Self-dual gauge fields, domain wall fermion zero modes and the Kugo-Ojima confinement criterion

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A new gauge fixing that imposes the U(1) real and quaternion real condition on the correlator of domain wall fermion on the left wall and right wall is proposed. The method is applied to the calculation of the mass function, the QCD effective coupling and the form factor of a proton. By assuming topological charge change $|Q| = 1$, the form factor of the nucleon is approximately fitted by the dipole form factor with cutoff $(0.71\text{GeV})^2$.

The difference of the Kugo-Ojima color confinement parameter in quenched simulation and in unquenched simulation is expected to be due to an anomaly caused by the vertex of two fermion zero modes and the Bose ghost/gluon.

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1. Introduction

The infrared (IR) QCD is characterized by the color confinement and the chiral symmetry breaking. We performed [1] lattice simulations of the domain wall fermion which preserves the chiral symmetry in the zero-mass limit using the full QCD gauge configurations of the RBC/UKQCD collaboration of $16^3 \times 32 \times 16$ lattice [2] and compared with results of staggered fermion of the MILC collaboration [3]. In these lattice simulations and in comparison with other works, we observed qualitative differences between quenched and unquenched simulations in the QCD effective running coupling, and in the Kugo-Ojima parameter [4] which is calculated numerically by the inverse of the Faddeev-Popov operator whose zero-modes subtracted at the zero momentum [5, 6, 7, 8]. The difference of the quenched and the unquenched simulation could be attributed to the fermion zero mode. In finite temperature hamiltonian QCD, zero-modes are localized around monopoles whose position is defined by the boundary condition on the time axis [9].

In Sect.2, we review the IR QCD and Gribov-Zwanziger-Kugo-Ojima theory, and in Sect.3, we show the method of gauge fixing using the boundary condition of fermions on the domain wall. In Sect.4, the quark-gluon coupling is presented and in Sect.5, the form factor of a nucleon is presented. Discussion and comments on the Kugo-Ojima parameter are given in Sect.6.

2. The IR QCD and Gribov-Zwanziger-Kugo-Ojima theory

The Landau gauge QCD or the Coulomb gauge QCD in the IR region suffers from the problem of gauge uniqueness. Gribov [10] defined the so called Gribov region Ω, which is convex and bounded in every direction and every gauge orbit passes through it at least once. Zwanziger [11] showed that the cut-off of the functional integral at the Gribov horizon $\partial\Omega$ may be replaced by the Boltzmann weight. He defined the free field action $S_0[A]$ and an ellipsoid in A space $Q[A]$ and replaced the measure inside the Gribov horizon

$$dA\,exp(-S_0)\delta(c - Q[A]) \rightarrow d\mu_\gamma = dA\,exp(-S_0)exp(-\gamma V Q[A])$$

where γ is defined from the self-consistency $\langle Q[A] \rangle_\gamma = c$, where the l.h.s is measured with use of $d\mu_\gamma$. When $\gamma \neq 0$, there appears in addition to the Fermi ghost c whose correlator $\langle c\bar{c} \rangle$ yields the Faddeev-Popov determinant, but the Bose ghost ϕ which couples to the quark and yields the linearing rising potential [12].

Kugo and Ojima [4] assumed $\gamma = 0$ and the BRST symmetry in the IR region and defined the color confinement parameter $u(k^2)$ as

$$\langle D_\mu c^a (A_\nu \times c)^b \rangle_k = (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\delta^{ab} u(k^2),$$
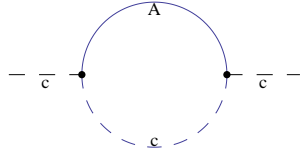
and $u(0) = -1$ as the color confinement condition. In this theory, the ghost dressing function $F(k)$ is expressed as
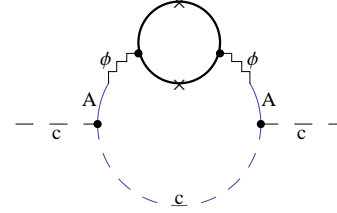
$$F(k) = \frac{1}{1 + u(k^2) + o(k^2)}$$

and the ghost propagator should be infrared divergent.

Quenched lattice simulations indicate, however, $u(0)$ is about -0.8 and unquenched lattice simulation indicates that $u(0) \sim -1$ [6], but the ghost propagator does not show the infrared divergence. Recently, violation of the BRST symmetry and infrared finiteness of the ghost propagator are proven [13, 14, 16], but whether $u(0)$ becomes -1 in unquenched simulation is not clear.

In perturbative QCD (pQCD), the ghost propagator is usually calculated via Fermi ghost-gluon coupling, but when $\gamma \neq 0$ there is a coupling of gluon to Bose ghost ϕ which couples to quarks. The fermionic zero mode should modify the measure $d\mu$ and we want to evaluate the zero mode effects using a specific gauge fixing.

Figure 1: The ghost-gluon loop contribution to the ghost propagator. Solid curves are gluons and dashed lines are Fermi ghosts.

Figure 2: The quark loop contribution in the ghost propagator. Thick solid curves are quarks, zig-zag lines are Bose ghosts.

3. The gauge fixing using the boundary condition of fermions on the domain wall

On the lattice, the expectation value of the quark propagator $S(p)$ consists of spin dependent $\mathscr{A} p$ part and spin independent $\mathscr{B}$ part. The propagator is written as

$$S(p) = \frac{-i\mathscr{A} p + \mathscr{B}}{\mathscr{A}(p^2 + \mathscr{M}\mathscr{M}^\dagger)} = \frac{-i\mathscr{A} p + \mathscr{B}}{\mathscr{A} p^2 + \mathscr{M}\mathscr{B}^\dagger}. \tag{3.1}$$

We specify the propagator of the left-handed quark by the suffix L and the right-handed quark by the suffix R. With use of the renormarization factor Z_A and Z_B, they are defined as

$$\mathrm{Tr}\langle \bar{\chi}(p,s)P_{L/R}\Psi(p,s)\rangle = Z_B(p)(2N_c)\mathscr{B}_{L/R}(p,s),$$

and

$$\mathrm{Tr}\langle \bar{\chi}(p,s)i\slashed{p}P_{L/R}\Psi(p,s)\rangle = Z_A(p)/(2N_c)i\mathbf{p}\mathscr{A}_{L/R}(p,s),$$

where $p_i = \frac{1}{a}\sin\frac{2\pi\bar{p}_i}{n_i}$ $(\bar{p}_i = 0,1,2,\cdots,n_i/4)$. The mass is defined as

$$\mathscr{M}(\hat{p}) = \frac{Re[\mathscr{B}_R(p,L_s/2)]}{Re[\mathscr{A}_R(p,L_s/2)]} \quad \text{and} \quad \mathscr{M}^\dagger(\hat{p}) = \frac{Re[\mathscr{B}_L(p,L_s/2)]}{Re[\mathscr{A}_L(p,L_s/2)]}.$$

3.1 The U(1) real condition

Since we adopt the representation of fermion spinors such that the γ_5 is diagonal, the fermion field ψ can be gauge transformed in the fifth dimension as

$$\psi \to e^{i\eta\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{-i\eta\gamma_5}, \tag{3.2}$$

3

such that on the left and on the right domain wall, the propagator becomes approximately real by choosing [1]

$$|e^{i\theta L}e^{i\eta} - 1|^2 + |e^{i\theta R}e^{-i\eta} - 1|^2$$

be minimum.

We sample wise measure the propagator by using the transformation matrix from the left boundary

$$a_x^0\sigma_1 + a_y^0\sigma_2 + a_z^0\sigma_3 + a_t^0\,iI = \begin{pmatrix} a_0 & b_0 \\ c_0 & d_0 \end{pmatrix}$$

which is given by the sample average of color diagonal components to the right boundary

$$a_x^{L_s-1}\sigma_1 + a_y^{L_s-1}\sigma_2 + a_z^{L_s-1}\sigma_3 + a_t^{L_s-1}\,iI = \begin{pmatrix} a_{L_s-1} & b_{L_s-1} \\ c_{L_s-1} & d_{L_s-1} \end{pmatrix}$$

3.2 The quaternion real condition

As the transformation matrix of the fermion at $s = 0$ to $s = L_s - 1$, we adopt the ansatz [15],

$$
\begin{aligned}
g_0 &= \begin{pmatrix} e^{-\nu} & 0 \\ 0 & e^{\nu} \end{pmatrix} \begin{pmatrix} \zeta^1 & \rho \\ 0 & \zeta^{-1} \end{pmatrix} \begin{pmatrix} e^{\mu} & 0 \\ 0 & e^{-\mu} \end{pmatrix} \\
&= \begin{pmatrix} e^{\gamma}\zeta^1 & f(\gamma,\zeta) \\ 0 & e^{-\gamma}\zeta^{-1} \end{pmatrix}.
\end{aligned}
\tag{3.3}
$$

In our five-dimensional domain wall fermion case, $\gamma = \mu - \nu$ and μ, ν contain the phase in the fifth direction $i\eta$.

$$2\mu = i\omega_2/\pi_2 - i\eta = (p_x + ip_y)\zeta + ip_t - p_z - i\eta \tag{3.4}$$

$$2\nu = i\omega_1/\pi_1 + i\eta = (p_x - ip_y)\zeta + ip_t + p_z + i\eta \tag{3.5}$$

The quaternion reality condition of the transformation matrix $g(\gamma,\zeta)$ gives

$$
\begin{pmatrix} a_{L_s-1} & b_{L_s-1} \\ c_{L_s-1} & d_{L_s-1} \end{pmatrix} \begin{pmatrix} \zeta^1 e^{\gamma} & f \\ 0 & \zeta^{-1}e^{-\gamma} \end{pmatrix} = \begin{pmatrix} \zeta^1 e^{-\gamma} & \bar{f} \\ 0 & \zeta^{-1}e^{\gamma} \end{pmatrix} \begin{pmatrix} a_0 & b_0 \\ c_0 & d_0 \end{pmatrix}
\tag{3.6}
$$

where $\bar{f} = \overline{f(\bar{\gamma}, -\dfrac{1}{\bar{\zeta}})}$.

The function f and $\bar{f}$ taken in [15] is.

$$f = \frac{d_0 e^{\gamma} - \frac{1}{a_{L_s-1}}e^{-\gamma}}{\psi}, \quad \bar{f} = \frac{\frac{1}{d_{L_s-1}}e^{\gamma} - a_0 e^{-\gamma}}{\psi}.$$

In general c_0 and c_{L_s-1} are polynomials of ζ, and they satisfy $c_{L_s-1}\zeta^1 = c_0\zeta^{-1}$. We define

$$\psi = \hat{c}_{-1}\zeta^{-1} + \hat{c}_1\zeta^1 + \delta$$

where $\hat{c}_{-1} = c_{L_s-1}$ and $\hat{c}_1 = c_0$ and δ is a constant, which is defined later.
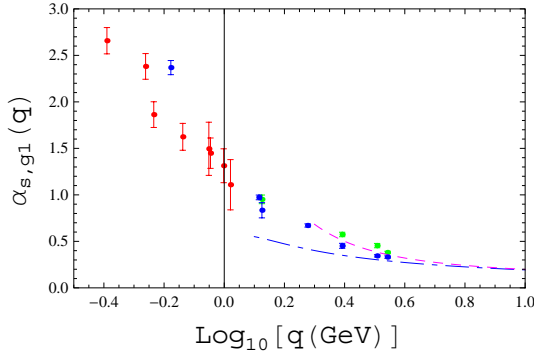
The difference

$$\Delta L/R = \begin{pmatrix} a_{L_s-1} & b_{L_s-1} \\ c_{L_s-1} & d_{L_s-1} \end{pmatrix}_{L/R} \begin{pmatrix} \zeta^1 e^\gamma & f \\ 0 & \zeta^{-1} e^{-\gamma} \end{pmatrix} - \begin{pmatrix} \zeta^1 e^{-\gamma} & \bar{f} \\ 0 & \zeta^{-1} e^\gamma \end{pmatrix} \begin{pmatrix} a_0 & b_0 \\ c_0 & d_0 \end{pmatrix}_{L/R} \tag{3.7}$$

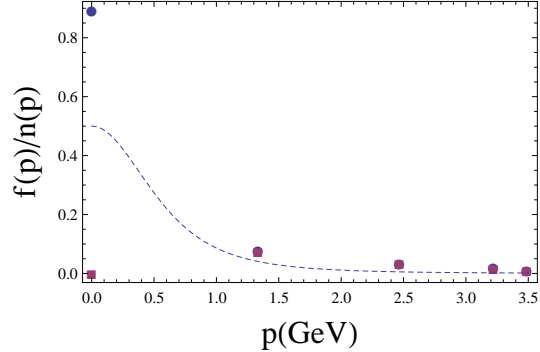should be small, if the left wall and the right wall are correlated by the self-dual gauge transformation.

We choose $\zeta^1 = \sqrt{\frac{c_0}{c_{L_s-1}}}$ to make $(\Delta L/R)_{21}$ consistent with 0 and obtain e^γ and δ, which approximately satisfy the quaternion reality. Corresponding to the sample-wise selection of larger absolute value of δ, i.e. a large zero-mode component, we assign a parameter $ind = 1$ for larger and 2 for smaller, and multiply the phase $(-1)^{ind}$ to the expectation value of the quark wave function that is used in the calculation of the effective mass of the quark. This phase factor can be absorbed in the phase $e^{i\eta}$ introduced to approximately satisfy the U(1) real condition.

4. The quark-gluon coupling

In Fig.1, the quark-gluon coupling in the Coulomb gauge of the domain wall fermion is compared with that of the staggered fermion. Red points below 1.2GeV are the experimental data of the JLab collaboration and the dark point is our result of momenta $q = 0.7$GeV/c. The dash-dotted line is the two loop pQCD result and the dashed line is the pQCD with A^2 condensates effect derived from the fitting the running coupling [7] calculated by using the gauge configuration of full-QCD staggered fermion of the MILC collaboration [3].

Figure 3: The effective coupling $\alpha_{s,g_1}(q)$ of domain wall fermion and the gluon without reality selection (green and with reality correction(magenta) in cylinder cut region. Dark low momentum points are that with reality correction.

Figure 4: The form factor of a nucleon. Dashed line is the phenomenological dipole fit of $1/(1 + p^2/M^2)^2$ with $M^2 = (0.71\text{GeV})^2$, multiplied by 1/2 to compare with the left-handed or right-handed contribution.

5. The form factor of a nucleon

We measure the form factor of a nucleon as a ratio of the three point function and the two-point function in momentum space. We calculate expectation values of G_{c+}^{NVN} etc by diagonalizing the

matrices whose coefficients of σ_1, σ_2 and σ_3 are real, and then evaluate e.g. $\langle (u_+^a C \gamma_5 d_-^b) u_+^c | \gamma_4 \gamma_\mu | (u_*^a C \gamma_5 d_*^b) u_*^c \rangle$ as

$$\frac{G^{NVN}{}_{c+}(p)}{G^{NN}_{c+}(p)} + \frac{G^{NVN}{}_{a-}(p)}{G^{NN}_{a-}(p)} + \frac{G^{NVN}{}_{b-}(p)}{G^{NN}_{b-}(p)} \tag{5.1}$$

The three point function $G^{NVN}_{c+}(p)$ is a product of

$$\sum_{a,b,c,c'} S(c_+, c_{+/-}, p) \langle V(c_{+/-}, c'_{-/+}, p) \rangle \gamma_5 S(c'_+ c'_{-/+}, p)^\dagger \gamma_5 \bar{S}(a, b, p).$$

where $\bar{S}(a, b, p)$ is the product of the propagator of the quark a and that of b.

$$\bar{S}(a, b, p) = S(a_-, a_-, p) \gamma_5 S(b_-, b_-, p)^\dagger \gamma_5$$

In the two point function $G^{NN}_{c+}(p)$, the factor before $\bar{S}(a, b, p)$ is replaced by the average of $S(c_+, c_{+/-}, p)$ and $S(c'_+ c'_{-/+}, p)^\dagger$.

In the product of $S(c_+, c_{+/-}, p) \gamma_5 S(c'_+ c'_{-/+}, p)^\dagger \gamma_5$ we assume contribution of self-dual field such that one propagator is that of left-handed and the other is right-handed. It means that we consider a change of the topological charge of one unit in the propagator. Of course it is an assumption which should be verified by comparison with experimental data.

The sum of the left-handed and the right-handed contributions are to be compared with the vector current form factor. The form factor at $p = 0$ is dominated by the $V(c_{+/-}, c'_{-/+}, p)$ of the left-handed fermion, while at finite p, the left-handed and the right-handed fermion contribute equally. The normalization at $p = 0$ is $1/36$ corresponding to $(3 \, \text{colors} \times 2 \, \text{spin projections})^2$. The data is taken from an average of 148 samples of RBC/UKQCD gauge configurations, and the dashed line is the phenomenological dipole fit with $M^2 = (0.71 \text{GeV})^2$, multiplied by $1/2$.

6. Discussion and comments on the Kugo-Ojima parameter

In the quenched calculation, whether the $u(0)$ becomes -1 or not depends on the treatment of the total derivative term [16]. On the lattice, however, this term is irrelevant. We think the difference of [13] and [16] exists in another kind of total derivative term. When the quark field contribution in the quark-quark-Bose ghost/gluon(ϕ/A) vertex [12]

$$g^3 \lambda^a \gamma_\mu S_{q\bar{q}} \gamma_\nu \lambda^b D_{AA,\mu\lambda} D^{bce}_{A\phi,\nu\kappa} f^{acd} k_\lambda$$

is replaced in the zero-mass limit by the product $(U^\dagger \partial_\mu U)(U^\dagger \partial_\nu U)$ where U are the quaternions of the fermion zero-modes, it becomes reminiscent of the anomalous current given in [17], which is expected to stabilize the IR fluctuation:

$$j_\kappa^e = -\frac{\varepsilon^2}{32\pi^3} \int \frac{d\Omega}{(\varepsilon z)^2} d^{abe} F_{\kappa\nu}^a(x) A_\nu^b(x, z)$$

where ε_κ is an arbitrary vector. The anomaly would be cancelled by that in the Baryon sector [18].

We think that the contribution of the Bose ghost that couples to quark zero-modes cannot be ignored in the unquenched Kugo-Ojima parameter calculation, and that the Nature of IR QCD possesses a kind of supersymmetry.

References

[1] Furui,S.:Propagator of the lattice domain wall fermion and the staggered fermion. Few-Body Syst. **45**, 51(2009). doi:10.1007/s00601-009-0008-9; arXiv:0801.0325[hep-lat].

[2] Allton,C. et al. : 2+1 flavor domain wall QCD on a $(2fm)^3$ lattice: light meson spectroscopy with $L_s = 16$. Phys. Rev. D**76**,014504 (2007); arXiv:hep-lat/0701013.

[3] Bernard,C et al.: QCD spectrum with three quark flavors. Phys. Rev. D**64**, 054506(2001).

[4] Kugo,T and Ojima,I. : Local covariant operator formalism of nonabelian gauge theories and quark confinement problem, Prog. Theor. Phys.(Kyoto)Suppl **66**,1 (1979).

[5] Furui, S. and Nakajima, H. : Infrared features of the Landau gauge QCD. Phys. Rev. D**69**,074505(2004) and references therein.

[6] Furui, S. and Nakajima, H. : What the Gribov copy tells about confinement and the theory of dynamical chiral symmetry breaking. Phys. Rev. D**70**,094504(2004), hep-lat/0403021.

[7] Furui,S. and Nakajima,H.: Infrared Features of Unquenched Landau Gauge QCD. Few-Body Syst. **40**, 101 (2006).

[8] Furui,S.: Roles of the color antisymmetric ghost propagator in the infrared QCD. Few-Body Syst. **43**,63(2009), doi:10.1007/s00601-008-0005-4; arXiv:0805.0680 [hep-lat].

[9] van Baal,P : Progress on Calorons, PoS 8th Conference Quark Confinement and Hadron Spectrum, arXiv:0901.2853, and references therein

[10] Gribov, V.N.:Quantization of non-abelian gauge theories, Nucl. Phys. **B139**, 1(1978)

[11] Zwanziger, D.: Local and renormalizable action from the Gribov horizon, Nucl. Phys. **B323**,513 (1989)

[12] Zwanziger, D.: Some exact infrared properties of gluon and ghost propagators and long-range force in QCD, arXiv:0904.2380.

[13] Dudal, D. et al.: Gribov no-pole condition, Zwanziger horizon condition, Kugo-Ojima confinrment criterion, boundary condition, BRST breaking and all that. arXiv:0904.0641.

[14] Kondo,K-I. : Kugo-Ojima color confinement criteroin and Gribov-Zwanziger horizon condition. Phys. Lett. B**678**(2009)322; arXiv:0904.4897[hep-th].

[15] Corrigan, E. and Goddard, P.: An n monopole solution with 4n-1 degrees of freedom. Comm. Math. Phys.**80**, 575(1981).

[16] Kondo,K-I. :Infrared behavior of the ghost propagator in the Landau gauge Yang-Mills theory, arXiv:0907.3249[hep-th].

[17] Gribov,V.N. : Anomalies and a possible solution of problems of zero-charge and infra-red instability, Phys. Lett. B**194**,119(1987).

[18] 't Hooft,G : in *Recent Developments in Gauge Theories*,eds 't Hooft,G, 135 (1980) Plenum Press.