The D_s and D^+ Leptonic Decay Constants from Lattice QCD

Fermilab Lattice and MILC Collaborations

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We present the leptonic decay constants f_{D_s} and f_{D^+} computed on the MILC collaboration's $2+1$ flavor asqtad gauge ensembles. We use clover heavy quarks with the Fermilab interpretation and improved staggered light quarks. The simultaneous chiral and continuum extrapolation, which determines both decay constants, includes partially-quenched lattice results at lattice spacings $a \approx 0.09$, 0.12 and 0.15 fm. We have made several recent improvements in our analysis: a) we include terms in the fit describing leading order heavy-quark discretization effects, b) we have adopted a more precise input r_1 value consistent with our other D and B meson studies, c) we have retuned the input bare charm masses based upon the new r_1. Our preliminary results are $f_{D_s} = 260 \pm 10$ MeV and $f_{D^+} = 217 \pm 10$ MeV.

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1. Introduction

We report on progress in the Fermilab Lattice and MILC Collaboration calculation of the D meson decay constants. This work is a continuation of the program that predicted the decay constants: $f_{D^+} = 201(3)(17)$ and $f_{D_s} = 249(3)(16)$ MeV [1], in good agreement with the CLEO-c value of $f_{D^+} = 205.8 \pm 8.5 \pm 2.5$ [2, 3]. We have since extended this calculation to two additional ensembles at our finest lattice spacing $a \approx 0.09$ fm and we have replaced a limited set of very coarse ($a \approx 0.18$ fm) ensembles with higher statistic ensembles at a somewhat finer spacing $a \approx 0.15$ fm. In our last update [4] we reported: $f_{D^+} = 207(11)$ and $f_{D_s} = 249(11)$, where f_{D_s} is about 0.6σ lower than the recent experimental average [5]. The value of f_{D_s} remains an pressing issue given that experimental average is about 2.1σ higher than the most precise lattice result from the HPQCD collaboration [6]. The apparent tension between experiment and lattice predictions has motivated suggestions of physics beyond the Standard Model [7].

Smaller statistical uncertainties and better control of systematic effects are key to resolving the f_{D_s} puzzle. In this report, we have doubled statistics on the most chiral of the $a \approx 0.09$ fm lattices; otherwise, statistics have not changed. A new generation of calculations, now underway, aims to increase statistics by a factor of four overall. Our progress includes: a) a better method of accounting for heavy-quark discretization effects b) a more precise input value for the scale parameter r_1, consistent with our other heavy quark studies and c) more precisely tuned input charm kappa values.

2. Staggered chiral perturbation theory for heavy-light mesons

We use the asqtad improved staggered action for both sea and light valence quarks. Leading discretization effects split the light pseudoscalar meson masses,

$$M^2_{ab,\xi} = (m_a + m_b)\mu + a^2\Delta_\xi \,, \tag{2.1}$$

where there are sixteen tastes in representations $\xi = P, A, T, V, I$.

Staggered chiral perturbation theory for heavy-light mesons accounts for such taste breaking effects [8]. At NLO in the chiral expansion, for $2+1$ flavors, and at leading order in the heavy quark expansion,

$$\phi_{H_q} = \Phi_0 \left[1 + \Delta f_H(m_q, m_l, m_h) + p_H(m_q, m_l, m_h) + c_L \alpha_V^2 a^2 \right] \,, \tag{2.2}$$

where $\phi_{H_q} = f_{H_q}\sqrt{m_{H_q}}$ and f_{H_q} is the decay constant of a heavy meson H_q consisting of a heavy quark and a light quark of mass m_q. The heavier sea quark has mass m_h and the two degenerate light sea quarks have mass m_l. The ϕ_{H_q}, in general, are partially quenched: $m_q \neq m_l$ and $m_q \neq m_h$. The chiral logarithm terms, Δf_H, are a dependent as a consequence of the mass splittings in Eq. (2.1) as well as from "hairpin" terms proportional to the low energy constants $a^2\delta'_A$ and $a^2\delta'_V$. The a dependence of the analytic terms, p_H, ensures that ϕ_{H_q} is unchanged by a change in the chiral scale, Λ_χ, of the logarithms. The expression in Eq. (2.2) is used in our combined chiral and continuum extrapolations. In practice, we add the NNLO analytic terms to the fit function in order to extend the fit up to $m_q \sim m_s$ and extract f_{D_s}. Priors for the parameters $a^2\delta'_A$ and $a^2\delta'_V$ as well as values of the physical light quark masses are obtained from the MILC analysis of f_π and f_K [9].

3. Discretization effects from clover heavy quarks

We use tadpole-improved clover charm quarks. At leading order, discretization errors are a combination of $\mathcal{O}(a^2\Lambda_{HQ}^2)$ and $\mathcal{O}(\alpha a\Lambda_{HQ})$ effects where α is the QCD coupling and Λ_{HQ} is the scale in the heavy quark expansion. Our past studies have estimated heavy quark discretization effects using such power counting arguments to bound the error at the smallest lattice spacing, taking $\Lambda_{HQ} \approx 700$ MeV. This rather crude method does not effectively use the data to guide the error estimate.

This study introduces a new procedure: the leading order heavy quark discretization errors are modeled to leading order as part of the combined chiral and continuum extrapolation. At tree-level, discretization effects arise from both the quark action and the (improved) current [10, 11, 12]. We add five extra terms to Equation 2.2:

$$\Phi_0\left[a^2\Lambda_{HQ}^2\left\{c_E f_E(am_Q)+c_X f_X(am_Q)+c_Y f_Y(am_Q)\right\}+\alpha_V a\Lambda_{HQ}\left\{c_B f_B(am_Q)+c_3 f_3(am_Q)\right\}\right] \tag{3.1}$$

The coefficients c_E, c_X, c_Y, c_B and c_3 are additional parameters determined in the fit while the f_i are (smooth) functions of the heavy quark mass, am_Q, known at tree level. We introduce priors for the coefficients constraining them to be $\mathcal{O}(1)$ while setting $\Lambda_{HQ} = 700$ MeV and $m_c \sim 1.2$ GeV. Currently the data are too noisy and the shapes of the functions f_i are too similar for the fit to prefer a particular Λ_{HQ}. Including the heavy-quark discretization terms increases the decay constants by a few MeV and increases the error from $\sim 1.8\%$ to $\sim 3.8\%$. The larger error now includes the residual heavy-quark discretization uncertainty in addition to residual light-quark discretization effects (encoded in Eq. (2.2)) as well as statistical errors. We quote a combined uncertainty from all the three sources of error.

4. Lattice spacing determination from r_1

The distance r_1 is a property of the QCD potential between heavy quarks. The ratio r_1/a, for lattice spacing a, has been computed for all of the MILC gauge ensembles. At intermediate stages of the decay constant analysis quantities are converted from lattice units to r_1 units using r_1/a. The value of r_1 must then be input in order to convert results to physical units. The r_1 value is also an input to the process of determining other quantities such as the bare charm quark masses as discussed in the next section.

Figure 1 depicts several r_1 determinations. The first two determinations historically (circa 2004–2005) are labeled "HPQCD $\Upsilon(2S\text{-}1S)$" [13] and "MILC $\Upsilon(2S\text{-}1S)$" [14]. They are both based on the same analysis of the Υ spectrum by the HPQCD Collaboration using a subset of the current MILC ensembles. The two determinations differ mainly in the details of the continuum extrapolation. The MILC Collaboration is also able to infer a value of r_1 based on the value of f_π they find in their analysis of the light mesons. Recent light-meson analyses include results from finer lattice spacings than the earlier Υ spectrum study and the resulting r_1 values are known to better precision. The figure shows the result of two recent analyses labeled 'MILC f_π 2007' [15] and "MILC f_π 2009" [9]. The (preliminary) 2009 result agrees at the 0.9σ level with the MILC Υ value but differs from the HPQCD Υ value at the 1.8σ level. As these proceedings were being
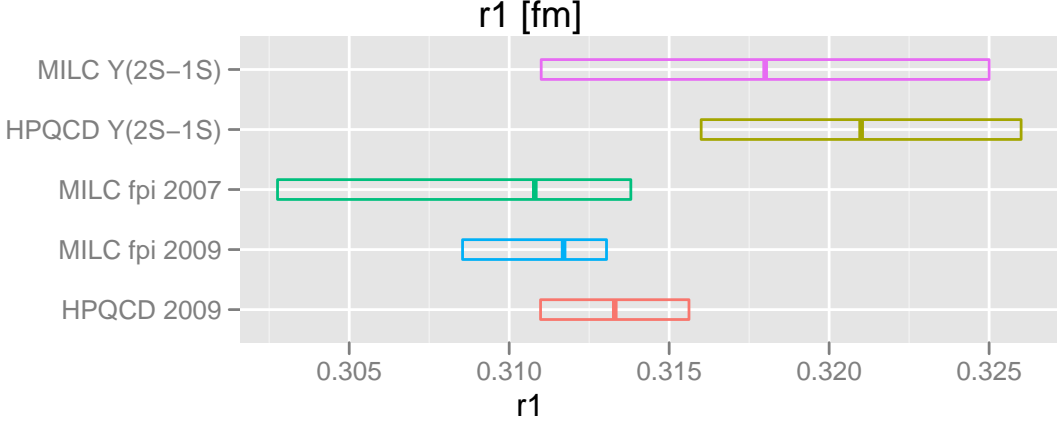
Figure 1: Values of scale parameter r_1 in fermi units. The "HPQCD $\Upsilon(2S\text{-}1S)$" value uses the HPQCD collaboration Υ spectrum results to set the physical value [13]. The "MILC $\Upsilon(2S\text{-}1S)$" value derives from essentially the same spectrum analysis [14]. MILC determines r_1 more precisely from their calculation of f_π: "MILC f_π 2007" [15] and "MILC f_π 2009" [9]. In a very recent update, "HPQCD 2009", several physical quantities, including recent Υ results, are used as inputs [16].

r_1 [fm]:		0.3108			0.318		
a	κ run	κ tune	$\delta\phi_s$	% ϕ_s	κ tune	$\delta\phi_s$	% ϕ_s
0.09	0.127	0.1272	-0.0043	-0.56	0.1267	$+0.0065$	$+0.84$
0.12	0.122	0.1222	-0.0036	-0.50	0.1215	$+0.0091$	$+1.26$
0.15	0.122	0.1222	-0.0031	-0.42	0.1213	$+0.0108$	$+1.47$
δf_{D_s} [MeV]		-1.8			$+1.3$		

Table 1: Tuning of κ charm at the three lattice spacings for two choices of r_1. The shift $\delta\phi_s$ is the change in ϕ at the strange quark mass when κ changes from the run value to tuned κ value. The corresponding change in extrapolated f_{D_s} is δf_{D_s}. In each case, all other extrapolation inputs are fixed to their appropriate (r_1 dependent) values.

prepared, HPQCD published a new value for r_1 [16], labeled "HPQCD 2009" in the figure, in much better agreement with MILC's recent r_1 values.

In this study, we use the MILC r_1 determinations from f_π to set the physical scale. Our central value for r_1 (the 2007 value) was also used in our studies of the semileptonic decays on the same lattices [17, 18]. The range of the 2009 MILC r_1 determination is used to set a symmetric uncertainty around the central value. Hence, we take $r_1 = 0.3108 \pm 0.0022$. Our previous decay constant studies used the MILC Υ value: $r_1 = 0.318 \pm 0.007$ as an input which is about one σ higher.

5. Retuning kappa charm

We determine the value of κ for the charm quark by requiring that the spin-averaged kinetic masses of the lattice pseudoscalar and vector mesons made from a heavy clover quark and strange

source	ϕ_{D_s}	ϕ_{D^+}	R_{D^+/D_s}
statistics and discretization effects	2.6	3.4	1.3
chiral extrapolation	2.0	2.5	0.8
inputs r_1, m_s, m_d and m_u	0.7	0.7	0.3
input m_c	1.0	1.2	0.2
Z_V^{cc} and Z_V^{qq}	1.4	1.4	0
higher-order ρ_{A_4}	0.3	0.3	0.2
finite volume	0.2	0.6	0.6
total	3.8	4.7	1.7

Table 2: Uncertainties as a percentage of ϕ and the ratio. The total combines all of the errors in quadrature.

asqtad valence quark equal the spin-averaged D_s meson mass. The tuning depends upon r_1 in the conversion between lattice and physical masses.

In the past year, we have conducted new kappa-tuning runs with at least four times the statistics of our older tunings. At each lattice spacing, we simulated mesons for three values of κ around charm and three light-quark masses around strange allowing us to retune κ for a given r_1.

Table 1 shows preliminary tunings for κ charm based upon the two input values: $r_1 = 0.3108$ fm (present value) and $r_1 = 0.318$ fm (past studies). For each r_1, the (preliminary) tuned kappa and the corresponding change $\delta\phi_s = \phi_s(\kappa \ tune) - \phi_s(\kappa \ run)$ is listed by lattice spacing. The "run" kappa values are those used in the decay constant simulations. We adjust each ϕ_q point by $\delta\phi_s$ prior to the chiral extrapolation to correct for the mistuning of kappa. The bottom row of the table shows the resulting change in f_{D_s}. The opposite signs of the differences show that keeping kappa tuned partly compensates the change in r_1. We find that changing r_1 from 0.318 fm to 0.3108 fm while keeping kappa charm tuned increases f_{D_s} by about 4.2 MeV.

6. The chiral and continuum extrapolation, results and uncertainty budget

We fit ϕ_{D_q} results from lattice simulations on eleven asqtad MILC ensembles [14] at the three lattice spacings: $a \approx 0.09$, 0.12 and 0.15 fm. Our valence quark masses are in the range $0.1m_s \lesssim m_q \lesssim m_s$. Since our last report, we have doubled the statistics at the most chiral of the $a \approx 0.09$ ensembles. The 3×4 panel of plots at the top in Fig. 2 shows the ϕ_{D_q} points and the fit where the fit function includes the lattice-spacing effects described in Sections 2 and 3. The plot at the bottom of Fig. 2 shows the extrapolations in the limit $a = 0$. The upper (D_s) curve shows $m_l \to \hat{m}$ setting $m_q = m_h = m_s$, while the lower (D^+) curve shows $m_q, m_l \to \hat{m}$ setting $m_h = m_s$. The physical quark mass inputs are from the MILC light meson analysis and $\hat{m} = (m_u + m_d)/2$. The points denoted by the red triangles correspond to physical f_{D_s} and f_{D^+}. Our preliminary results are:

$$f_{D^+} = 217 \pm 10 \ \text{MeV}, \quad f_{D_s} = 260 \pm 10 \ \text{MeV} \quad \text{and} \quad f_{D_s}/f_{D^+} = 1.20 \pm 0.02 . \tag{6.1}$$

We have combined the statistical and the systematic uncertainties listed in Table 2 in quadrature. Our largest uncertainty is the combined uncertainty from statistical and residual discretization effects. The second largest uncertainty, chiral extrapolation, is an estimate of chiral expansion effects not included in the fit function and effects from variation in the extrapolation procedure. The
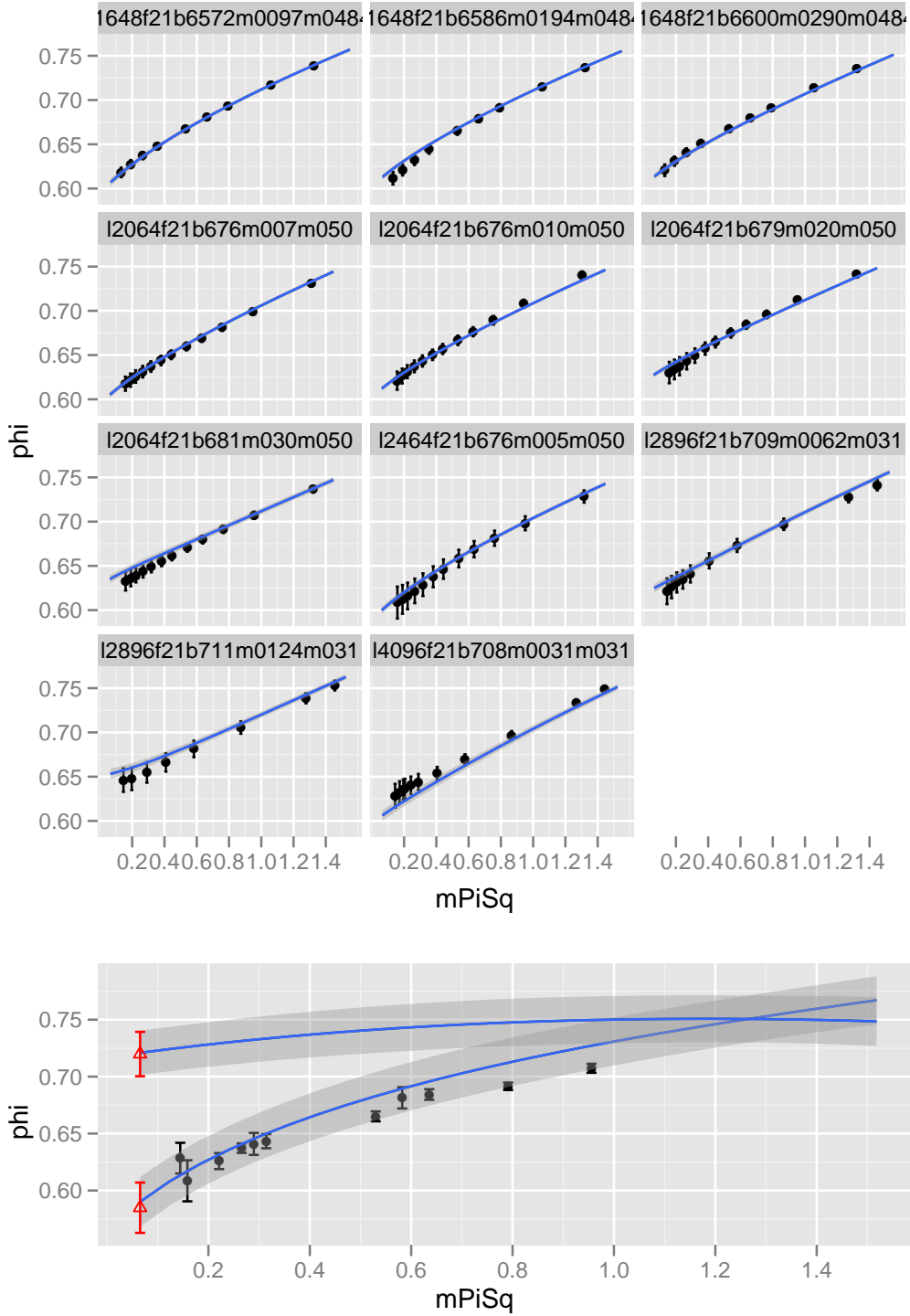
Figure 2: The preliminary *D* meson chiral extrapolation. The 3×4 matrix of plots (top) show the ϕ data and corresponding fit including a^2 effects. Reading from left to right and top to bottom, plots correspond to $(a, m_l/m_h)$ values of $(0.15, 0.2)$, $(0.15, 0.4)$, $(0.15, 0.6)$, $(0.12, 0.14)$, $(0.12, 0.2)$, $(0.12, 0.4)$, $(0.12, 0.6)$, $(0.12, 0.1)$, $(0.09, 0.2)$, $(0.09, 0.4)$ and $(0.09, 0.1)$. The larger plot (bottom) shows an overlay of the f_{D_s} and f_{D+} extrapolations. The extrapolated curves are the fit (with error bands) taking $a^2 \to 0$ and fixing/extrapolating the light quarks to their physical masses. The extrapolations are not expected to go though any of the points which are computed at finite *a*. None of the data points having m_q near m_s seen the upper panel are visible in the D_s extrapolation.
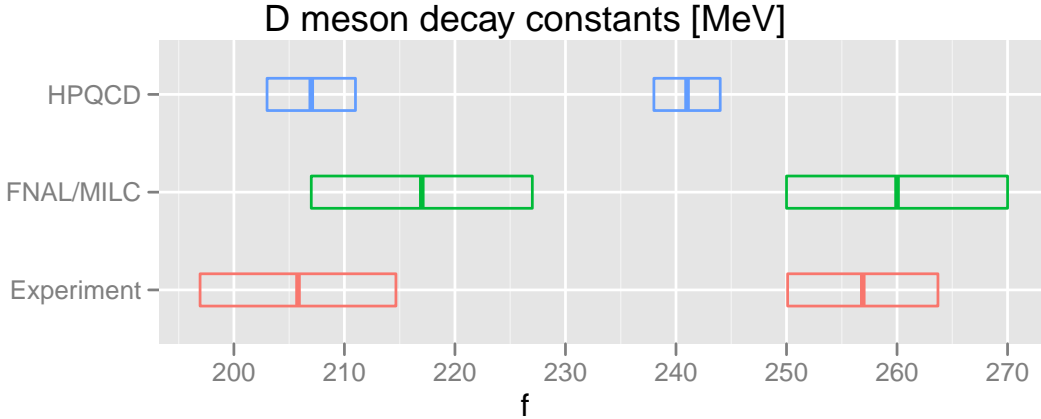
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D meson decay constants [MeV]

Figure 3: Comparisons of the Fermilab/MILC values of f_{D+} and f_{D_s} to values from the HPQCD Collaboration [6] and recent experimental values [3][5].

third largest error is the statistical error in the nonperturbative calculation of the current renormalizations Z_V^{cc} and Z_V^{qq}. The value of f_{D_s} is about eleven MeV (one sigma) higher than our earlier value. Using nominal kappa values rather than tuned values at the previous r_1 value accounts for about 1.3 MeV of the difference. Changing to the new r_1 while keeping kappa tuned results in a 4.2 MeV increase. Incorporating heavy quark effects into the fit increases f_{D_s} by about 2 MeV. Higher statistics on the most chiral of the $a \approx 0.09$ fm lattice increases f_{D_s} by about 1 MeV. These changes combine nonlinearly in the fit to yield the net increase.

Figure 3 compares the Fermilab and MILC Collaboration values for the decay constants with the HPQCD Collaboration [6] values and with the experimental results. The experimental result for f_{D+} is from CLEO [3] while the f_{D_s} value is the Heavy Flavor Averaging Group average [5] of determinations by CLEO, BaBar and Belle. The Fermilab / MILC results remain in agreement with experiment. The total error on the experimental average for f_{D_s} is now smaller that our error providing a challenge for future lattice determinations. The apparent discrepancy between the HPQCD value of f_{D_s} and the other two f_{D_s} values is most striking. The HPQCD value is lower by about 1.8–2.1σ. The source of this difference may be clarified by further lattice simulations.

7. Summary and future plans

We have made several improvements in our analysis: a) discretization effects from both heavy and light quarks are modeled in our extrapolation function, b) we adopted a more precise r_1 value which derived from the MILC f_π analysis rather than the r_1 value related to early Υ spectrum results c) we have improved the tuning of kappa charm. These improvements to the analysis will be more crucial in our next generation of decay constant study. We will increase statistics by a factor of four and extend the analysis to the finer lattice spacings $a \approx 0.06$ and 0.045 fm which will reduce our combined statistical plus discretization error as well as help reduce uncertainties attributed to chiral extrapolation procedures. In addition, a new high-statistics computation of the nonperturbative part of the current renormalization aims for an error below the 0.5% level.

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