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Topological configurations of Yang-Mills field responsible for magnetic-monopole loops as quark confiner

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We have given a new description of the lattice Yang-Mills theory a la Cho-Faddeev-Niemi-Shabanov, which has enabled us to confirm in a gauge-independent manner "Abelian"-dominance and magnetic-monopole dominance in the Wilson loop average, yielding a gauge-independent dual superconductor picture for quark confinement. In particular, we have given a new procedure (called reduction) for obtaining a gauge-independent magnetic monopole from a given Yang-Mills field. In this talk, we demonstrate how some of known topological configurations in the SU(2) Yang-Mills theory such as merons and instantons generate closed loops of magnetic-monopole current as the quark confiner, both of which are characterized by the gauge-invariant topological index, topological charge (density) and magnetic charge (density), respectively. We also try to detect which type of topological configurations exist in the lattice data involving magnetic-monopole loops generated by Monte Carlo simulation. Here we apply a new geometrical algorithm based on "computational homology" to discriminating each closed loop from clusters of magnetic-monopole current, since the magnetic-monopole current on a lattice is integer valued.

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1. Introduction

It is interesting to study the confinement mechanism of QCD. The dual superconductivity picture is a promising mechanism for quark confinement. It is known that the string tension of the Abelian part and the monopole part in Yang-Mills (YM) fields reproduce the original one, so called Abelian dominance and monopole dominance in the string tension. There is another approach that the center vortex can explain the string tension. However, the dominances of these objects has been observed only in the special gauges such as the maximal Abelian (MA) gauge or the maximal center gauge, but not in other gauges.

We have given a new description of the Yang-Mills (YM) theory on a lattice, which is expected to give an efficient framework to explain quark confinement based on the dual superconductivity picture in the gauge independent manner. In the case of the $SU(2)$ Yang-Mills theory, it is constructed as a lattice version of the Cho-Faddeev-Niemi-Shabanov (CFNS) decomposition[1] in a continuum theory[2][3][4]. By performing numerical simulations we have demonstrated that the gauge-invariant magnetic monopoles can be constructed and they reproduces the string tension [3], and shown the infrared “Abelian" dominance in the propagator, i.e., the extracted "Abelian" propagator is dominant in the infrared region[4]. We have extended the framework to the $SU(N)$ YM theory [5]. In fact, we have demonstrated the numerical simulation in the $SU(3)$ case, which has two options corresponding to its stability subgroups. One is the maximal option with the stability subgroup $\tilde{H} = U(1) \times U(1)$, which is the gauge-independent reformulation of the Abelian projection represented by the conventional MA gauge. We have demonstrated the gauge independent study of "Abelian" dominance in the conference lattice2007 [8]. The other is the minimal one with the stability group $\tilde{H} = U(2)$, which is a new type of formulation and derives a non-Abelian magnetic monopole [6]. We have demonstrated the non-Abelian magnetic monopole dominance in the conference lattice2008 [9].

In this talk, we apply this method to investigate the role of magnetic monopole for the confinement in the gauge independent manner. In what follows, we restrict to the $SU(2)$ case. We summarize the result of the lattice formulation of the CFNS decomposition, which can extract the dominant degrees of freedom that are relevant to quark confinement in the Wilson criterion in such a way that they reproduce almost all the string tension in the linear inter-quark potential. Combined with non-Abelian Stokes’s theorem, the magnetic monopoles can be extracted from the decomposed relevant part of YM field in the gauge invariant manner. The monopole dominance suggests that the magnetic monopole plays a central role in quark confinement. So, it is interesting to investigate the magnetic monopole as a quark confiner. We demonstrate the analysis for some known topological configurations as well as lattice data, where we can extract magnetic monopole loops directly from YM configuration, which are the gauge invariant physical objects.

2. New variables on a lattice

We summarize the new description of the $SU(2)$YM theory on a lattice[3][4] (See also [11]): The YM field $\mathbb{A}_{x',\mu}$ is represented as a link variable

$$U_{x,\mu} = \exp\left(-ig\int_{x}^{x+\hat{\mu}\varepsilon} dx^{\mu}\mathbf{A}_{\mu}(x)\right) = \exp\left(-ig\varepsilon\mathbb{A}_{x',\mu}\right), \tag{2.1}$$

which is supposed to be decomposed into the product of new variables, $X_{x,\mu}, V_{x,\mu} \in SU(2)$,

$$U_{x,\mu} = X_{x,\mu}V_{x,\mu} \tag{2.2a}$$

$$V_{x,\mu} = \exp\left(-ig\varepsilon\mathbb{V}_{x',\mu}\right), \tag{2.2b}$$

$$X_{x,\mu} = \exp\left(-ig\varepsilon\mathbb{X}_{x,\mu}\right), \tag{2.2c}$$

such that the decomposed variables are transformed by a full gauge transformation $\Omega_x \in SU(2)$:

$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}, \tag{2.3a}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}, \qquad X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_{x}, \tag{2.3b}$$

where $V_{x,\mu}$ is defined on a link $\langle x, x+\varepsilon\mu \rangle$ like $U_{x,\mu}$, and $X_{x,\mu}$ on a site. So, the corresponding (Lie algebra-valued) gauge filed is evaluated at the midpoint, $x' = x + \varepsilon\mu/2$, for $U_{x,\mu}$ and $V_{x,\mu}$, and at the site x for $X_{x,\mu}$.To define the decomposition, a color field, $\mathbf{n}_x = n_x^k \sigma^k/2$ is introduced as a site variable, where σ^k is the Pauli matrix and n_x^k $(k = 1,2,3)$ is a unit vector. The color field is transformed adjointly by an independent gauge transformation $\Theta_x \in SU(2)$ as $\mathbf{n}_x \to \Theta_x \mathbf{n}_x \Theta_x^\dagger$. The decomposition is determined by solving the defining equations:

$$D^\varepsilon_\mu[V]\mathbf{n}_x := \mathbf{n}_x V_{x,\mu} - V_{x,\mu}\mathbf{n}_{x+\mu} = 0, \tag{2.4a}$$

$$\mathrm{tr}(\mathbf{n}_x X_{x,\mu}) = 0, \tag{2.4b}$$

and the solution is obtained in terms of $U_{x,\mu}$ and $\mathbf{n}_x$

$$V_{x,\mu} = \tilde{V}_{x,\mu} \Big/ \sqrt{\frac{1}{2}\mathrm{tr}\left(\tilde{V}_{x,\mu}\tilde{V}^\dagger_{x,\mu}\right)}, \qquad \tilde{V}_{x,\mu} = U_{x,\mu} + 4\mathbf{n}_x U_{x,\mu}\mathbf{n}_{x+\mu}, \tag{2.5a}$$

$$X_{x,\mu} = U_{x,\mu} V^\dagger_{x,\mu}. \tag{2.5b}$$

The reduction condition must be introduced in order that the theory written in terms of new variables is equipollent to the original YM theory, i.e., the symmetry extended by introducing the color field, $SU(2)_\Omega \times [SU(2)/U(1)]_\Theta$, must be reduced to the same symmetry as the original YM theory, $SU(2)_{\Omega=\Theta}$. The reduction condition is given by the minimization of the functional, which is invariant under the gauge transformation, $SU(2)_{\Omega=\Theta}$. Here, we use the following functional:

$$F_{\mathrm{Red.}} = \sum\nolimits_{x,\mu} \mathrm{tr}\left(\left(D^\varepsilon_\mu[U]\mathbf{n}_x\right)\left(D^\varepsilon_\mu[U]\mathbf{n}_x\right)^\dagger\right)/\mathrm{tr}(\mathbf{1}). \tag{2.6}$$

Note that the stationary condition with respect to the color field, $\partial F_{\mathrm{Red.}}/\partial n_x^k = 0$, gives the differential form corresponding to the continuum theory, and it determines the color field for a given YM field. The algorithm solving the reduction condition is given by Ref.[11].

3. Gauge independent magnetic monopole

The Wilson loop operator on a lattice is given by the path ordered product of link variables,

$$W_C[U] := \mathrm{tr}\left[P\prod\nolimits_{<x,x+\mu>\in C} U_{x,\mu}\right]/\mathrm{tr}(\mathbf{1}) = \mathrm{tr}\left[P\exp\left(-ig\oint_C dx^\mu \mathbf{A}_\mu(x)\right)\right]/\mathrm{tr}(\mathbf{1}), \tag{3.1}$$

where the path C is defined along the relevant links, and we have used the definition of the link variable, $U_{x,\mu}$, in the 2nd equality. Following the paper [6], the Wilson loop operator in the fundamental representation in the continuum theory is rewritten into

$$\begin{aligned} W_C[\mathbf{A}] &:= \mathrm{tr}\left[P\exp\left(-ig\oint_C dx^\mu \mathbf{A}_\mu(x)\right)\right]/\mathrm{tr}(\mathbf{1}) \\ &= \int d\mu[\xi]_C \exp\left\{-ig\oint_C dx^\mu \mathbf{V}_\mu(x)\right\} \\ &= \int d\mu[\xi]_\Sigma \exp\left\{-ig\int_{\Sigma:\partial\Sigma=C} dS^{\mu\nu}\mathscr{F}_{\mu\nu}[\mathbf{V}]\right\} \end{aligned} \tag{3.2}$$

where $\mathbf{V}_\mu(x)$ is given by

$$\mathbf{V}_\mu(x) = \mathrm{tr}(\mathbf{A}_\mu(x)\mathbf{n}(x))\mathbf{n}(x) + \frac{1}{ig}\left[\partial_\mu \mathbf{n}(x), \mathbf{n}(x)\right]. \tag{3.3}$$

Following the paper [7], it is turn out that the defining equation eq(2.4) gives the decomposition which reproduces "Abelian" (V) dominance for the Wilson loop operator on a lattice even with a finite lattice spacing ε ;

$$W_C[U] \cong \mathrm{const}.W_C[V], \qquad W_C[V] := \mathrm{tr}\left[P\prod\nolimits_{<x,x+\mu>\in C} V_{x,\mu}\right]/\mathrm{tr}(\mathbf{1}) \tag{3.4}$$

and we can identify the $\mathbb{V}_{x',\mu}$ in eq(2.2b) with$\mathbf{V}_\mu(x')$ at the midpoint of the link. The field strength of $\mathbf{V}$ is given by

$$V_{x,\alpha}V_{x+\alpha,\beta}V^\dagger_{x+\beta,\alpha}V^\dagger_{x,\beta} = \exp\left(-ig\varepsilon\mathrm{tr}\left(\mathscr{F}_{x'',\mu\nu}[\mathbb{V}]\mathbf{n}_{x'}\right)\mathbf{n}_{x'}\right). \tag{3.5}$$

By using the Hodge decomposition, eq(3.2) is further rewritten into

$$W_C[\mathbf{A}] = \int d\mu[\xi]_\Sigma \exp\left\{-ig(K,\Xi_\Sigma) - ig(J,N_\Sigma)\right\}, \tag{3.6}$$

where we have used $K := \delta^* F$, $J := \delta F$ for F being the field strength 2-form $\mathscr{F}_{\mu\nu}$, and $\Xi_\Sigma := \delta^* S_\Sigma \Delta^{-1}$ and $N_\Sigma := \delta\Theta_\Sigma\Delta^{-1}$ with the four-dimensional Laplacian, $\Delta = d\delta + \delta d$. Here S_Σ is the vorticity tensor defined by $S^{\mu\nu}_\Sigma = \int_\Sigma dS^{\mu\nu}(X(\sigma))\delta(x - X(\sigma))$ on the surface $\Sigma : \partial\Sigma = C$ supported by the Wilson loop C. Therefore, the monopole dominance for the lattice Wilson loop is given by

$$\langle W_C[U]\rangle \cong \langle W_C[V]\rangle = \left\langle \exp\left(-ig\varepsilon\sum\nolimits_{x,\mu} k_{x,\mu}\Xi_{x,\mu}\right)\right\rangle, \tag{3.7a}$$

where the lattice magnetic monopole current $K_{x,\mu}$ and $\Xi_{x,\mu}$ are given by

$$\Xi_{x,\mu} = \sum\nolimits_s \frac{1}{2}\varepsilon_{\mu\alpha\beta\gamma}\partial^s_\alpha\Delta^{-1}(x-s)S^J_{\beta\gamma}(s+\mu),\ \partial_\alpha S^J_{\alpha\beta} = J_\beta \ \in C \tag{3.8}$$

$$k_{x,\mu} = 2\pi\bar{k}_{x,\mu} = \frac{1}{2\varepsilon^2}\varepsilon_{\mu\lambda\alpha\beta}\partial_\lambda\Theta_{x,\alpha\beta}[V], \quad \Theta_{x,a\beta}[V] := \arg\mathrm{tr}\left((\mathbf{1}+\mathbf{n}_x)V_{x,\alpha}V_{x+\alpha,\beta}V^\dagger_{x+\beta,\alpha}V^\dagger_{x,\beta}\right), \tag{3.9}$$

where ∂_λ denotes the forward difference (lattice derivative) in the $\hat{\lambda}$ direction: $\varepsilon\partial_\lambda f(x) := f(x+\varepsilon\hat{\lambda}) - f(x)$. It should be noticed that the magnetic monopole current $k_{x,\mu}$ is gauge invariant, as can be seen from the transformation law of the new variables.

4. Analysis of the magnetic monopoles as quark confiners

We have demonstrated the "Abelian" dominance and the magnetic-monopole dominance in the gauge invariant way[11]. The static potential calculated from the decomposed variable V is reproduced by one from the Yang-Mills field, i.e., the string tension obtained by V field 94% of one from YM field. The string tension calculated only from magnetic monopoles reproduces 93% of it. This shows that the proposed new description of YM theory extract the degrees of freedom relevant for the quark confinement from YM field, and the magnetic monopole which is the decomposed part of $V_{x,\mu}$, plays a central role for the quark confinement. So we investigate the magnetic monopoles as quark confiners.

The magnetic monopole current is a integer-valued, $\bar{k}_{x,\mu} \in \{0, \pm 1, \pm 2\}$, and can be identified with the link variables on the dual lattice $\langle \tilde{x} - \varepsilon\hat{\mu}, \tilde{x}\rangle$ with $\tilde{x} = (x_1 + \varepsilon/2, x_2 + \varepsilon/2, x_3 + \varepsilon/2, x_4 + \varepsilon/2)$. So non-zero monopole currents can be identified with directional edges which construct a directional graph. The current conservation,

$$\varepsilon\partial_\mu k_{x,\mu} = 2\pi\sum\nolimits_\mu \left(\bar{k}_{x+\mu,\mu} - \bar{k}_{x,\mu}\right) = 0,$$

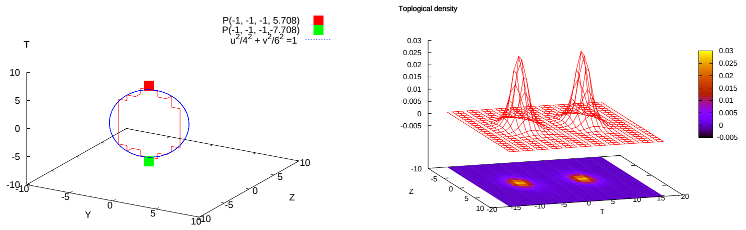

Figure 1: (Left panel) The plot of a magnetic-monopole loop generated by a pair of (smeared) merons in 4-dimensional Euclidean space. The 3-dimensinal plot is obtained by projecting the 4-dimensional dual lattice space to the 3-dimensional one, i.e., $(x,y,z,t) \to (y,z,t)$. The positions of two meron sources are described by solid boxes, and the monopole loop by red solid line. (Right panel) The plot of the topological charge density for $z-t$ plane (slice of $x=y=0$). Two peaks of the topological charge density are located at the positions of two merons.

implies that there is no source and no sink. When we identify a monopole current of charge $\overline{k}_{x,\mu} = \pm 2$ with a double edges of a single charge, each vertex (site of dual lattice) has the same number of incoming and outgoing edges. Therefore, the monopole currents construct the loops. Here, some loops are connected each other at dual lattice sites, or share links of the dual lattice carrying the double monopole charge. Thus the analysis of the monopole configuration is converted to a geometrical problem. In the magnetic part of the Wilson loop operator eq(3.6) or eq(3.7a), the term (K,Ξ_Σ) represents the 4-dimentional version of the Gauss's linking number formula, that is, the linking number between a single loop K and a two-dimensional surface Σ whose boundary is the Wilson loop C. The conjecture on translation the contribution of center vortex and of magnetic monopole loops to the Wilson loops in Ref. [6] can be checked by using the lattice data.

Now, let us apply this method to topological configurations of Yang-Mills field, which has been considered to play the central role for confinement. The topological solution can be translated to the lattice variable by using the definition of the link variable:

$$U_{x,\mu} = P\exp\left(-ig\int_x^{x+\hat{\mu}\varepsilon} dx^\mu \mathbf{A}_\mu(x)\right) \simeq P\prod_{n=1}^{N} \exp\left(-ig\frac{\varepsilon}{N}\mathbf{A}_\mu(x+(n-1/2)\frac{\varepsilon}{N})\right),$$

where the integral along the link $\langle x, x+\hat{\mu}\varepsilon\rangle$ is calculated by the path ordered product of the exponentiated gauge potential. Here, we demonstrate the two-merons case. Figure 1 shows the detected magnetic monopole. The numerical analysis on the lattice reproduces the result of analytical study in Ref.[12], even though the analysis is done by using the finite volume lattice. It is interesting to investigate other topological configurations such as two-instantons and calorons. We find a monopole loop for the JNR type of two-instantons solution, while no monopole loop for one-instanton and two-instantons of 't Hooft type. The detail analysis of two-instantons solutions will appear in the subsequent paper[13].

Then, we return to the analysis of lattice data. It is very hard to manipulate monopole configurations directly, since they contain more than 35000 non-zero monopole currents (see the left panel of Figure 2). Therefore, we introduce the algebraic algorithm for topology. The CHomP homology software, provided by the computational homology project[14], computes topological invariants called the Betti numbers of a collection and their generators in the algebraic way. The Betti numbers are part of the information contained in the homology groups of a topological space, which intuitively measure the number of connected components, the number of holes, and the number of enclosed cavities in low dimensions. In our case, the generators of the dimension-one homology group correspond to magnetic monopole loops.

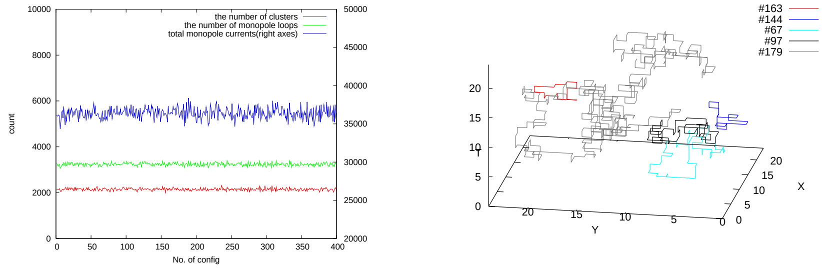

Figure 2: (Left panel) The analysis of magnetic monoples for 400 configulations in 24^4 lattice with the parameter $\beta = 2.4$. The blue line shows the number of the non-zero magnetic monoole currents (right virtical axes). The red line an dgreen line show the number of clusters of connected loops (the Betti number of dimension zero) and the number of loops (the Betti number of dimension one) for each configuration, respectively (see left virtical axes). (Right panel) The 3-dimensional polt of detected magbetc monopole loops, where the graph in 4dimensional Euclid space is projected to the 3-dimensional space, i.e., $(x,y,z,t) \to (x,y,z)$.

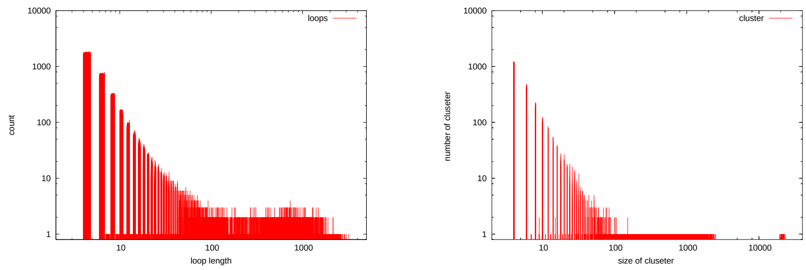

Figure 3: (Left panel) The histgram for the length of monopole loops, i.e., for each configuration the number of monopole loops with length n is counted, and is polted as a truss of poles with the same length. (Right panel) The histogram of the cluster size.

We apply the method to the lattice data of 24^4 lattice with periodic boundary condition whose configurations are generated by using the standard Wilson action with the parameter $\beta = 2.4$. The left panel of Figure 2 shows data of detected magnetic monopole currents for 400 configurations. The blue line shows the number of non-zero charge currents, i.e., non-zero magnetic monopole currents share about 3% of links. The red line and green line show the number of clusters and the number of loops for each configurations, respectively. The right panel of Figure 2 shows an example of detected magnetic monopoles, which are plotted in the 3-dimensional space projected from the 4-dimendional Euclidian space. Figure 3 shows detail of the magnetic monopole configurations. Then, we are ready to investigate the magnetic monopole contribution to the static potential by using extracted monopole loops.

5. Summary and discussions

We have given a new description of Yang-Mills theory on a lattice, which gives the decomposition of the Yang-Mills field for extracting the relevant degrees of freedom for quark confinement in the gauge independent manner based on the dual superconductivity picture. The extracted magnetic monopoles explain the string tension and they should play the central role for quark confinement. It is interesting to investigate the magnetic monopole as a quark confiner. The implication of the quark confinement by the configurations topological charge can be convert to the geometrical relations of the topological charge density and distribution of the magnetic monopole loops, since the magnetic monopole and topological charge density are given as a gauge invariant object (physical object). We have applied the method to the topological configurations and the lattice data, and extracted the magnetic-monopole loops directly. Using these magnetic-monopole configurations, we investigate the relations among the topological charge density, magnetic monopoles and static potential. It is also interesting to investigate the implication of the magnetic monopole loops for the confinement and deconfinement phase transition.

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