the physical point and to improve statistics.

The nucleon mass M_N is obtained by a fit to the nucleon correlator $C(t)$ and as such can be thought of as a complicated function of the correlator at different times: $M_N = f(C(t_1), C(t_2), C(t_3)...)$. The crucial idea is that one can use the chain rule for differentiation to rewrite the derivative:

$$\frac{\partial M_N}{\partial m_q} = \frac{\partial M_N}{\partial C(t_1)}\frac{\partial C(t_1)}{\partial m_s} + \frac{\partial M_N}{\partial C(t_2)}\frac{\partial C(t_2)}{\partial m_s} + \frac{\partial M_N}{\partial C(t_3)}\frac{\partial C(t_3)}{\partial m_s ...} \tag{3.1}$$

The partial derivatives $\frac{\partial M_N}{\partial C(t_i)}$ can be evaluated most simply by applying a small perturbation to the nucleon correlator and examining the change in the fit result. The other partial derivative $\frac{\partial P(t_i)}{\partial m_s}$ can be evaluated by an application of the Feynman-Hellman theorem in reverse to relate it to $\langle P(t_i)\bar{s}s\rangle - \langle P(t_i)\rangle\langle\bar{s}s\rangle$.

Here we take advantage of the fact that, whenever the MILC code generates or reads a lattice for analysis, it prints a stochastic estimator for $\int d^4x\,\bar{s}s$. The number of estimators used in this work, 6-16 per lattice, is sufficient for their fluctuation to contribute significantly to the statistical error. These values can be used to compute $\left\langle C(t)\int d^4x\,\bar{s}s\right\rangle - \langle C(t)\rangle\left\langle\int d^4x\,\bar{s}s\right\rangle$ by simple evaluation with no additional use of computer time. By doing this for each t used in the fit to determine M_N, it is possible to evaluate the chain rule sum and determine $\frac{\partial M_N}{\partial m_s}$.

This double use of the Feynman-Hellman theorem to access the nuclear strangeness might seem redundant: as we have the values of $\langle\bar{s}s\rangle$, why not simply evaluate the matrix element $\langle N|s\bar{s}|N\rangle$ directly? The problem is that the lattice operator used to create and annihilate the nucleon overlaps with many other three-quark states; the nucleon is simply the lowest-lying three-quark state. Furthermore, the normalization of the nucleon state created this way is unknown. Using this operator along with the $\langle\bar{s}s\rangle$ data to evaluate $\langle N|s\bar{s}|N\rangle$ directly would give the sea quark content of some superposition of states of unknown normalization, *not* of the nucleon itself. The fitting procedure used to extract the nucleon mass provides a way of extracting information about the nucleon alone, and the double use of the Feynman-Hellman theorem provides a way to relate it to $\langle N|s\bar{s}|N\rangle$.

Statistical errors on the result for $\frac{\partial M_N}{\partial m_s}$ were calculated using the jackknife method with blocks of size 10; this is large enough to take into account autocorrelations in the lattice data. Use of larger jackknife blocks produces an insignificant change in the size of the errors.

4. Choice of fit range

Choosing a lower minimum distance in the nucleon mass fits will result in lower statistical error but may introduce systematic biases from pollution by excited states. The minimum fit distance required to avoid significant systematic error in $\langle N|s\bar{s}|N\rangle$ can be smaller than the one used for precision measurements of M_N. When extracting M_N, any pollution of the correlator by the excited states in the fit range will cause an incorrectly high value for the mass. However, the lowest-lying excited state is the delta. As we expect the effect of the delta on the strange quark condensate to be broadly similar to that of the nucleon, a small amount of delta pollution will not create substantial systematic error.

The minimum distances chosen should be consistent in physical units between lattice spacings. The minimum distance chosen should also be in a region where the result at successive minimum distances does not differ greatly. Both looking at the result ensemble-by-ensemble and looking at fits to all ensembles in the same nominal lattice spacing suggest the use of $t_{min} = 0.6$ fm. This choice is also suggested by the signal-to-noise ratio in $\frac{\partial C(t)}{\partial m_s}$; see Figure 1. In lattice units, this gives $t_{min} = 5a$ ($a = 0.13$fm), $t_{min} = 7a$ ($a = 0.09$fm), and $t_{min} = 10a$ ($a = 0.06$fm). By examining the dependence of $\frac{\partial M_N}{\partial m_s}$ on t_{min}, we conservatively estimate the systematic error due to excited state pollution as 10%. However, the nucleon mass itself can be computed with much lower statistical error at higher t_{min}; mass fits at these higher minimum distances differ from those at $t_{min} = 0.6$ fm by only 1% to 5%. Thus 10% is potentially an overly-pessimistic estimate.

The result is quite insensitive to the maximum distance used for the fits; this is expected, since the signal-to-noise ratio of the correlator is very poor there.

The values of $\frac{\partial M_N}{\partial m_s}$ obtained on each ensemble using these minimum distances are tabulated here and shown in the first panel of Figure 2. In the table, both the bare value (in the lattice regularization), and the value converted to the $\overline{ms}(2\text{ GeV})$ regularization and shifted to the correct strange quark mass (see below) are given.

β	am_l	am_s	a (fm)	N_{lats}	$\langle NssN\rangle$ (bare)	$\langle MssN\rangle$ (adj)
6.81	0.30	0.50	0.117	552	0.676(190)	0.620(147)
6.79	0.20	0.50	0.118	2067	0.702(98)	0.639(76)
6.76	0.10	0.50	0.119	2278	0.779(137)	0.696(106)
6.76	0.07	0.50	0.118	2098	0.867(214)	0.766(166)
6.76	0.05	0.50	0.117	2033	0.753(299)	0.679(230)
6.75	0.06	0.30	0.117	4566	0.884(171)	0.645(132)
7.08	0.0031	0.031	0.084	1013	1.232(339)	0.955(249)
7.085	0.00465	0.031	0.084	599	0.500(369)	0.417(271)
7.09	0.0062	0.031	0.084	1943	0.705(158)	0.568(116)
7.10	0.0093	0.031	0.084	1137	1.093(183)	0.853(134)
7.11	0.0124	0.031	0.084	1993	0.936(109)	0.739(80)
7.18	0.0310	0.031	0.081	496	0.530(161)	0.449(118)
7.10	0.0062	0.0186	0.082	948	0.776(218)	0.500(159)
7.46	0.0018	0.018	0.059	823	0.556(366)	0.375(255)
7.465	0.0025	0.018	0.059	798	0.848(494)	0.579(345)
7.47	0.0036	0.018	0.058	658	0.561(268)	0.379(187)
7.475	0.0054	0.018	0.059	616	0.813(443)	0.554(309)
7.48	0.0072	0.018	0.059	620	1.352(249)	0.929(160)
7.46	0.0036	0.0108	0.058	550	0.643(438)	0.328(305)

5. Analysis

5.1 Renormalization

Since in the continuum extrapolation it is necessary to compare quantities measured at different

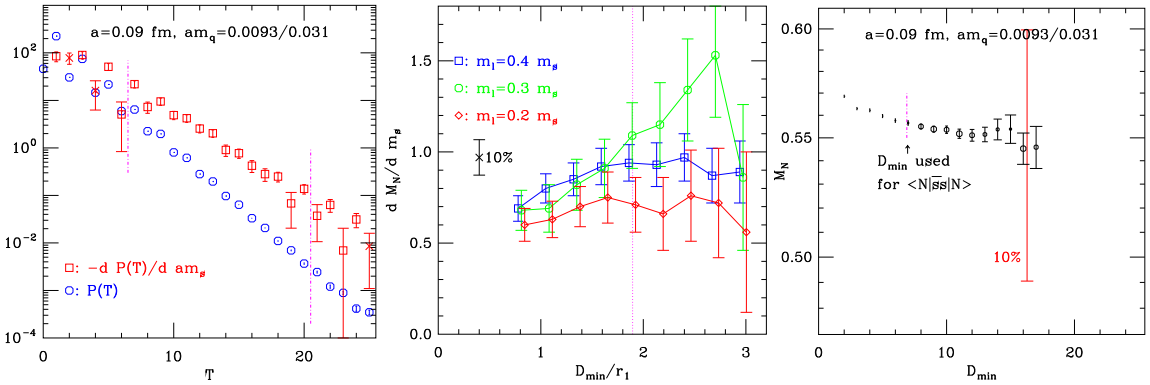

Figure 1: The nucleon correlator and the derivative of this correlator with respect to m_s for the ensemble with $am_l = 0.0093$ and $am_s = 0.031$ (first panel). For the derivative, the squares are points where the derivative is negative, and crosses are points where it is positive. The vertical lines show the range used in fitting the correlator. The second panel shows $\frac{\partial M_N}{\partial m_s}$ for three ensembles with $a \approx 0.9$fm as a function of the minimum distance used in the fitting, and the third panel shows the fitted nucleon mass itself versus t_{min}. The error bars labelled "10%" in the second and third panels show the size of the ten percent systematic error estimate from excited state contamination.

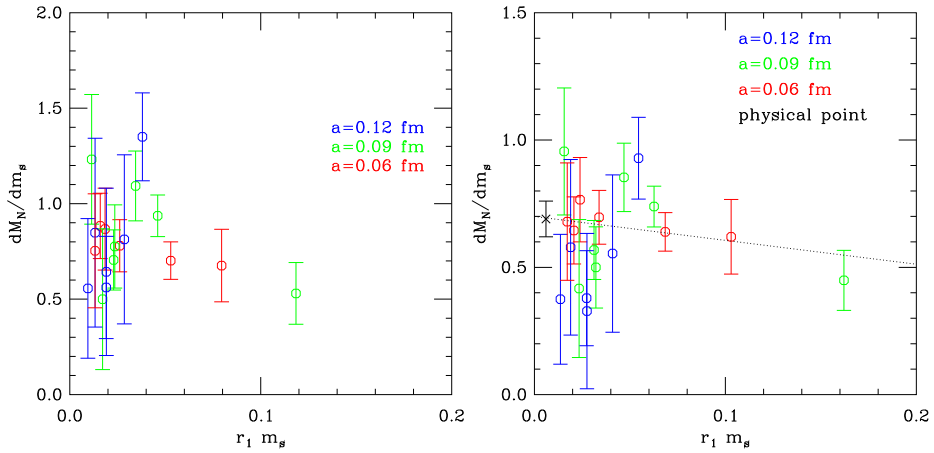

Figure 2: $\frac{\partial M_N}{\partial m_s}$ evaluated on the MILC ensembles. The first panel shows the "bare" results; the second panel shows the results converted to the $\overline{ms}(2\text{ GeV})$ renormalization and adjusted to the physical strange quark mass. The dotted line shows the chiral and continuum fit; the black point at $m_l = .005$ shows this fit evaluated at the physical point.

lattice spacings and thus different regularization schemes, it is necessary to convert all of the data to a common renormalization scheme since $\frac{\partial m_N}{\partial m_s}$ is a renormalization dependent quantity. Since the final result will be presented in the $\overline{MS}$ renormalization scheme at a scale of 2 GeV so it will be most useful to the broader physics community, it is preferable to begin the analysis by converting all values to this renormalization scale. The Z-factors for this conversion are known [12].

5.2 Strange quark mass dependence

Since the value of m_s in lattice units is not known prior to a lattice run, all of the ensembles

were run at strange quark masses different than the physical one; on some ensembles this error is large (20%). It is thus necessary to determine the dependence of $\frac{\partial m_N}{\partial m_s}$ on m_s (that is, $\frac{\partial^2 m_N}{\partial m_s^2}$) to perform an extrapolation to the physical m_s.

As MILC has run one or two ensembles at each nominal lattice spacing with m_s set to 60% of the guess at the physical value, it should be possible to determine $\frac{\partial^2 m_N}{\partial m_s^2}$ by examining $\frac{\partial m_N}{\partial m_s}$ on both these ensembles and those with the heavier strange quarks. However, since these "light strange" ensembles are short, better results for this second derivative can be obtained by a different method.

On many ensembles, the light quark is quite heavy; in some cases, it is actually closer to the physical value of m_s than the lattice heavy quark. Thus the behavior of the "light" quark condensate on these ensembles is similar to that of a strange quark condensate with a lower strange quark mass.

These ensembles with a heavier light quark allow for the measurement of the behavior of a "heavy quark condensate" at two different quark masses, and allow us to estimate the second derivative $\frac{\partial^2 M_N}{\partial m_s^2} = (\frac{\partial M_N}{\partial m_s} - \frac{\partial M_N}{\partial m_l})/(m_s - m_l)$. Since $\frac{\partial M_N}{\partial m_s}$ and $\frac{\partial M_N}{\partial m_l}$ are measured on the same lattices, their values are correlated, so the error on $\frac{\partial M_N}{\partial m_s} - \frac{\partial M_N}{\partial m_l}$ (determined by jackknife) is reduced.

By examining five ensembles with a large number of configurations and relatively heavy light quarks, it is possible to determine $\frac{\partial^2 M_N}{\partial m_s^2} = -2.2$ by a weighted average of its value on each, estimated as above. This value is then used to extrapolate each data point to $m_{s,phys}$. The second panel of Figure 2 shows the data converted to $\overline{MS}(2GeV)$ and adjusted to the correct m_s.

5.3 Light quark mass dependence and continuum extrapolation

Similarly, the value of $\frac{\partial M_N}{\partial m_s}$ may depend on m_l, and we are most interested in evaluating it at the physical light quark mass. In this case the data set contains results for $\frac{\partial M_N}{\partial m_s}$ at many different values of m_l, so it is possible to determine the dependence on m_l with a simple fit. Examination of the χPT form for $\frac{\partial M_N}{\partial m_s}$ reveals a constant plus linear fit in m_l is good enough; no χPT terms at higher-order are relevant at the level of statistical accuracy provided by the present data [13].

It is also necessary to extrapolate to the continuum. In the Asqtad fermion formulation, the leading-order errors in the action are proportional to a^2. Thus, the leading-order effect on $\frac{\partial M_N}{\partial m_s}$ will likewise be proportional to a^2; this effect can be determined by adding such a term to the fit form. However, such a term will be poorly constrained, since the better statistics in our dataset are from the $a = .12$ fm ensembles, and since the effect from lattice spacing is small. Other hadronic quantities calculated using the MILC Asqtad data show roughly a 10% effect between the coarse lattices and the continuum, so we use a Bayesian prior with a central value of 0 and a width corresponding to a 10% effect to constrain the lattice spacing dependent term in the fit.

As both the light quark mass dependence and the lattice spacing dependence will be computed in the same fit, the proper fit form is $\frac{\partial M_N}{\partial m_s} = A + Bm_l + Ca^2$, with a Bayesian constraint on C.

6. Result and error budget

Evaluating the fit above at the physical value of m_l and in the continuum, we find $\frac{\partial M_N}{\partial m_s} = 0.69 \pm 0.07_{statistical}$. We estimate the systematic error due to excited states present in the nucleon correlator, as discussed above, at 10%. The extrapolation to the physical light quark mass involves higher order terms in χPT which were not considered here. To estimate the size of this effect, we

note that if the nucleon mass itself is fit to a constant-plus-linear form over the range considered here, the result is seven percent off from the result obtained when two more orders in m_π are added to the fit. We thus estimate the effect of higher order terms in χPT as 7%. In one case where a spatial volume larger than the one used here has been run, the nucleon mass computed on the larger volume differs by 1%. Since the effect on the strange quark condensate is potentially more sensitive to finite volume effects, we estimate the systematic error due to finite volume effects as 3%. Finally, Ref. [12] quotes an error in Z_m as 4%. If these errors are combined in quadrature we thus estimate the total systematic error as 0.09.

The renormalization-invariant quantity $m_s \frac{\partial M_N}{\partial m_s}$ is also commonly quoted. Using a similar fit, we calculate $m_s \frac{\partial M_N}{\partial m_s} = 59(6)(8)$ MeV. This quantity does not include uncertainty in Z_m, as this cancels, but includes a lattice systematics error of nearly the same size, coming from the 2% uncertainty in the lattice scale and the uncertainty in the lattice strange quark mass.

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