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Dynamical lattice computation of the Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$

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We perform a two-flavor dynamical lattice computation of the Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$ at zero recoil in the static limit. We find $\tau_{1/2}(1) = 0.297(26)$ and $\tau_{3/2}(1) = 0.528(23)$ fulfilling Uraltsev's sum rule by around 80%. We also comment on a persistent conflict between theory and experiment regarding semileptonic decays of B mesons into orbitally excited P wave D mesons, the so-called "$1/2$ versus $3/2$ puzzle", and we discuss the relevance of lattice results in this context.

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1. Introduction

We are concerned with semileptonic decays of B mesons (B and B^*) into orbitally excited P wave D mesons (collectively denoted as D^{**}'s): $B^{(*)} \rightarrow D^{**} \, l \, \nu$. These decays are of particular interest, because there is a persistent conflict between theory and experiment, the so-called "$1/2$ versus $3/2$ puzzle": while experimental results indicate that a decay into "$1/2$ P wave D^{**}'s" is more likely, theory favors the decay into "$3/2$ P wave D^{**}'s" (for recent reviews cf. [1, 2]).

1.1 Heavy-light mesons

A heavy-light meson is made from a heavy quark (b, c) and a light quark (u, d), i.e. $B = \{\bar{b}u \, , \, \bar{b}d\}$ and $D = \{\bar{c}u \, , \, \bar{c}d\}$.

In the static limit ($m_b, m_c \rightarrow \infty$) there are no interactions involving the static quark spin. Therefore, it is appropriate to classify states according to parity $\mathscr{P}$ and the total angular momentum of the light quarks and gluons j (cf. the left column of Table 1).

If m_b, m_c are finite, j is not a good quantum number anymore. States have to be classified according to parity $\mathscr{P}$ and total angular momentum J (cf. the right column of Table 1). Although j is not a "true quantum number" anymore, it is still an approximate quantum number justifying the notation D_J^j. The above mentioned P wave D^{**}'s are $\{D_0^* \, , D_1' \, , D_1 \, , D_2^*\} = \{D_0^{1/2} \, , D_1^{1/2} \, , D_1^{3/2} \, , D_2^{3/2}\}$.

$j^{\mathscr{P}}$	$J^{\mathscr{P}}$
$(1/2)^- \equiv S$	$0^- \equiv B,D$
	$1^- \equiv B^*,D^*$
$(1/2)^+ \equiv P_-$	$0^+ \equiv D_0^* \equiv D_0^{1/2}$
	$1^+ \equiv D_1' \equiv D_1^{1/2}$
$(3/2)^+ \equiv P_+$	$1^+ \equiv D_1 \equiv D_1^{3/2}$
	$2^+ \equiv D_2^* \equiv D_2^{3/2}$

Table 1: Classification of heavy-light mesons (left: static limit; right: finite heavy quark masses).

1.2 The $1/2$ versus $3/2$ puzzle

Experiments (ALEPH, BaBar, BELLE, CDF, DELPHI, DØ), which have studied the semileptonic decay $B \rightarrow X_c \, l \, \nu$ (where X_c is some hadronic part containing a c quark), find the following composition of X_c:

- $\approx 75\%$ D and D^*, i.e. S wave states (which is in agreement with theory).
- $\approx 10\%$ $D_1^{3/2}$ and $D_2^{3/2}$, i.e. $j = 3/2$ P wave states (which is in agreement with theory).
- For the remaining $\approx 15\%$ the situation is rather vague: a natural candidate would be $D_0^{1/2}$ and $D_1^{1/2}$, i.e. $j = 1/2$ P wave states. This, however, would imply $\Gamma(B \rightarrow D_{0,1}^{1/2} \, l \, \nu) > \Gamma(B \rightarrow D_{1,2}^{3/2} \, l \, \nu)$, which is in conflict with theory. This conflict between experiment and theory is called the $1/2$ versus $3/2$ puzzle.

On the theory side most statements are made in the static limit $m_b, m_c \to \infty$. In this limit the eight matrix elements relevant for decays $B \to D^{**} \, l \, \nu$ can be parameterized by two form factors, the Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$ [3]. Here we only list two of these matrix elements:

$$\langle D_0^{1/2}(v') | \bar{c} \gamma_5 \gamma_\mu b | B(v) \rangle \ \propto \ \tau_{1/2}(w) (v - v')_\mu \tag{1.1}$$

$$\langle D_2^{3/2}(v', \varepsilon) | \bar{c} \gamma_5 \gamma_\mu b | B(v) \rangle \ \propto \ \tau_{3/2}(w) \Big((w+1) \varepsilon^*_{\mu\alpha} v^\alpha - \varepsilon^*_{\alpha\beta} v^\alpha v^\beta v'_\nu \Big), \tag{1.2}$$

where v and v' are the four velocities associated with the B and the D meson respectively,
$w = (v' \cdot v)$ and ε is the polarization tensor of the D meson.

By means of operator product expansion (OPE) a couple of sum rules has been derived in the static limit [4, 5]. The most prominent in this context is the Uraltsev sum rule,

$$\sum_n \left(\left| \tau_{3/2}^{(n)}(1) \right|^2 - \left| \tau_{1/2}^{(n)}(1) \right|^2 \right) \ = \ \frac{1}{4}, \tag{1.3}$$

where $\tau_{1/2} \equiv \tau_{1/2}^{(0)}$, $\tau_{3/2} \equiv \tau_{3/2}^{(0)}$ and the sum is over all $1/2$ and $3/2$ P wave states respectively. From experience with sum rules one expects approximate saturation from the ground states, i.e.

$$\left| \tau_{3/2}^{(0)}(1) \right|^2 - \left| \tau_{1/2}^{(0)}(1) \right|^2 \ \approx \ \frac{1}{4}, \tag{1.4}$$

which implies $|\tau_{1/2}(1)| < |\tau_{3/2}(1)|$. This in turn strongly suggests
$\Gamma(B \to D_{0,1}^{1/2} \, l \, \nu) < \Gamma(B \to D_{1,2}^{3/2} \, l \, \nu)$, which, as already mentioned, is in conflict with experiment.

Phenomenological models [6, 7] give the same qualitative picture, even when considering finite heavy quark masses [8].

Possible explanations to resolve the $1/2$ versus $3/2$ puzzle include the following:

- The experimental signal for the remaining 15% of X_c is rather vague; therefore, only a small part might actually be $D_0^{1/2}$ and $D_1^{1/2}$.
- Sum rules like (1.3) might not be saturated by the ground states.
- Sum rules derived by OPE hold in the static limit and might change for finite heavy quark masses.
- Sum rules make statements about the zero recoil situation ($w = 1$), where the B and the D meson have the same velocity; to obtain decay rates, however, one has to integrate over w.

With a dynamical lattice computation of $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$ in the static limit, which is presented in the following section, we attempt to shed some light on this puzzle.

2. Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$

For a more detailed presentation of this computation we refer to [9]. We use a method, which was proposed and tested in the quenched case in [10].

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Since the "Isgur-Wise relations" (1.1) and (1.2) are not directly useful to compute $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$ (the right hand sides vanish at zero recoil), they have to be rewritten as shown in [11]:

$$\langle D_0^{1/2}(v)|\bar{c}\gamma_5\gamma_j D_k b|B(v)\rangle = -ig_{jk}\Big(m(D_0^{1/2}) - m(B)\Big)\tau_{1/2}(1) \tag{2.1}$$

$$\langle D_2^{3/2}(v,\varepsilon)|\bar{c}\gamma_5\gamma_j D_k b|B(v)\rangle = +i\sqrt{3}\varepsilon_{jk}\Big(m(D_2^{3/2}) - m(B)\Big)\tau_{3/2}(1). \tag{2.2}$$

We compute $\tau_{1/2}$ by means of (2.1) and an "effective form factor":

$$\tau_{1/2}(1) = \lim_{t_0-t_1\to\infty,\, t_1-t_2\to\infty} \tau_{1/2,\mathrm{effective}}(t_0-t_1, t_1-t_2) \tag{2.3}$$

$$\tau_{1/2,\mathrm{effective}}(t_0-t_1,t_1-t_2) = \\ = \frac{1}{Z_{\mathscr{D}}}\left|\frac{N(P_-)N(S)\ \Big\langle\Big(\mathscr{O}^{(P_-)}(t_0)\Big)^\dagger\ (\bar{Q}\gamma_5\gamma_3 D_3 Q)(t_1)\ \mathscr{O}^{(S)}(t_2)\Big\rangle}{\Big(m(P_-)-m(S)\Big)\ \Big\langle\Big(\mathscr{O}^{(P_-)}(t_0)\Big)^\dagger \mathscr{O}^{(P_-)}(t_1)\Big\rangle\ \Big\langle\Big(\mathscr{O}^{(S)}(t_1)\Big)^\dagger \mathscr{O}^{(S)}(t_2)\Big\rangle}\right|. \tag{2.4}$$

To this end we need static-light meson creation operators $\mathscr{O}^{(S)}$, $\mathscr{O}^{(P_-)}$ and $\mathscr{O}^{(P_+)}$, static-light meson masses $m(S)$, $m(P_-)$ and $m(P_+)$, 2-point and 3-point functions, and norms $N(S)$, $N(P_-)$ and $N(P_+)$. $Z_{\mathscr{D}}$ is a perturbatively computed renormalization constant, whose derivation is explained in detail in [12, 9]. The computation of $\tau_{3/2}$ is analogous. Explicit formulae can be found in [9].

2.1 Simulation setup

We use $L^3 \times T = 24^3 \times 48$ gauge configurations produced by the European Twisted Mass Collaboration (ETMC). The gauge action is tree-level Symanzik improved and the fermionic action $N_f = 2$ Wilson twisted mass at maximal twist yielding automatic $\mathscr{O}(a)$ improvement of physical quantities. The lattice spacing is $a = 0.0855\,\mathrm{fm}$. To be able to extrapolate our results to physical light quark masses, we consider three different bare quark masses μ_q corresponding to "pion masses" m_PS, which are listed in Table 2. For more details regarding these gauge configuration we refer to [13, 14].

μ_q	m_PS in MeV	number of gauge configurations
0.0040	314(2)	1400
0.0064	391(1)	1450
0.0085	448(1)	1350

Table 2: Bare quark masses, pion masses and number of gauge configurations.

2.2 Static-light meson creation operators

The meson creation operators we use are latticized versions of the continuum expression

$$\mathscr{O}^{(\Gamma)}(\mathbf{x}) = \bar{Q}(\mathbf{x})\int d\hat{\mathbf{n}}\,\Gamma(\hat{\mathbf{n}})U(\mathbf{x};\mathbf{x}+r\hat{\mathbf{n}})\psi^{(u)}(\mathbf{x}+r\hat{\mathbf{n}}), \tag{2.5}$$

where $\bar{Q}(\mathbf{x})$ creates a static antiquark at position $\mathbf{x}$, $\psi^{(u)}(\mathbf{x}+r\hat{\mathbf{n}})$ creates a light quark separated by a distance r from the static antiquark, U is a gauge covariant parallel transporter and Γ a combination of spherical harmonics and γ matrices yielding well defined parity $\mathscr{P}$ and total angular momentum of the light degrees of freedom j. The operators are collected in Table 3.

$\Gamma(\hat{\mathbf{n}})$	$J^{\mathscr{P}}$	$j^{\mathscr{P}}$	$\mathrm{O_h}$	lattice $j^{\mathscr{P}}$	notation
γ_5	0^-	$(1/2)^-$	A_1	$(1/2)^-$, $(7/2)^-$, ...	S
1	0^+	$(1/2)^+$		$(1/2)^+$, $(7/2)^+$, ...	P_-
$\gamma_1\hat{n}_1 - \gamma_2\hat{n}_2$ (cyclic)	2^+	$(3/2)^+$	E	$(3/2)^+$, $(5/2)^+$, ...	P_+
$\gamma_5(\gamma_1\hat{n}_1 - \gamma_2\hat{n}_2)$ (cyclic)	2^-	$(3/2)^-$		$(3/2)^-$, $(5/2)^-$, ...	$D_\pm$

Table 3: J: total angular momentum; j: total angular momentum of the light degrees of freedom; $\mathscr{P}$: parity.

2.3 2-point functions, static-light meson masses, norms of meson states

With meson creation operators (2.5) at hand it is straightforward to compute the 2-point functions

$$\mathscr{C}^{(\Gamma)}(t) \;=\; \left\langle \left(\mathscr{O}^{(\Gamma)}(t)\right)^\dagger \mathscr{O}^{(\Gamma)}(0)\right\rangle \quad , \quad \Gamma \;\in\; \{\gamma_5\,,1\,,\gamma_1\hat{n}_1 - \gamma_2\hat{n}_2\}. \tag{2.6}$$

From these 2-point functions we extract the meson masses $m(S)$, $m(P^-)$ and $m(P^+)$ via effective mass plateaus. To illustrate the quality of our data we show effective masses for $\mu_\mathrm{q} = 0.0040$ in Figure 1. For details regarding the computation of the low lying static-light meson spectrum within our twisted mass setup we refer to [15, 16].

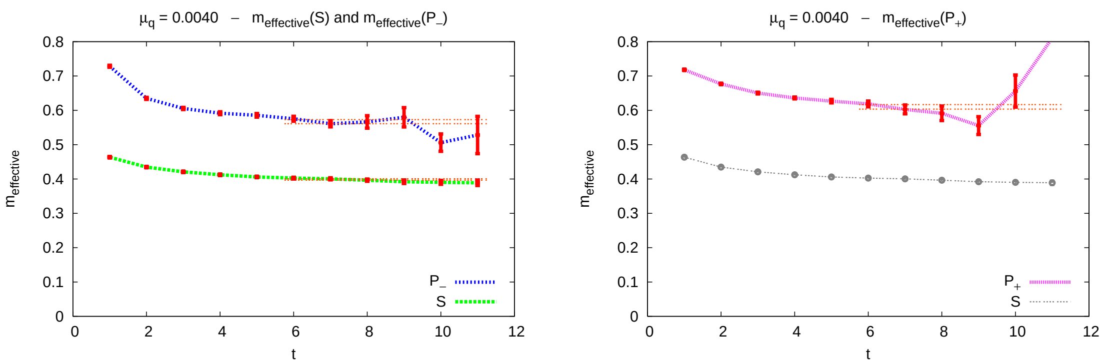

Figure 1: Effective masses for S, P_- and P_+ for $\mu_\mathrm{q} = 0.0040$.

Moreover, we obtain the ground state norms $N(S)$, $N(P_-)$ and $N(P_+)$ by fitting exponentials to the 2-point functions (2.6) at large temporal separations.

2.4 3-point functions

The computation of the 3-point functions is again straightforward. We chose to represent the covariant derivative inside the heavy-heavy current in a symmetric way by a single spatial link in positive and negative direction.

2.5 Results

In Figure 2a we show the effective form factors $\tau_{1/2,\mathrm{effective}}$ (eqn. (2.4)) and $\tau_{3/2,\mathrm{effective}}$ for $t_0 - t_2 = 10$ as functions of $t_0 - t_1$ for $\mu_\mathrm{q} = 0.0040$ (plots for the other two quark masses look qualitatively identical). We extract $\tau_{1/2}$ and $\tau_{3/2}$ by fitting constants to the central three data points as indicated by the dashed lines. Results are collected in Table 4.

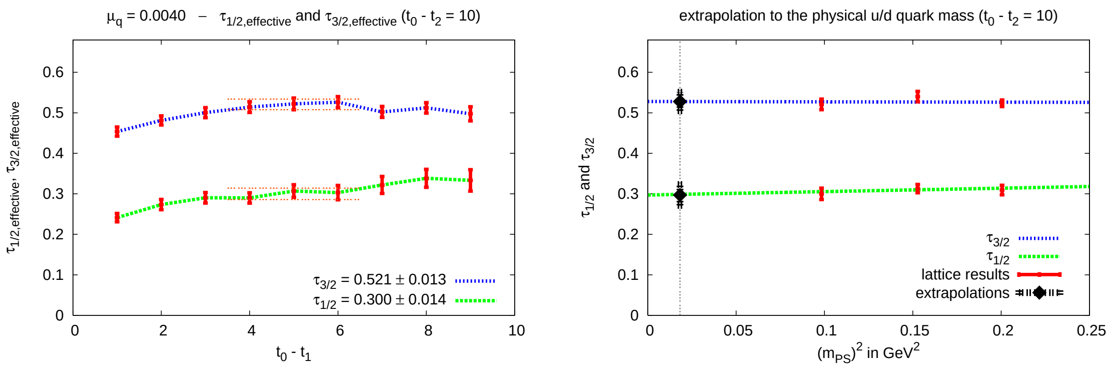

Figure 2: a) Effective form factors $\tau_{1/2,\mathrm{effective}}$ and $\tau_{3/2,\mathrm{effective}}$ for $t_0 - t_2 = 10$ and $\mu_\mathrm{q} = 0.0040$. **b)** Linear extrapolation of $\tau_{1/2}$ and $\tau_{3/2}$ in $(m_\mathrm{PS})^2$ to the physical u/d quark mass.

μ_q	$\tau_{1/2}(1)$	$\tau_{3/2}(1)$	$(\tau_{3/2})^2 - (\tau_{1/2})^2$
0.0040	0.300(14)	0.521(13)	0.181(16)
0.0064	0.313(10)	0.540(13)	0.194(13)
0.0085	0.309(12)	0.524(8)	0.178(9)

Table 4: $\tau_{1/2}$ and $\tau_{3/2}$ and their contribution to the Urlatsev sum rule.

As expected from sum rules $\tau_{3/2}$ is significantly larger than $\tau_{1/2}$. Moreover, we find that the ground states fulfill the Uraltsev sum rule (1.3) by around 80%.

We use our results at three different values of the pion mass to linearly extrapolate $\tau_{1/2}$ and $\tau_{3/2}$ in $(m_\mathrm{PS})^2$ to the physical u/d quark mass ($m_\mathrm{PS} = 135\,\mathrm{MeV}$; cf. Figure 2b). Our final result is

$$\tau_{1/2}^{m_\mathrm{phys}}(1) \; = \; 0.297(26) \quad , \quad \tau_{3/2}^{m_\mathrm{phys}}(1) \; = \; 0.528(23). \tag{2.7}$$

3. Conclusions

Our result (2.7) confirms the sum rule expectation that $\tau_{3/2}(1) \gg \tau_{1/2}(1)$ in the static limit. When comparing to the experimentally measured form factors ($\tau_{1/2}^{\mathrm{exp}}(1) = 1.28$ and $\tau_{3/2}^{\mathrm{exp}}(1) = 0.75$ [17]) we find fair agreement for $\tau_{3/2}$ but a strong discrepancy for $\tau_{1/2}$.

In our opinion this discrepancy calls for action both on the theoretical and the experimental side: it would be highly desirable to have a first principles lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ beyond the zero recoil situation and also for finite heavy quark masses; on the other hand a thoroughly refined experimental analysis of the decay into $1/2$ D^{**}'s, for which the signal is rather faint, seems to be necessary.

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