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Finite Size Scaling and Universality in SU(2) at Finite Temperature

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We study the 4-th Binder cumulant on $4\times {N_\sigma}^3$ lattices for a pure SU(2) gauge theory. We use 20 data points for a sequence of N_σ in β intervals shrinking when N_σ increases, in order to reduce the nonlinear effects. Using a log-log fit of the slope versus N_σ, we obtain the preliminary result $\nu = 0.637(11)$ in reasonably good agreement with the value for the $3D$ Ising model universality class. The corrections due to irrelevant directions appear to be dominated by a term proportional to ${N_\sigma}^{-2.03(4)}$ which seems compatible with the breaking of rotational symmetry.

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1. Introduction

Finite Size Scaling (FSS) provides a powerful tool which is used to extrapolate information to infinite volume. It can also be used to study the critical behavior and calculate critical exponents. When the system is close to a 2nd order phase transition, some long distance properties can be determined by the global features of the system, such as the symmetries and the dimensionality. Universality enables us to classify seemingly different systems into certain classes. Svetitsky and Yaffe [1] pointed out that, for $d+1$ dimensional $SU(N)$ pure gauge model, "if any portion of the boundary is second-order, then the critical behavior will be described by some fixed point of d-dimensional, $Z(N)$ invariant spin systems." For a pure gauge $SU(2)$ theory in 3+1 dimension, we expect the universality class of the finite temperature transition to be the same as the 3 dimensional Ising model. Existing results on FSS for $SU(2)$ [2, 3, 4, 5, 6] agree well with this expectation.

In the following, we work on $SU(2)$ gauge theory in 3+1 dimensions: $N_\tau \times N_\sigma^3$, where $N_\tau = 4$, and $N_\sigma = 2,4,6,8,\cdots 16$, with periodic boundary conditions. We focus on direct estimations of the exponents ν and ω. For $N_\tau = 4$, the only direct estimate of ν we are aware is $0.65(4)$ obtained in Ref. [2], which is compatible with the accurate value $0.6298(5)$ obtained in Ref. [7] for the 3-dimensional Ising model. We would like: 1) to improve the accuracy of the estimate of Ref. [2] and 2) resolve the corrections due to the irrelevant directions. Part 2) is largely unexplored and a better understanding of these corrections could help us design methods to reduce these effects as done in Ref. [7] for the 3 dimensional Ising model. In the existing work for $N_\tau = 4$ in Refs. [2, 3, 6], a fixed β interval procedure was used. This means the β interval is fixed for different volumes. In these proceedings, we shrink the interval in order to reduce the nonlinear effects [9] and use a finer β resolution.

2. Binder cumulants and FSS

In the following, we define the 4th order Binder cumulant [8], g_4, as

$$g_4 = 1 - \frac{\langle P^4 \rangle}{3\langle P^2 \rangle^2}, \quad P = \frac{1}{N_\sigma^3} \sum_{\vec{x}} \frac{1}{2} \mathrm{Tr} \prod_{\tau=1}^{N_\tau} U_{\tau,\vec{x};0} \tag{2.1}$$

Related definitions appear in the literature, such as $B_4 = \frac{\langle P^4 \rangle}{\langle P^2 \rangle^2}$. We assume that there is no external field and that the g_4 depends on the scaling variables as

$$g_4 = g_4(u_\kappa N_\sigma^{1/\nu}, u_1 N_\sigma^{-\omega}, \ldots) \tag{2.2}$$

with

$$\begin{aligned} u_\kappa &= \kappa + u_\kappa^{(2)} \kappa^2 + \ldots \\ u_1 &= u_1^{(0)} + u_1^{(1)} \kappa + \ldots \end{aligned}$$

where u_κ is the only relevant scaling variable, u_1 is the first irrelevant scaling variable, and κ is the reduced quantity $\kappa = (\beta - \beta_c)/\beta_c$. Expanding up to the first nonlinear corrections, we obtain

$$g_4(\beta, N_\sigma) = g_4(\beta_c, \infty) + f_1 \kappa N_\sigma^{1/\nu} + f_2 \kappa^2 N_\sigma^{2/\nu} + (c_0 + c_1 \kappa N_\sigma^{1/\nu}) N_\sigma^{-\omega} + \cdots \tag{2.3}$$

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This expansion is accurate if $|\kappa|N_\sigma^{1/\nu}$ is small enough. In addition we would like the nonlinear effects of f_2 and c_1 to be negligible. It is easy to estimate f_2 from numerical data for intermediate values of κ [9] . However, c_1 is more difficult to resolve from an already small effect and its effect will be ignored. In the following, we will work with values of $|\kappa|N_\sigma^{1/\nu}$ such that the effects of f_2 are within the numerical errors of g_4 that we now proceed to discuss.

3. Error Analysis

For each volume, we worked on the region near β_c (we discuss β_c later). We generated 50,000 Polyakov loops with 20 different seeds for each β. Therefore, for each β, we have 1,000,000 values. However, the data is correlated and we will need to remove the correlations. The autocorrelation

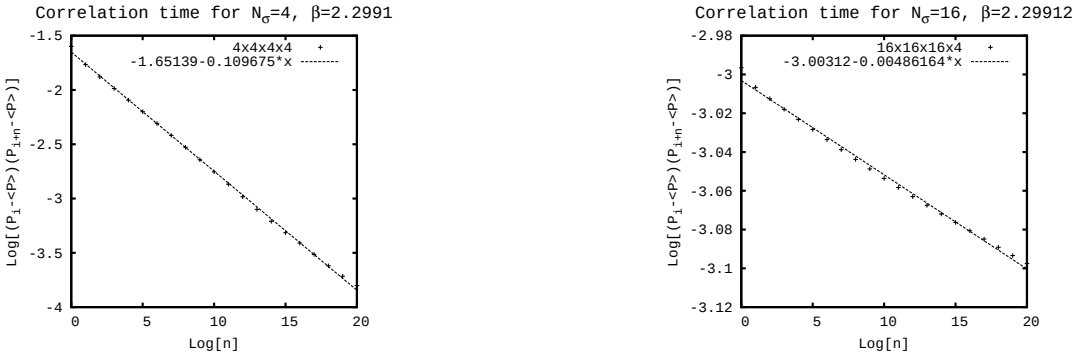

Figure 1: These two graphs show that the autocorrelation time changes with the volume. For N_σ=4, the autocorrelation time is around 10. For N_σ=16, the autocorrelation time is around 200 illustrating the critical slowing down.

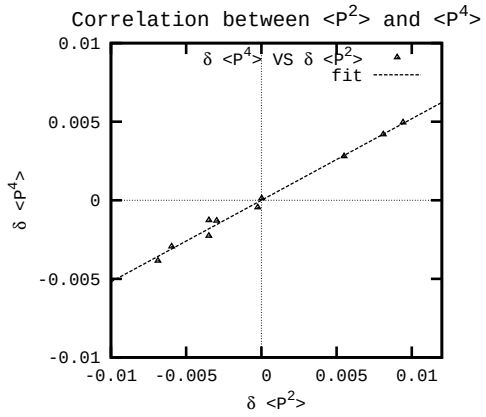

Figure 2: variation of P^4 and P^2 from the ensemble average of for 10 different seeds .

times change with β and the volume. Fig. 1 shows two autocorrelation functions. In order to remove the autocorrelation among the data, we tried the bootstrap and jackknife methods. For the jackknife method, we skipped every τ data in order to remove the autocorrelation, where τ is the autocorrelation time. Comparisons with other methods to estimate the errors will be discussed

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elsewhere. We should note here that one should not calculate the error of P^4 and P^2 first and then use the error propagation to get the error of g_4. This is simply because P^4 and P^2 are highly correlated, see Fig. 2. The jackknife method can remove the correlation between P^4 and P^2.

4. Determination of the critical exponent ν

We now focus on the estimation of ν. If we are reasonably close to β_c, we can use the linear form:

$$g_4(\beta, N_\sigma) \simeq g_4(\beta_c, \infty) + c_0 N_\sigma^{-\omega} + f_1 \kappa N_\sigma^{1/\nu} \tag{4.1}$$

This expression contains 6 unknown parameters: $g_4(\beta_c,\infty)$, c_0, ω, f_1, β_c and $1/\nu$ and we will use a new strategy to attack this difficult problem [10]. A first observation is that the dependence on ν can be isolated from the other parameters by studying the linear dependence in β. However, one should keep in mind that the slope of this linear fit depends implicitly on the choice of the center of the interval which should be as close as possible to β_c. In order to guarantee that nonlinear effects are under control, we will consider β intervals of the form

$$|\beta - (\beta_c)_{app}| < 0.015 \times (4/N_\sigma)^{(1/\nu)_{app}} \tag{4.2}$$

We will start with reasonable values for $(\beta_c)_{app}$ and $(1/\nu)_{app}$ and then show that the effect of their variations is small. The factor 0.015 has been chosen following a procedure described in Ref. [9] and guarantees an approximate linear relation between g_4 and β at fixed N_σ:

$$g_4 \simeq a_{N_\sigma} + b_{N_\sigma} \times \beta \ . \tag{4.3}$$

This is illustrated in the left side of Fig. 3 for $(\beta_c)_{app} = 2.299$, $(1/\nu)_{app} = 1.6$ and $N_\sigma = 10$. We see that in the chosen interval the deviation from linearity seem mostly due to statistical errors rather than a systematic curvature that would be observed if we had chosen a broader interval. The volume dependence of the parameters of the linear fit are

$$a_{N_\sigma} \simeq g_4(\beta_c,\infty) + c_0 N_\sigma^{-\omega} - f_1 N_\sigma^{1/\nu} \tag{4.4}$$

$$b_{N_\sigma} \simeq f_1 N_\sigma^{1/\nu}/\beta_c \tag{4.5}$$

Once we have b_{N_σ} for different N_σ, we can do a log-log fit to determine the inverse critical exponent $1/\nu$. One should note that the slope can be determined independently of β_c or $g_4(\beta_c,\infty)$. This is illustrated in the right side of Fig. 3. From the log-log fit with $N_\sigma \geq 6$, we obtain $1/\nu = 1.56(4)$. We plan to model and explain the deviations from linearity at low N_σ. We need to address the dependence on $(\beta_c)_{app}$ and $(1/\nu)_{app}$. We used the central value $\beta_c = 2.2991$ from Ref. [6] as the critical value and changed the center of the interval $(\beta_c)_{app}$, between 2.297 and 2.301 and $(1/\nu)_{app}$ between 1.4 and 1.8. We calculated $1/\nu$ for a set of 189 values of $((\beta_c)_{app},\ (1/\nu)_{app})$. The histogram is shown in Figure 4 left. We can see that the values appear in a rather large range between 1.468 and 1.631. The average of this set of results is $\overline{1/\nu} = 1.570$ with $\sigma = 0.027$ which will be our preliminary result. At the conference, a larger value was found, but with low statistics at large N_σ. In the meantime, we collected more data at large N_σ. If we choose the center of the data in a smaller range, namely with $(\beta_c)_{app}$ from 2.298 and 2.300 and the same range for $(1/\nu)_{app}$, we

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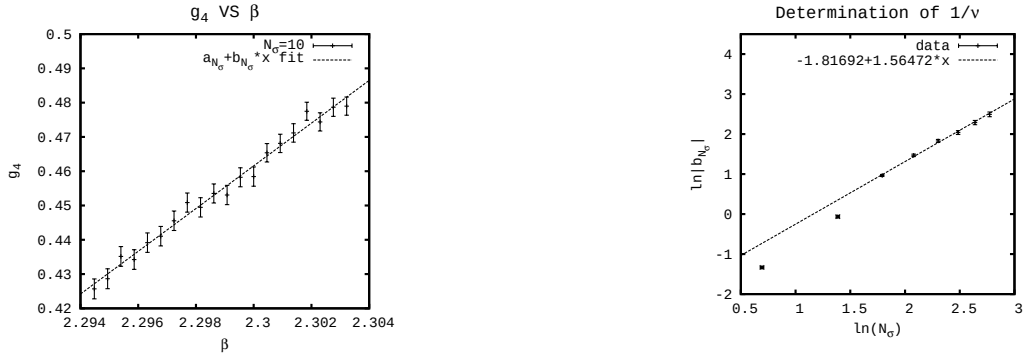

Figure 3: Left: linear fit of g_4 near β_c for $N_\sigma = 10$, $(\beta_c)_{app} = 2.299$ and $(1/\nu)_{app} = 1.6$. We get a_{N_σ} and b_{N_σ} from the fit for different volumes. Right: determination of $1/\nu$ from the log-log fit discussed in the text. $b_L \simeq f_1 \times N_\sigma^{1/\nu}/\beta_c$. By taking $Log[|b_{N_\sigma}|]$ vs. $Log[N_\sigma]$, we can get the slope which is just $1/\nu$. We only did this fit with N_σ between 6 and 16. The N_σ = 2, and 4 points deviate from the linear approximation.

Figure 4: Left Fig.: $(\beta_c)_{app}$ changes from 2.297 to 2.301; $(1/\nu)_{app}$ changes from 1.4 to 1.8. $\overline{1/\nu} = 1.570$; $\sigma = 0.027$. Right Fig.: $(\beta_c)_{app}$ changes from 2.298 to 2.300; $(1/\nu)_{app}$ changes from 1.4 to 1.8. $\overline{1/\nu} = 1.571$; $\sigma = 0.028$.

get a slightly different distribution shown on the right side of 4, but the average is essentially the same. It seems thus possible to average over $(\beta_c)_{app}$ and $(1/\nu)_{app}$ in order to get a more accurate value of $1/\nu$. For reference, $1/\nu_{Ising}$ is estimated as 1.5887(85) in Ref. [11] and 1.5878(12) in Ref. [7] and our preliminary result here is consistent with these more accurate values. This is preliminary, we plan to go to larger N_σ and to determine β_c independently using the method of Ref. [6].

5. Determination of the critical exponent ω

Unless we determine β_c and $1/\nu$ very precisely, it is very difficult to subtract the effects of the third term of Eq. (4.1). If we can work at β_c, this term is absent:

$$g_4(\beta_c, N_\sigma) = g_4(\beta_c, \infty) + c_0 N_\sigma^{-\omega} \tag{5.1}$$

Consistently with the previous section and the rest of the literature, we assume the universal value $g_4(\beta_c, \infty) = 0.46575$ as found in Ref. [7]. $Log[|g_4 - g_4(\beta_c, \infty)|]$ vs. $Log[N_\sigma]$ should be linear right

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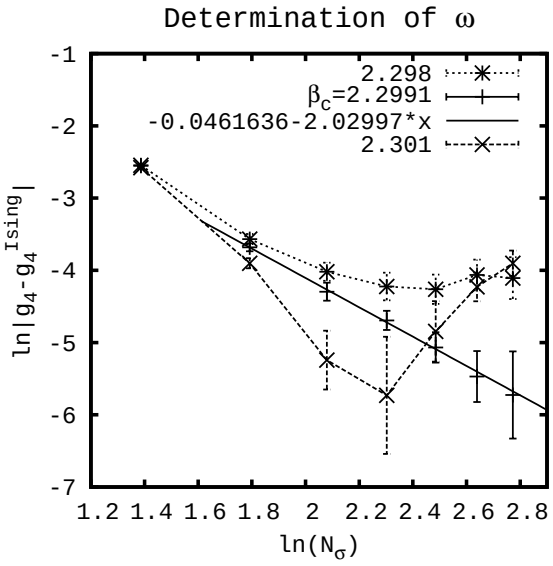

Figure 5: For β_c=2.2991, the behavior is approximately linear: $g_4 \simeq g_4(\beta_c,\infty) + c_0 \times N_\sigma^{-\omega}$.

at β_c and nonlinear for all the other βs. This is shown in Figure 5. At the same time, the slope is $-\omega$. The result we obtained from this analysis is $\omega = 2.030(36)$. This is very different from $\omega_{Ising} = 0.812$ [11]. It is possible that the coefficient of the $N_\sigma^{-\omega}$ is very small and the exponent we extrapolated is a sub-subleading exponent. For a detail discussion of the subleading corrections, see Ref. [7]. The most plausible explanation seems that this exponent is related to the irrelevant direction associated with the breaking of rotational symmetry [12] and which is close to 2.

6. Conclusion and perspective

By using methods discussed in Refs. [9, 10], we analyzed the 4-th order Binder cumulant of pure SU(2) lattice gauge theory and estimated the critical exponent ν and ω. $1/\nu$ is in good agreement with the value for the universality class of the 3D Ising model, while the corrections to scaling seem dominated by anisotropic effects [12].

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References

[1] B. Svetitsky and L. G. Yaffe, *Critical behavior at finite temperature confinement transitions, Nucl. Phys.*, **B210** 423 (1982).

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[2] Engels J, Fingberg J and Weber M, *Finite size scaling of SU(2) lattice gauge theory in 3+1 dimensions*, *Nucl. Phys.* **B332** 737 (1990).

[3] Fingberg J, Heller U M and Karsch F, *Scaling and asymptotic scaling in the SU(2) gauge theory*, *Nucl. Phys.* **B392** 517 (1993).

[4] I.L. Bogolubsky, V.K. Mitrjushkin, A.V. Sergeev, M. Muller-Preussker, H. Stuben, *Polyakov loops and Binder cumulants in SU(2) theory on large lattices*, *Nucl.Phys.Proc.Suppl.***129** 611 (2004).

[5] A. Papa, C. Vena, *Finite size scaling and deconfinement transition: The Case of 4-D SU(2) pure gauge theory*, *Int.J.Mod.Phys.***A19** 3209 (2004).

[6] A. Velytsky, *Finite temperature SU(2) gauge theory: Critical coupling and universality class*, *Int. J. Mod. Phys.* ,**C19** 1079 (2008).

[7] M. Hasenbusch, K. Pinn, and S. Vinti *Critical exponents of the three dimensional Ising universality class from finite-size scaling with standard and improved actions*, *Phys. Rev.* ,**B59** 11471 (1999).

[8] K. Binder, *Critical properties from Monte Carlo coarse graining and renormalization*, *Phys. Rev. Lett.*,**47** 693 (1981).

[9] Y. Meurice, *How to control nonlinear effects in Binder cumulants*, arXiv:0712.1190 [hep-lat].

[10] Y. Meurice, preprint in preparation.

[11] A. M. Ferrenberg and D. P. Landau *Critical behavior of the three dimensional Ising model: A high-resolution Monte Carlo study*, *Phys. Rev.* **B44**, 5081 (1991)

[12] M. Campostrini, A. Pelissetto, P. Rossi, and E. Vicari, *Two-point correlation function of three-dimensional $O(N)$ models: The critical limit and anisotropy*, *Phys. Rev* **E57**, 184 (1998).

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