PROCEEDINGS OF SCIENCE

$b \to s\ell^+\ell^-$ in the high q^2 region at two-loops

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We report on the first analytic NNLL calculation for the matrix elements of the operators O_1 and O_2 for the inlusive process $b \to X_s l^+ l^-$ in the kinematical region $q^2 > 4m_c^2$, where q^2 is the invariant mass squared of the lepton-pair.

RADCOR 2009 - 9th International Symposium on Radiative Corrections (Applications of Quantum Field Theory to Phenomenology)
October 25-30 2009
Ascona, Switzerland

*Speaker.

PoS(RADCOR2009)029

1. Introduction

In the Standard Model, the flavor-changing neutral current process $b \to X_s l^+ l^-$ only occurs at the one-loop level and is therefore sensitive to new physics. In the kinematical region where the lepton invariant mass squared q^2 is far away from the $c\bar{c}$-resonances, the dilepton invariant mass spectrum and the forward-backward asymmetry can be precisely predicted using large m_b expansion, where the leading term is given by the partonic matrix element of the effective Hamiltonian

$$\mathscr{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i(\mu). \tag{1.1}$$

We neglect the CKM combination $V_{us}^* V_{ub}$ and the operator basis is defined as in [1]. In [2] we published the first analytic NNLL calculation of the high q^2 region of the matrix elements of the operators

$$O_1 = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L), \qquad O_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)\,, \tag{1.2}$$

which dominate the NNLL amplitude numerically. Earlier these results were only available analytically in the region of low q^2 [3, 4]. Using equations of motion the NNLL matrix elements of the effective operators take the form

$$\langle s\ell^+\ell^- | O_i | b \rangle_{\text{2-loops}} = -\left(\frac{\alpha_s}{4\pi}\right)^2 \left[F_i^{(7)} \langle O_7 \rangle_{\text{tree}} + F_i^{(9)} \langle O_9 \rangle_{\text{tree}} \right], \tag{1.3}$$

where $O_7 = e/g_s^2 m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$ and $O_9 = e^2/g_s^2 (\bar{s}_L \gamma_\mu b_L) \sum_l (\bar{l} \gamma^\mu l)$.

2. Calculations

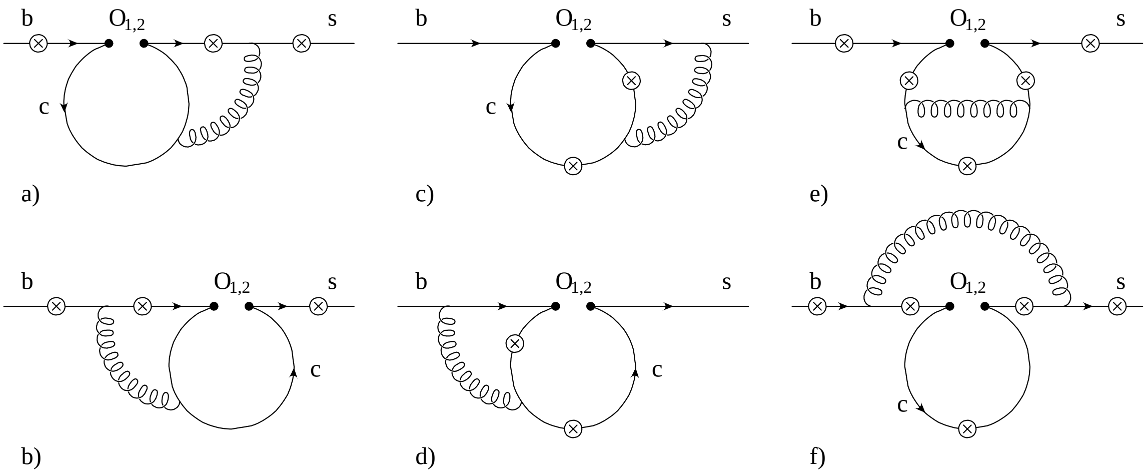

Figure 1: Diagrams that have to be taken into account at order α_s. The circle-crosses denote the possible locations where the virtual photon is emitted (see text).

The diagrams contributing at order α_s are shown in Figure 1. We set $m_s = 0$ and define

$$\hat{s} = \frac{q^2}{m_b^2} \quad \text{and} \quad z = \frac{m_c^2}{m_b^2}, \tag{2.1}$$

where q is the momentum of the virtual photon. After reducing occurring tensor-like Feynman integrals [5] the remaining scalar integrals can be further reduced to master integrals using integration by parts (IBP) identities [6]. Considering the region $\hat{s} > 4z$, we expanded the master integrals in z and kept the full analytic dependence in $\hat{s}$.

For power expanding Feynman integrals we use a combination of *method of regions* [7] and *differential equation techniques* [8, 9]: Consider a set of Feynman integrals $I_1,\ldots,I_n$ depending on the expansion parameter z and related by a system of differential equations obtained by differentiating I_α with respect to z and applying IBP identities:

$$\frac{d}{dz}I_\alpha = \sum_\beta h_{\alpha\beta}I_\beta + g_\alpha, \tag{2.2}$$

where g_α contains simpler integrals which pose no serious problems. Expanding both sides of (2.2) in ε, z and $\ln z$

$$I_\alpha = \sum_{i,j,k} I^{(j,k)}_{\alpha,i}\varepsilon^i z^j (\ln z)^k, \qquad h_{\alpha\beta} = \sum_{i,j} h^{(j)}_{\alpha\beta,i}\varepsilon^i z^j, \qquad g_\alpha = \sum_{i,j,k} g^{(j,k)}_{\alpha,i}\varepsilon^i z^j (\ln z)^k, \tag{2.3}$$

and inserting (2.3) into (2.2) we obtain algebraic equations for the coefficients $I^{(j,k)}_{\alpha,i}$

$$0 = (j+1)I^{(j+1,k)}_{\alpha,i} + (k+1)I^{(j+1,k+1)}_{\alpha,i} - \sum_\beta\sum_{i'}\sum_{j'} h^{(j')}_{\alpha\beta,i'} I^{(j-j',k)}_{\beta,i-i'} - g^{(j,k)}_{\alpha,i}. \tag{2.4}$$

This enables us to recursively calculate higher powers of z once the leading powers are known. In practice this means that we need the $I^{(0,0)}_{\alpha,i}$ and sometimes also the $I^{(1,0)}_{\alpha,i}$ as initial condition to (2.4). These initial conditions can be computed using method of regions. A non trivial check is provided by the fact that the leading terms containing logarithms of z can be calculated by both method of regions and the recurrence relation (2.4).

The summation index j in (2.3) can take integer or half-integer values, depending on the specific set of integrals I_α. In order to determine the possible powers of z and $\ln(z)$ we used the algorithm described in [9]. A given D-dimensional L-loop Feynman integral $I(z)$ reads in Feynman parameterization

$$I(z) = (-1)^N\left(\frac{i}{(4\pi)^{D/2}}\right)^L \Gamma(N-LD/2)\int d^Nx\,\delta(1-\sum_{n=1}^N x_n)\frac{U^{N-(L+1)D/2}}{(zF_1+F_2)^{N-LD/2}}, \tag{2.5}$$

where U, F_1 and F_2 are polynomials in x_n. Using Mellin-Barnes representation (2.5) can be cast into the following form

$$\begin{aligned} I(z) =& (-1)^N\left(\frac{i}{(4\pi)^{D/2}}\right)^L \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} ds\, z^s\Gamma(-s)\Gamma(s+N-LD/2) \\ &\times\int d^Nx\,\delta(1-\sum_{n=1}^N x_n)U^{N-(L+1)D/2}F_1^sF_2^{-s-N+LD/2}. \end{aligned} \tag{2.6}$$

By closing the integration contour over s to the right hand side the poles on the positive real axis turn into powers of z. If we apply the technique of *sector decomposition* [10] to (2.6) we end up with terms of the following form

$$\sum_{l=1}^N\sum_k \int_0^1 d^{N-1}t\left(\prod_{j=1}^{N-1} t_j^{A_j-B_j\varepsilon-C_js}\right)U_{lk}^{N-(L+1)D/2}F^s_{1,lk}F^{-s-N+LD/2}_{2,lk}, \tag{2.7}$$

where U_{lk}, $F_{1,lk}$ and $F_{2,lk}$ contain terms that are constant in $\vec{t}$. From (2.7) we can read off that the poles in s are located at:

$$s_{jn} = \frac{1+n+A_j - B_j\varepsilon}{C_j}, \tag{2.8}$$

where $n \in \mathbb{N}_0$.

Additionally, the procedure described above allows us to evaluate the coefficients of the expansion in z numerically which we used to again test the initial conditions of the differential equations.

3. Results

In order to get accurate results we keep terms up to z^{10}. Our results agree with the previous numerical calculation [11] within less than 1% difference. To demonstrate the convergence of the power expansions, we show in Figure 2 the form factors defined in (1.3) as functions of $\hat{s}$, where we include all orders up to z^6, z^8 and z^{10}. We use as default value $z = 0.1$ such that the $c\bar{c}$-threshold is located at $\hat{s} = 0.4$. One sees from the figures that far away from the $c\bar{c}$-threshold, i.e. for $\hat{s} > 0.6$, the expansions for all form factors are well behaved.

The impact of our results on the perturbative part of the high q^2-spectrum [3]

$$R(\hat{s}) = \frac{1}{\Gamma(B \to X_c e^- \bar{\nu}_e)} \frac{d\Gamma(\bar{B} \to X_s \ell^+\ell^-)}{d\hat{s}} \tag{3.1}$$

is shown in Figure 3 (left), where we used the same parameters as in [2]. The finite bremsstrahlung corrections calculated in [4] are neglected. From Figure 3 (left) we conclude that for $\mu = m_b$ the contributions of our results lead to corrections of the order $10\% - 15\%$. Integrating $R(\hat{s})$ over the high $\hat{s}$ region, we define

$$R_{\text{high}} = \int_{0.6}^{1} d\hat{s} R(\hat{s}). \tag{3.2}$$

Figure 3 (right) shows the dependence of the perturbative part of R_{high} on the renormalization scale. We obtain

$$R_{\text{high,pert}} = (0.43 \pm 0.01(\mu)) \times 10^{-5}, \tag{3.3}$$

where we determined the error by varying μ between 2 GeV and 10 GeV. The corrections due to our results lead to a decrease of the scale dependence to 2%.

Acknowledgments

This work is partially supported by the Swiss National Foundation, by EC-Contract MRTNCT-2006-035482 (FLAVIAnet) and by the Helmholtz Association through funds provided to the virtual institute "Spin and strong QCD" (VH-VI-231). The Albert Einstein Center for Fundamental Physics is supported by the "Innovations- und Kooperationsprojekt C-13 of the Schweizerische Universitätskonferenz SUK/CRUS".

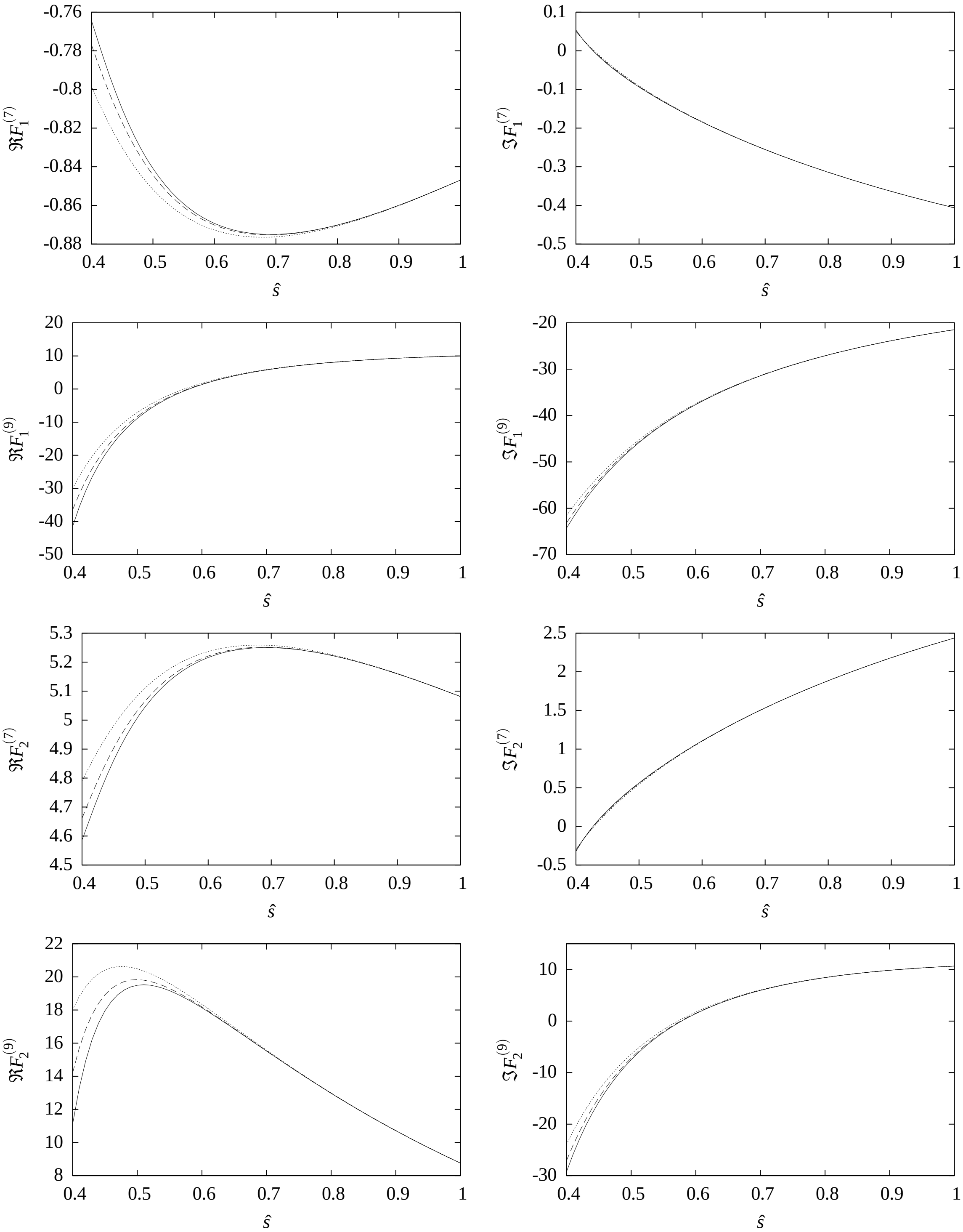

Figure 2: Real and imaginary parts of the form factors $F_{1,2}^{(7,9)}$ as functions of $\hat{s}$. To demonstrate the convergence of the expansion in z we included all orders up to z^6, z^8 and z^{10} in the dotted, dashed and solid lines respectively. We put $\mu = m_b$ and used the default value $z = 0.1$.

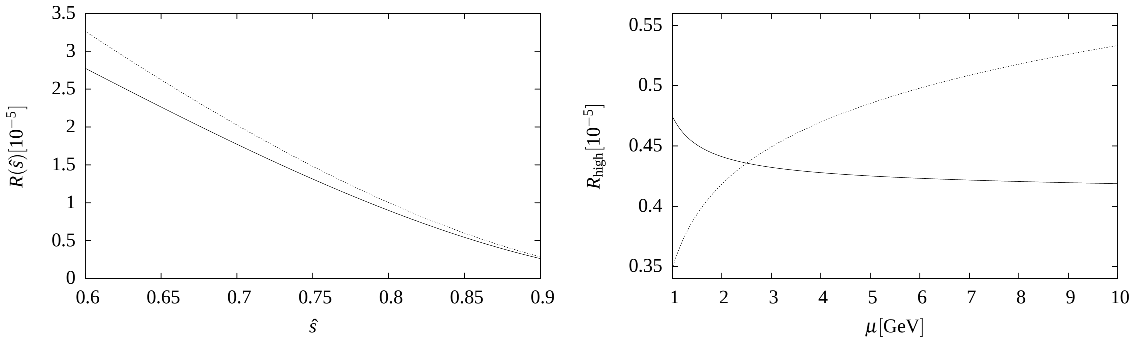

Figure 3: Perturbative part of $R(\hat{s})$ (left) and R_{high} (right) at NNLL. The solid lines represents the NNLL result, whereas in the dotted lines the order α_s corrections to the matrix elements associated with $O_{1,2}$ are switched off. In the left figure we use $\mu = m_b$. See text for details.

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