Kaon oscillations in the Standard Model and Beyond using $N_f = 2$ dynamical quarks

ETM Collaboration

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We compute non-perturbatively the B-parameters of the complete basis of four-fermion operators needed to study the Kaon oscillations in the SM and in its supersymmetric extension. We perform numerical simulations with two dynamical maximally twisted sea quarks at three values of the lattice spacing on configurations generated by the ETMC. Unwanted operator mixings and $O(a)$ discretization effects are removed by discretizing the valence quarks with a suitable Osterwalder-Seiler variant of the Twisted Mass action. Operators are renormalized non-perturbatively in the RI/MOM scheme. Our preliminary result for $B_{\mathrm{K}}^{\mathrm{RGI}}$ is 0.73(3)(3).

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1. Introductory Remarks and Calculation Setup

We will present the main features of the method and preliminary results of the bag parameter calculation for the K meson oscillations at three values of the lattice spacing using the $N_f = 2$ dynamical quark configurations produced by the ETM collaboration.

ETMC dynamical configurations have been produced with the tree-level Symmanzik improved action in the gauge sector while the dynamical quarks have been regularized by employing the twisted mass (tm) formalism [1]. It has been demonstrated that with the condition of *maximal twist* this formalism provides automatic $O(a)$-improved physical quantities [2].

In the so called physical basis the fermion lattice action concerning the sea sector is written

$$S_{sea} = a^4 \sum_x \bar{\psi}(x)(\gamma\tilde{\nabla} - i\gamma_5\ \tau_3\ W_{cr} + \mu_{sea})\psi(x)\ , \tag{1.1}$$

with $W_{cr} = -\frac{a}{2}\sum_\mu \nabla^*_\mu \nabla_\mu + M_{cr}(r=1)$; $\psi = (u\ \ d)^T$ is a doublet of degenerate light sea quarks while $\mu_{sea} = \mathrm{diag}(\mu_u\ \ \mu_d)$. We should also note that the tm formalism offers a simpler renormalisation pattern with comparison to the standard Wilson regularization. This is true for some important physical quantities calculated on the lattice, as for example the pseudoscalar decay constant and the chiral condensate.

It has been shown that the use of the tm regularization can simplify the renormalization pattern properties of the four-fermion operators which enter in the calculation of certain phenomenologically important weak matrix elements such as B_K [1, 3, 4]. In order to achieve both $O(a)$ improvement and a continuum-like renormalization pattern in the evaluation of B_K we introduce the valence quarks with Osterwalder-Seiler lattice action and allow for replica of the down (d, d') and strange (s, s') flavours [5], viz.

$$S_{val} = a^4 \sum_x \sum_{f=d,d',s,s'} \bar{q}_f(x)\left(\gamma\tilde{\nabla} - i\gamma_5\ r_f\ W_{cr} + \mu_f\right) q_f(x)\ , \qquad -r_s = r_d = r_{d'} = r_{s'} = 1\,. \tag{1.2}$$

The valence sector action above is written (unlike eq. (1.1)) in the so called physical quark basis with the field q_f representing just one individual flavour. While the four fermion operator relevant for B_K (see eq. (2.1)) is chosen to contain all the four valence flavours in eq. (1.2), the interpolating fields for the external (anti)Kaon states are made up of a tm-quark pair ($\bar{d}\gamma_5 s$, with $-r_s = r_d$) and a OS-quark pair ($\bar{d'}\gamma_5 s'$, with $r_{d'} = r_{s'}$). This mixed action setup with maximally twisted Wilson-like quarks has been studied in detail in Ref. [5], allows for an easy matching of sea and valence quark masses and leads to unitarity violations that vanish as a^2 as the continuum limit is approached. In the present case, however, the quark mass matching is incomplete because we are neglecting the sea strange quark (i.e. partially quanched computation), thereby inducing some (possibly small) O(a^0) systematic error. We notice that the proposed method for obtaining automatic $O(a)$ improved results has already been tested successfully in the calculation of B_K with fully quenched quarks [6].

In Table 1 we give the simulation details concerning the mass values of the sea and the valence quarks for each value of the gauge coupling for the calculation presented in this work. The smallest sea quark mass corresponds to a pion of about 270 MeV for the case of $\beta = 3.90$. For $\beta = 4.05$ the lightest pion weights 300 MeV while for $\beta = 3.80$ the lowest pion mass is around 400 MeV. The highest sea quark mass for the three values of the lattice spacing is about half the strange quark

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mass. For the inversions in the valence sector we have made use of the stochastic method (one–end trick of ref. [7]) in order to increase the statistical information. Propagator sources have been located at randomly chosen timeslices. For more details on the dynamical configurations and the stochastic method application see Refs [8, 9].

β	$a^{-4}(L^3 \times T)$	$a\mu_\ell = a\mu_{sea}$	$a\mu_h$
3.80	$24^3 \times 48$	0.0080 0.0110	0.0200, 0.0250
($a \sim 0.1$ fm)			0.0300, 0.0360
3.90	$24^3 \times 48$	0.0040, 0.0064	0.0150, 0.0220
		0.0085, 0.0100	0.0270, 0.0320
3.90	$32^3 \times 64$	0.0030, 0.0040	0.0220, 0.0270
($a \sim 0.085$ fm)			
4.05	$32^3 \times 64$	0.0030, 0.0060	0.0150, 0.0180
($a \sim 0.065$ fm)		0.0080	0.0220, 0.0260

Table 1: Simulation details

2. The K-meson bag parameter

We recall that in our mixed action setup all the physical quantities are evaluated with no O(a) discretization effects (see Ref. [5]) and moreover the four fermion operator relevant for B_K, which reads

$$\left[V_\mu V_\mu + A_\mu A_\mu\right]^{\text{phys-basis}}_{\text{bare}} = [(\bar{q}_s\gamma_\mu q_d)(\bar{q}_{s'}\gamma_\mu q_{d'}) + (\bar{q}_s\gamma_\mu\gamma_5 q_d)(\bar{q}_{s'}\gamma_\mu\gamma_5 q_{d'})] + [d \leftrightarrow d']\,. \tag{2.1}$$

is multiplicatively renormalizable. This can be easily understood by noting that in the (unphysical) tm basis, where the Wilson term enters the valence action in the standard way (with no $i\gamma_5$-twist) and the operator renormalization properties are the same of the standard Wilson fermionic action, the operator (2.1) takes the form

$$\left[V_\mu A_\mu + A_\mu V_\mu\right]^{\text{tm-basis}}_{\text{bare}} = [(\bar{\chi}_s\gamma_\mu \chi_d)(\bar{\chi}_{s'}\gamma_\mu\gamma_5 \chi_{d'}) + (\bar{\chi}_s\gamma_\mu\gamma_5 \chi_d)(\bar{\chi}_{s'}\gamma_\mu \chi_{d'})] + [d \leftrightarrow d']\,, \tag{2.2}$$

Here $\chi_f = \exp^{-i\gamma_5\pi/4} q_f$ and $\bar{\chi}_f = \bar{q}_f \exp^{-i\gamma_5\pi/4}, f = d, d', s, s'$ are the tm basis valence quark fields. The operator (2.2) is known to be protected from mixing under renormalisation due to *CPS* symmetry [10]. In summary we have ("R" stands for "renormalized")

$$\left[V_\mu V_\mu + A_\mu A_\mu\right]^{\text{phys-basis}}_{\text{R}} = Z_{VA+AV}\left[V_\mu V_\mu + A_\mu A_\mu\right]^{\text{phys-basis}}_{\text{bare}} = Z_{VA+AV}\left[V_\mu A_\mu + A_\mu V_\mu\right]^{\text{tm-basis}}_{\text{bare}}\,, \tag{2.3}$$

where the name of the renormalization constant is chosen so as to be consistent with the notation used in the standard Wilson fermion literature.

In order to estimate the B_{K}-parameter we calculate a three-point correlation function where a four-fermion operator is free to move in lattice time t whereas two "K-meson walls" consisting of noisy sources are imposed at fixed time separation $t_R - t_L = T/2$. The t_L value changes randomly from configuration to configuration. In our simulations we consider the time reversed case too and we average them properly. The plateau signal is taken for $t_L \ll t \ll t_R$. We extract B_K from the ratio:

$$R_{B_\mathrm{K}} = \frac{C^{(3)}_{KOK}(t-t_L, t-t_R)}{C^{(2)}_K(t-t_L)C^{(2)}_K(t-t_R)} \xrightarrow{t_L \ll t \ll t_R} B_\mathrm{K} \tag{2.4}$$

In our analysis all correlation functions satisfy the condition $a\mu_l = a\mu_\mathrm{sea}$ while the valence strange-like quark mass values are given in Table 1. An important remark is in order: the mixed regularization set-up that we have used leads at finite lattice spacing to different values for the decay constant and the pseudoscalar masses of the two K-mesons employed in the calculation. We find that the discretisation effects are negligible for the decay constant while happen to be significant in the case of the pseudoscalar mass. For this reason we normalize the four fermion matrix element by dividing with $(8/3)m_K^{OS}\, m_K^{tm}\, f_K^{OS}\, f_K^{tm}$. Moreover, as expected, the cutoff effects diminish drastically towards the CL. So this kind of systematic error is well under control.

The fits to the light quark mass behaviour are performed using the $SU(2)$ Partially Quenched Chiral Perturbation Theory formula of refs [11, 12]. In our case the fit ansatz is:

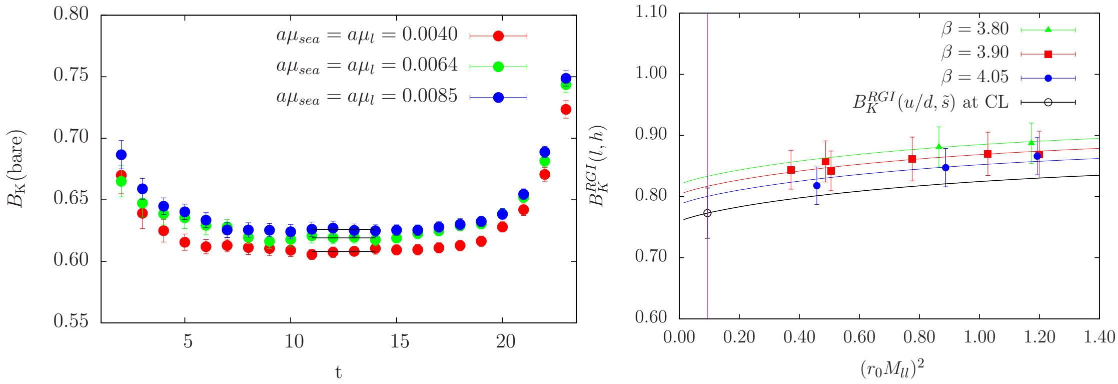
$$B(\mu_h) \;=\; B_\chi(\mu_h)\left[1 + b(\mu_h)\,\frac{2B_0}{f^2}\,\mu_l \,-\, \frac{2B_0}{32\pi^2 f^2}\mu_l\,\ln\left(\frac{2B_0\mu_l}{\Lambda_\chi^2}\right)\right] + D(\mu_h)a^2 \tag{2.5}$$

where μ_h denotes the quark mass values around the strange quark (see Table 1). Thus, the fit procedure consists of a combined fit of chiral and continuum extrapolation. We find that the cutoff effects on our data are well described by a μ_l-independent (but μ_h-dependent) $O(a^2)$ term.

Two methods of analysis have been followed. The first method relies on using the information for the physical mass values of the up/down and strange quarks in the continuum limit, as they have been estimated in a recent ETMC computation [13]. Note that the implementation of this method requires the knowlegde of the quark mass renormalization constant [14]. The second method consists of employing the pseudoscalar masses instead of the quark masses. In this case we choose a set of three values of reference pseudoscalar masses made out of two strange-like quarks, M_{hh}; keeping each of them fixed we perform the chiral fits in terms of the light pseudoscalar mass. In the end of the procedure we estimate B_K via an interpolation at the physical point defined by the formula $M_{ss}^2 = 2M_K^2 - M_\pi^2$. Both methods give compatible final results within less than one standard deviation.

In Figure 1(a) the quality of the plateau is shown for $\beta = 3.90$, for three values of the light quark mass and for one typical value of μ_h; in Figure 1(b) we present an example of a combined chiral plus continuum fit (three value of the lattice spacing) for $B_\mathrm{K}^\mathrm{RGI}(l,h)$ versus the light pseudoscalar mass squared in units of r_0; the value of M_{hh} is in the vicinity of the physical one.

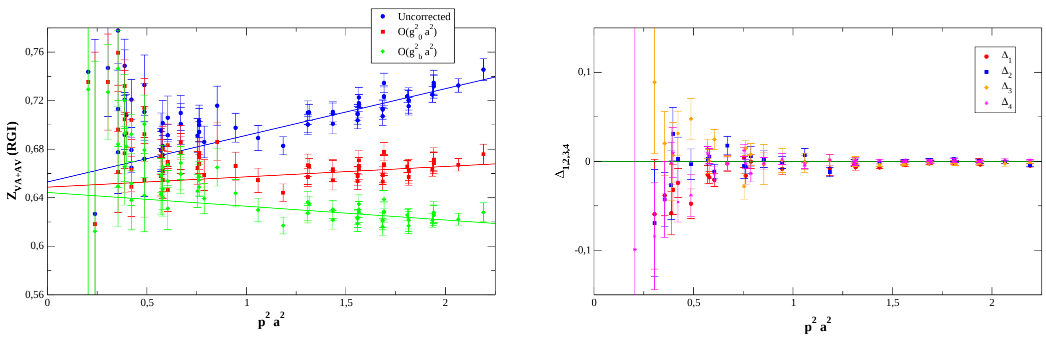
The two point renormalisation constants for the axial and vector current have been calculated using the RI-MOM method [15]. We recall that the physical axial current made up of OS quarks is normalized by Z_A while the one consisting of tm quarks is normalized by Z_V [14]. The RI-MOM method has also been employed for the calculation of the renormalisation constant of the four-fermion operator [16]. In Figure 2(a) we show the behaviour of the renormalisation constant as a function of the momentum squared in lattice units $(ap)^2$ for $\beta = 3.90$ at the valence chiral limit



(a) (b)

Figure 1: (a) The quality of the plateau for three values of the light quark mass for $\beta = 3.90$; (b) Combined chiral and continuum fit for $B_{\rm K}^{\rm RGI}(l,h)$ versus $(r_0 M_{ll})^2$. The empty black circle gives the value at the continuum limit for the case of $(r0M_{hh}) = 1.50$;

and for $a\mu_{sea} = 0.0040$. Discretization effects of $O(a^2)$ have been evaluated at one loop [17] and subtracted from the relevant correlation functions. Thus, the leading discretization effects on our RI-MOM determination of the renormalization constant are of $O(g^4a^2, g^2a^4)$. We show three types of results; two of them correspond to two estimates of the subtracted perturbative contributions. The amount of the subtraction depends on the choice of the value for the gauge coupling. We have considered two cases for the gauge coupling, the naive (g_0) and the boosted one (g_b). We also show the result for the Z_{VA+AV}(RGI) without considering any perturbative subtractions (indicated as "uncorrected" in the figure). In the right panel of the same figure we illustrate the absence of mixing with "wrong chirality" operators; in fact the mixing coefficients are vanishing.



(a) (b)

Figure 2: (a) RI/MOM computation of the multiplicative renormalization factor Z_{VA+AV}(RGI) at $\beta = 3.90$; (b) Mixing coefficients Δ_k $(k = 1, \cdots, 4)$ with other four-fermion operators with "wrong chirality".

Our preliminary result for $B_{\rm K}$ in the RGI scheme in the continuum limit is

$$B_{\rm K}^{\rm RGI} = 0.73(3)(3)$$

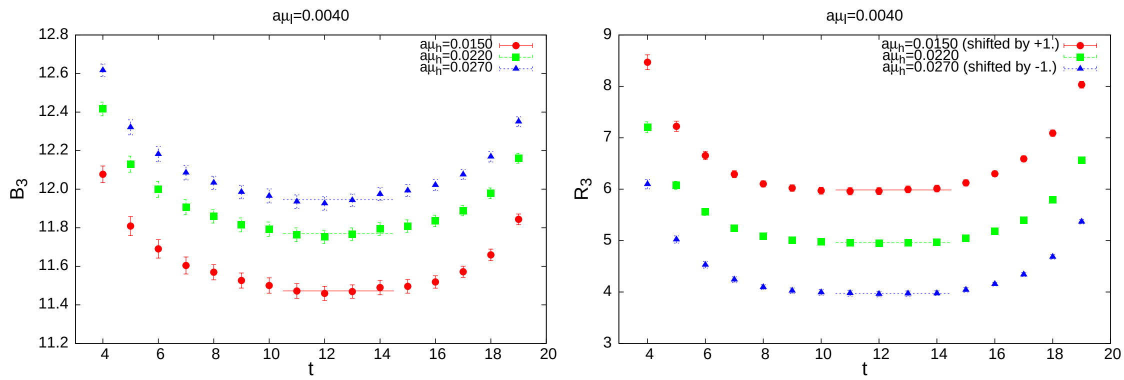
The first error includes the uncertainty coming from the correlators and from the fit procedure (chiral plus continuum) while the second one is due to the uncertainties in the calculation of the renormalisation constants. We are currently attempting to reduce the latter uncertainty.

3. The K-bag meson parameter beyond the SM

Interactions beyond the SM including supersymmetry furnish new diagrams in the calculation of the $\Delta S = 2$ process. Their effect expressed in the OPE expansion is to enrich the set of the local operators to be considered in the low energy regime, see e.g. [18]. Therefore one has to calculate on the lattice the matrix elements of five parity even four-fermion operators namely $O_1 = O_{VV+AA}, O_2 = O_{SS+PP}, O_3 = O_{\tilde{T}T}, O_4 = O_{SS-PP}$ and $O_5 = O_{VV-AA}$ [19, 20, 21].

It is well known that the renormalisation pattern of the parity-even four-fermion operators becomes complicated because of mixings as soon as the regularization breaks the chiral symmetry; this is certainly the case of Wilson fermions. However using the proposal of Ref. [5] this problem is bypassed; due to the axial rotation mapping of the parity-even to parity-odd operators in the tm basis the renormalisation pattern becomes continuum-like [16]. It is worth mentioning that, as in the case of the SM four-fermion operator, the lattice estimates of the matrix elements of $O_2, \ldots, O_5$ are automatically $O(a)$-improved.

First results regarding the signal quality for the case of $\beta = 3.90$ are given in Figure 3. We depict the plateaus for the B_3 bag parameter (left panel) and for the quantity $R_3 \sim \frac{\langle \bar{K}|O_3|K\rangle}{\langle \bar{K}|O_1|K\rangle}$ (right panel). Both figures refer to the same value of the light quark mass for three different choices of the strange-like quark mass. Computation at the other two values of the lattice spacing as well as a full determination of the renormalisation constant matrix is still in progress.



(a) (b)

Figure 3: (a) and (b): the quality of the signal for the quantities B_3 and R_3 respectively at three values of the strange-like quark mass using $a\mu_l = 0.0040$ at $\beta = 3.90$.

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