PROCEEDINGS OF SCIENCE

Higgs $+2$ jets: Compact Analytic Results

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This report describes the recent efforts to compute analytic formulae for the Next-to-Leading-Order (NLO) QCD corrections to Higgs plus two jet production at hadron colliders. In these calculations the Higgs boson couples to gluons via a top-quark loop which is integrated out to form an effective vertex. The amplitudes are further simplified by splitting the real Higgs scalar into the sum of two complex scalars ϕ and $\phi^\dagger$. Four-dimensional unitarity is used to construct the cut-containing pieces of the amplitude, while a variety of bootstrap and Feynman diagram techniques are used to construct the rational pieces. The results described here are valid in the limit of a large top quark mass and when the transverse momenta of the jets are less than m_t.

RADCOR 2009 - 9th International Symposium on Radiative Corrections (Applications of Quantum Field Theory to Phenomenology) ,
October 25 - 30 2009
Ascona, Switzerland

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[†] Based upon work done in collaboration with Simon Badger, John Campbell, Keith Ellis, Nigel Glover and Pierpaolo Mastrolia

PoS(RADCOR2009)025

1. Introduction

With the LHC now taking data and the Tevatron regularly setting new luminosity records, the search for the Higgs boson is entering a crucial phase. A discovery, or at the very least improved exclusion limits [1], is to be expected in the near future. In order to maximise the effectiveness of the experimental search strategies it is crucial to have excellent theoretical predictions for both Higgs signal and background processes.

One such process is the production of the Higgs boson in association with two jets. Here the signal provided by electroweak induced vector-boson-fusion (VBF) [2] is essential for a measurement of the Yukawa coupling of the W and Z vector bosons. The VBF process is under good theoretical control, with both strong and electroweak one-loop calculations completed.

On the other hand one of the main background processes, Higgs boson production in association with two jets via gluon fusion has had less theoretical attention. One-loop QCD corrections to this process have been calculated semi-numerically [3, 4], which allowed some phenomenology to be done. Over the last couple of years much theoretical effort has been devoted to completing the analytic calculation of the Higgs plus two jet process, with the hope of producing faster code for phenomenological studies.

Gluon fusion proceeds via a top-quark loop, calculations which include the full top quark mass are difficult. However, by integrating out the top quark from the loop one can simplify the calculation. This procedure leads to an effective Lagrangian to express the coupling of gluons to the Higgs field [5],

$$\mathscr{L}_H^{\text{int}} = \frac{C}{2} H \,\text{tr}\, G_{\mu\nu} G^{\mu\nu} . \tag{1.1}$$

The effective Lagrangian approximation is valid in the limit $m_H < 2m_t$. To $\mathscr{O}(\alpha_S^2)$ the coefficient C is given by [6, 7],

$$C = \frac{\alpha_S}{6\pi v}\left(1 + \frac{11}{4\pi}\alpha_S\right) + \mathscr{O}(\alpha_S^3) . \tag{1.2}$$

Here v is the vacuum expectation value of the Higgs field (246 GeV). The trace in Eq. (1.1) is over the colour degrees of freedom which, since SU(3) generators in the fundamental representation are normalised such that $\text{tr}\, T^a T^b = \delta^{ab}$. Introducing a complex scalar field [8, 9], $\phi = \frac{1}{2}(H + iA)$, $\phi^\dagger = \frac{1}{2}(H - iA)$, results in the following expression for the effective Lagrangian, Eq. (1.1),

$$\mathscr{L}_{H,A}^{\text{int}} = \frac{C}{2}\left[H \,\text{tr}\, G_{\mu\nu} G^{\mu\nu} + iA \,\text{tr}\, G_{\mu\nu}\, {}^*G^{\mu\nu}\right] = C\left[\phi \,\text{tr}\, G_{SD\,\mu\nu} G_{SD}^{\mu\nu} + \phi^\dagger \,\text{tr}\, G_{ASD\,\mu\nu} G_{ASD}^{\mu\nu}\right] ,$$

where the gluon field strength has been separated into a self-dual and an anti-self-dual component,

$$G_{SD}^{\mu\nu} = \frac{1}{2}(G^{\mu\nu} + {}^*G^{\mu\nu}), \quad G_{ASD}^{\mu\nu} = \frac{1}{2}(G^{\mu\nu} - {}^*G^{\mu\nu}), \quad {}^*G^{\mu\nu} \equiv \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma} G_{\rho\sigma} . \tag{1.3}$$

Calculations performed in terms of the field ϕ are simpler than the calculations for the Higgs boson and, moreover, the amplitudes for $\phi^\dagger$ can be obtained by parity. In the final stage, the full Higgs boson amplitudes are then written as a combination of ϕ and $\phi^\dagger$ components:

$$A(H, \{p_k\}) = A(\phi, \{p_k\}) + A(\phi^\dagger, \{p_k\}) , \tag{1.4}$$

2. Method

One-loop amplitudes contain two pieces referred to as the cut-constructible and rational pieces. In massless gauge theories the cut-constructible part of one-loop amplitudes can be written asa sum over constituent basis integrals,

$$C_4(\phi,1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4}) = \sum_i C_{4;i}I_{4;i} + \sum_i C_{3;i}I_{3;i} + \sum_i C_{2;i}I_{2;i}. \tag{2.1}$$

Here $I_{j;i}$ represents a j-point scalar basis integral, with a coefficient $C_{j;i}$. The sum over i represents the sum over the partitions of the external momenta over the j legs of the basis integral.

Multiple cuts isolate different integral functions and allow the construction of a linear system of equations from which the coefficients can be extracted. When considering quadruple cuts of one-loop amplitudes, one is forced to consider complex momenta in order to fulfill the on-shell constraints [10]. The four on-shell constraints are sufficient to isolate each four-point (box) configuration by freezing the loop momentum, thereby allowing the determination of the corresponding coefficient by a purely algebraic operation. To isolate the coefficients of lower-point integrals, one needs to cut fewer than four lines. In this case the loop momenta is no longer completely determined, but, according to the number of cuts, some of its components are free variables. In this case the computation of the three- (triangle) and two-point (bubble) coefficients can also be reduced to algebraic procedures by exploiting the singularity structure of amplitudes in the complex-plane [11]. Alternatively one can extract the coefficients of bubble- and triangle-functions by employing the spinor-integration technique [12, 13]. This method has recently inspired a novel technique for evaluating the double-cut phase-space integrals *via* Stokes' Theorem applied to functions of two complex-conjugated variables [14].

In addition to the cut-constructible pieces one-loop QCD amplitudes contain pieces which cannot be reconstructed by four dimensional cuts. As such additional techniques have been used to calculate the one-loop ϕ + 4 parton amplitudes. For the ϕ-MHV, ϕ-all minus, $\phi q\overline{q}$ -MHV and $\phi q\overline{q}Q\overline{Q}$ helicity amplitudes the unitarity bootstrap method was employed [15]. In this approach one calculates the rational piece of the amplitudes from four-dimensional BCFW recursion relations [16]. For the ϕ-NMHV and $\phi q\overline{q}$-NMHV amplitudes the rational piece was obtained from the reduction of Feynman diagrams.

3. Higgs plus four gluon amplitudes

This section summarises the calculation of the ϕ plus four gluon amplitudes in the limit of a large top quark mass. The tree level amplitudes linking a ϕ with n gluons can be decomposed into colour ordered amplitudes as [17],

$$\mathscr{A}_n^{(0)}(\phi,\{k_i,\lambda_i,a_i\}) = iCg^{n-2}\sum_{\sigma\in S_n/Z_n}\mathrm{tr}(T^{a_{\sigma(1)}}\cdots T^{a_{\sigma(n)}})A_n^{(0)}(\phi,\sigma(1^{\lambda_1},..,n^{\lambda_n})). \tag{3.1}$$

Here S_n/Z_n is the group of non-cyclic permutations on n symbols, and j^{λ_j} labels the momentum p_j and helicity λ_j of the j^{th} gluon, which carries the adjoint representation index a_i. The one-loop amplitudes follow the same colour ordering as the pure QCD amplitudes [18],

$$\mathscr{A}_n^{(1)}(\phi,\{k_i,\lambda_i,a_i\}) = iCg^n \sum_{c=1}^{[n/2]+1} \sum_{\sigma\in S_n/S_{n;c}} G_{n;c}(\sigma) A_n^{(1)}(\phi,\sigma(1^{\lambda_1},\ldots,n^{\lambda_n})) \tag{3.2}$$

where $G_{n;1}(1) = N_c\,\mathrm{tr}(T^{a_1}\cdots T^{a_n})$ and $G_{n;c}(1) = \mathrm{tr}(T^{a_1}\cdots T^{a_{c-1}})\,\mathrm{tr}(T^{a_c}\cdots T^{a_n}) c > 2$. The sub-leading terms can be computed by summing over various permutations of the leading colour amplitudes [18]. Table 1 indicates the references in which the various helicity contributions to $Hgggg$ have first been calculated analytically.

H amplitude	ϕ amplitude	$\phi^\dagger$ amplitude
$\mathscr{A}(H,1^+,2^+,3^+,4^+)$	$\mathscr{A}(\phi,1^+,2^+,3^+,4^+)$ [22]	$\mathscr{A}(\phi^\dagger,1^+,2^+,3^+,4^+)$ [19]
$\mathscr{A}(H,1^-,2^+,3^+,4^+)$	$\mathscr{A}(\phi,1^-,2^+,3^+,4^+)$ [22]	$\mathscr{A}(\phi^\dagger,1^-,2^+,3^+,4^+)$ [23]
$\mathscr{A}(H,1^-,2^-,3^+,4^+)$	$\mathscr{A}(\phi,1^-,2^-,3^+,4^+)$ [20]	$\mathscr{A}(\phi^\dagger,1^-,2^-,3^+,4^+)$ [20]
$\mathscr{A}(H,1^-,2^+,3^-,4^+)$	$\mathscr{A}(\phi,1^-,2^+,3^-,4^+)$ [21]	$\mathscr{A}(\phi^\dagger,1^-,2^+,3^-,4^+)$ [21]

Table 1: ϕ and $\phi^\dagger$ amplitudes needed to construct a given one-loop $Hgggg$ amplitude, together with the references where they can be obtained. In all cases the $\phi^\dagger$ amplitudes are constructed from the ϕ amplitudes given in the reference using the parity operation. Results for all helicity combinations are also written, in uniform notation, in ref. [23].

4. Higgs plus four parton amplitudes

The colour decomposition of the $H\bar{q}qgg$ amplitudes is exactly the same as for the case $\bar{q}qgg$ which was written down in ref. [24]. At tree-level there are two colour stripped amplitudes,

$$\mathscr{A}_4^{(0)}(\phi,1_{\bar{q}},2_q,3,4) = Cg^2 \sum_{\sigma\in S_2} (T^{a_{\sigma(3)}}T^{a_{\sigma(4)}})_{i_2}^{\bar{\imath}_1} A_4^{(0)}(\phi,1_{\bar{q}},2_q,\sigma(3),\sigma(4))\,. \tag{4.1}$$

At one-loop level the colour decomposition is,

$$\begin{aligned}\mathscr{A}_4^{(1)}(\phi,1_{\bar{q}},2_q,3,4) \;=\; & Cg^4 c_\Gamma \Bigg[N_c \sum_{\sigma\in S_2} (T^{a_{\sigma(3)}}T^{a_{\sigma(4)}})_{i_2}^{\bar{\imath}_1} A_{4;1}(\phi,1_{\bar{q}},2_q,\sigma(3),\sigma(4)) \\ & + \delta^{a_3a_4}\,\delta_{i_2}^{\bar{\imath}_1} A_{4;3}(\phi,1_{\bar{q}},2_q;3,4)\Bigg]\,. \end{aligned} \tag{4.2}$$

The colour stripped amplitudes $A_{4;1}$ and $A_{4;3}$ can further be decomposed into primitive amplitudes,

$$A_{4;1}(\phi,1_{\bar{q}},2_q,3,4) \;=\; A_4^L(\phi,1_{\bar{q}},2_q,3,4) - \frac{1}{N_c^2} A_4^R(\phi,1_{\bar{q}},2_q,3,4) + \frac{n_f}{N_c} A_4^{\mathrm{f}}(\phi,1_{\bar{q}},2_q,3,4)\,, \tag{4.3}$$

and,

$$\begin{aligned}A_{4;3}(\phi,1_{\bar{q}},2_q;3,4) \;=\; & A_4^L(\phi,1_{\bar{q}},2_q,3,4) + A_4^R(\phi,1_{\bar{q}},2_q,3,4) + A_4^L(\phi,1_{\bar{q}},3,2_q,4) \\ & + A_4^L(\phi,1_{\bar{q}},2_q,4,3) + A_4^R(\phi,1_{\bar{q}},2_q,4,3) + A_4^L(\phi,1_{\bar{q}},4,2_q,3)\,. \end{aligned} \tag{4.4}$$

All of these colour decomposition equations, namely Eqs. (4.1, 4.2, 4.3, 4.4) are equally valid if the ϕ is replaced by a $\phi^\dagger$ or a Higgs boson H. Table 2 indicates the references in which the various helicity contributions to $Hgggg$ have first been calculated analytically. The amplitudes for a Higgs boson with four quarks was first calculated analytically in [3] (with explicit helicity amplitudes in [25]).

H amplitude	ϕ amplitude	$\phi^\dagger$ amplitude
$\mathscr{A}(H,1_{\bar{q}}^-,2_q^+,3^+,4^+)$	$\mathscr{A}(\phi,1_{\bar{q}}^-,2_q^+,3^+,4^+)$ [22]	$\mathscr{A}(\phi^\dagger,1_{\bar{q}}^-,2_q^+,3^+,4^+)$ [26]
$\mathscr{A}(H,1_{\bar{q}}^-,2_q^+,3^-,4^-)$	$\mathscr{A}(\phi,1_{\bar{q}}^-,2_q^+,3^-,4^-)$ [26]	$\mathscr{A}(\phi^\dagger,1_{\bar{q}}^-,2_q^+,3^-,4^-)$ [22]
$\mathscr{A}(H,1_{\bar{q}}^-,2_q^+,3^+,4^-)$	$\mathscr{A}(\phi,1_{\bar{q}}^-,2_q^+,3^+,4^-)$ [25]	$\mathscr{A}(\phi^\dagger,1_{\bar{q}}^-,2_q^+,3^+,4^-)$ [25]
$\mathscr{A}(H,1_{\bar{q}}^-,2_q^+,3^-,4^+)$	$\mathscr{A}(\phi,1_{\bar{q}}^-,2_q^+,3^-,4^+)$ [25]	$\mathscr{A}(\phi^\dagger,1_{\bar{q}}^-,2_q^+,3^-,4^+)$ [25]

Table 2: ϕ and $\phi^\dagger$ amplitudes needed to construct a given one-loop $H\bar{q}qgg$ amplitude, together with the references where they can be obtained. In all cases the $\phi^\dagger$ amplitudes are constructed from the ϕ amplitudes given in the reference, using the parity operation.

5. Conclusion

This report summarises the recent completion of the analytic results for the process $pp \to H+2j$. The results were obtained by using the unitarity method to calcaulte the cut-constructible pieces of the amplitudes. The rational pieces were calculated with various techniques, for the non-NMHV helicity configurations the unitarity-bootstrap technique was used. For the most recent NMHV calculations the rational pieces were obtained from Feynman diagrams. The calculations were performed using an effective Lagrangian which simplifies the full theory in which the top quark loops are included. The effective theory provides an accurate prescription of the physics provided that $m_H < 2m_t$. It is hoped that the analytic formulae described here will aid in producing fast phenomenological studies of Higgs physics at hadron colliders.

Acknowledgements

I would like to thank my collaborators Simon Badger, John Campbell, Keith Ellis, Nigel Glover and Pierpaolo Mastrolia. I would also like to thank Lance Dixon for useful discussions. This work was funded by an STFC studentship.

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