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Precise Charm- and Bottom-Quark Masses: Recent Updates

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Recent theoretical and experimental improvements in the determination of charm and bottom quark masses are discussed. The final results, $m_c(3\,\text{GeV}) = 986(13)\,\text{MeV}$ and $m_b(m_b) = 4163(16)\,\text{MeV}$ represent, together with a closely related lattice determination $m_c(3\text{ GeV}) = 986(10)\,\text{MeV}$, the presently most precise determinations of these two fundamental Standard Model parameters. A critical analysis of the theoretical and experimental uncertainties is presented.

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The past years have witnessed significant improvement in the determination of charm and bottom quark masses as a consequence of improvements in experimental techniques as well as theoretical calculations. Quark mass determinations can be based on a variety of observables and theoretical calculations. The one presently most precise follows an idea advocated by the ITEP group more than thirty years ago [1], and has gained renewed interest after significant advances in higher order perturbative calculations [2] have been achieved. In particular the four-loop results (i.e. the coefficients C_n discussed below) are now available for the Taylor coefficients of the vacuum polarization, analytically up to $n = 3$ and numerically up to $n = 10$. The method exploits the fact that the vacuum polarization function $\Pi(q^2)$ and its derivatives, evaluated at $q^2 = 0$, can be considered short distance quantities with an inverse scale characterized by the distance between the reference point $q^2 = 0$ and the location of the threshold $q^2 = (3\text{ GeV})^2$ and $q^2 = (10\text{ GeV})^2$ for charm and bottom, respectively. This idea has been taken up in [3] after the first three-loop evaluation of the moments became available [4, 5, 6] and has been further improved in [7] using four-loop results [8, 9] for the lowest moment. An analysis which is based on the most recent theoretical [10, 11, 12] and experimental progress has been performed in [13] and will be reviewed in the following.

Let us recall some basic notation and definitions. The vacuum polarization $\Pi_Q(q^2)$ induced by a heavy quark Q with charge Q_Q (ignoring in this short note the so-called singlet contributions), is an analytic function with poles and a branch cut at and above $q^2 = M^2_{J/\psi}$. Its Taylor coefficients $\bar{C}_n$, defined through

$$\Pi_Q(q^2) \equiv Q_Q^2 \frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n z^n \tag{1}$$

can be evaluated in pQCD, presently up to order α_s^3. Here $z \equiv q^2/4m_Q^2$, where $m_Q = m_Q(\mu)$ is the running $\overline{\text{MS}}$ mass at scale μ. Using a once-subtracted dispersion relation

$$\Pi_Q(q^2) = \frac{1}{12\pi^2} \int_0^\infty ds \frac{R_Q(s)}{s(s-q^2)} \tag{2}$$

(with R_Q denoting the familiar R-ratio for the production of heavy quarks with flavour Q), the Taylor coefficients can be expressed through moments of R_Q. Equating perturbatively calculated and experimentally measured moments,

$$\mathscr{M}_n^{\text{exp}} = \int \frac{ds}{s^{n+1}} R_Q(s) \tag{3}$$

leads to an (n-dependent) determination of the quark mass

$$m_Q = \frac{1}{2} \left(\frac{9Q_Q^2}{4} \frac{C_n}{\mathscr{M}_n^{\text{exp}}} \right)^{\frac{1}{2n}} . \tag{4}$$

Significant progress has been made in the perturbative evaluation of the moments since the first analysis of the ITEP group. The $\mathscr{O}(\alpha_s^2)$ contribution (three loops) has been evaluated more than 13 years ago [4, 5, 6], as far as the terms up to $n = 8$ are concerned, recently even up to $n = 30$ [14, 15]. About ten years later the lowest two moments ($n = 0, 1$) of the vector correlator were evaluated in $\mathscr{O}(\alpha_s^3)$, i. e. in four-loop approximation [8, 9]. The corresponding two lowest

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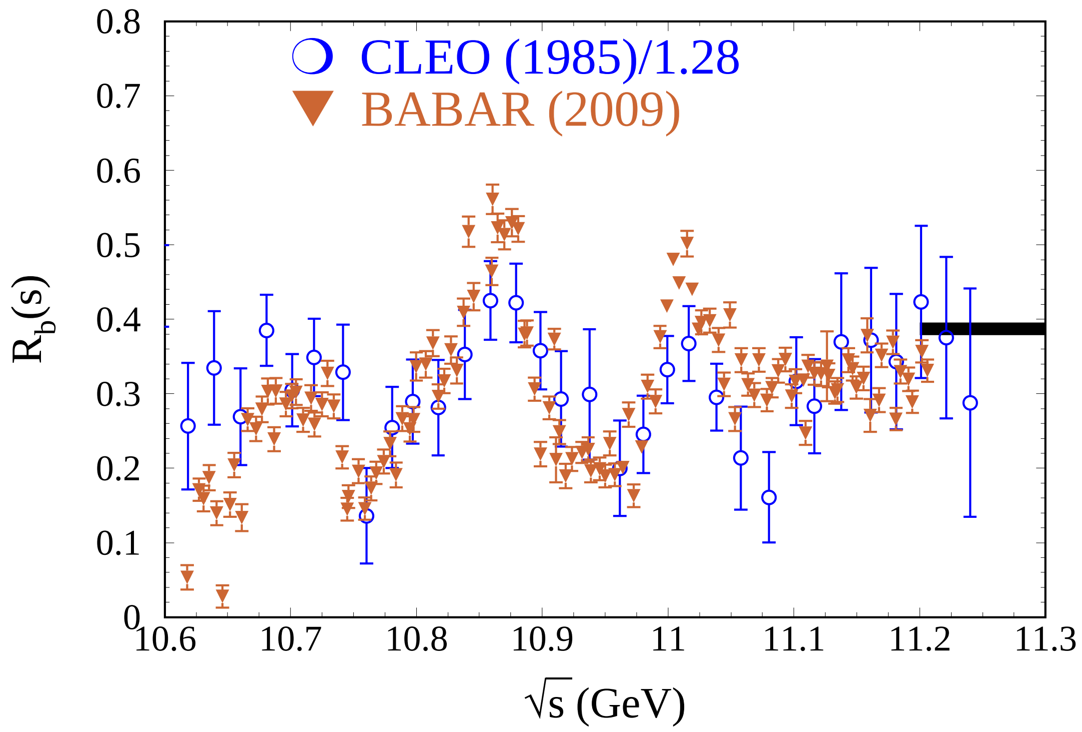

Figure 1: Comparison of rescaled CLEO data for R_b with BABAR data. [13, 19]. The black bar on the right corresponds to the theory prediction [20].

moments for the pseudoscalar correlator were obtained in [16] in order to derive the charmed quark mass from lattice simulations [17]. In [10, 11] the second and third moments were evaluated for vector, axial and pseudoscalar correlators. Combining, finally, these results with information about the threshold and high-energy behaviour in the form of a Padé approximation, the full q^2-dependence of all four correlators was reconstructed and the next moments, from four up to ten, were obtained with adequate accuracy [12].

Most of the experimental input had already been compiled and exploited in [7], where it is described in more detail. However, until recently the only measurement of the cross section above but still close to the B-meson threshold was performed by the CLEO collaboration more than twenty years ago [18]. Its large systematic uncertainty was responsible for a sizable fraction of the final error on m_b. This measurement has been recently superseded by a measurement of BABAR [19] with a systematic error between 2 and 3%. In [13] the radiative corrections were unfolded and used to obtain a significantly improved determination of the moments. The final results for $m_c(3\text{ GeV})$ and $m_b(10\text{ GeV})$ are listed in Table 1. Despite the significant differences in the composition of the errors, the results for different values of n are perfectly consistent. For charm the result from $n = 1$ has the smallest dependence on the strong coupling and the smallest total error, which we take as our final value

$$m_c(3\text{ GeV}) = 986(13)\text{ MeV}\,, \tag{5}$$

n	$m_c(3\text{ GeV})$	exp	α_s	μ	np	total	$m_b(10\text{ GeV})$	exp	α_s	μ	total
1	986	9	9	2	1	13	3597	14	7	2	16
2	976	6	14	5	0	16	3610	10	12	3	16
3	978	5	15	7	2	17	3619	8	14	6	18
4	1004	3	9	31	7	33	3631	6	15	20	26

Table 1: Results for $m_c(3\text{ GeV})$ and for $m_b(10\text{ GeV})$ in MeV. The errors are from experiment, α_s, the variation of μ and (for m_c) the gluon condensate.

and consider its consistency with $n = 2$, 3 and 4 as additional confirmation. Transforming this to the scale-invariant mass $m_c(m_c)$ [21], including the four-loop coefficients of the renormalization group functions one finds [13] $m_c(m_c) = 1279(13)$ MeV. Let us recall at this point that a recent study [17], combining a lattice simulation for the data for the pseudoscalar correlator with the perturbative three- and four-loop result [6, 16, 11] has led to $m_c(3\text{ GeV}) = 986(10)$ MeV in remarkable agreement with [7, 13].

The treatment of the bottom quark case proceeds along similar lines. However, in order to suppress the theoretically evaluated input above 11.2 GeV (which corresponds to roughly 60% for the lowest, 40% for the second and 26% for the third moment), the result from the second moment has been adopted as our final result,

$$m_b(10\text{ GeV}) = 3610(16)\text{ MeV}, \tag{6}$$

corresponding to $m_b(m_b) = 4163(16)$MeV. The explicit α_s dependence of m_c and m_b can be found in [13]. When considering the ratio of charm and bottom quark masses, part of the α_s and of the μ dependence cancels

$$\frac{m_c(3\text{ GeV})}{m_b(10\text{ GeV})} = 0.2732 - \frac{\alpha_s - 0.1189}{0.002} \cdot 0.0014 \pm 0.0028\,, \tag{7}$$

which might be a useful input in ongoing analysis of bottom decays.

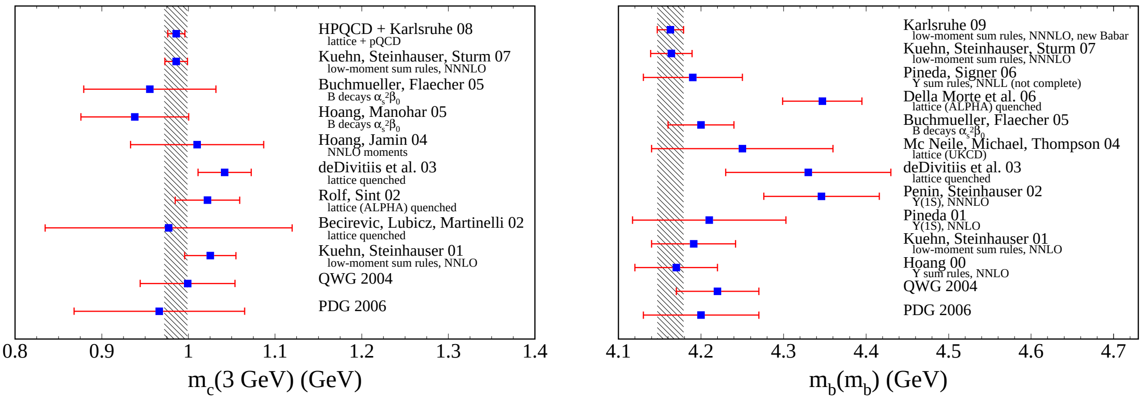

Figure 2: Comparison of recent determinations of $m_c(3\text{ GeV})$ and $m_b(m_b)$.

In Fig. 2 the results of this analysis are compared to others based on completely different methods. The m_c value is well within the range suggested by other determinations. In case of m_b our result is somewhat towards the low side, although still consistent with most other results.

The results presented in [13] constitute the most precise values for the charm- and bottom-quark masses available to date. Nevertheless it is tempting to point to the dominant errors and thus identify potential improvements. In the case of the charmed quark the error is dominated by the parametric uncertainty in the strong coupling $\alpha_s(M_Z) = 0.1185 \pm 0.002$. Experimental and theoretical errors are comparable, the former being dominated by the electronic width of the narrow resonances. In principle this error could be further reduced by the high luminosity measurements at BESS III. A further reduction of the (already tiny) theory error, e. g. through a five-loop calculation looks difficult. Further confidence in our result can be obtained from the comparison with the forementioned lattice evaluation.

Improvements in the bottom quark mass determination could originate from the experimental input, e. g. through an improved determination of the electronic widths of the narrow Υ resonances or through a second, independent measurement of the R ratio in the region from the $\Upsilon(4S)$ up to 11.2 GeV. As shown in Fig. 1, there is a slight mismatch between the theory prediction above 11.2 GeV and the data in the region below with their systematic error of less than 3%.

In this connection it may be useful to collect the most important pieces of evidence supporting this remarkably small error. Part of the discussion is applicable to both charm and bottom, part is specific to only one of them. In particular for charm, but to some extent also for bottom, the μ-dependence of the result increases for the higher moments, starting with $n = 4$, and dominates the total error. We will therefore concentrate on the moments $n = 1$, 2, and 3 which were used for the mass determination, results for $n = 4$ will only be mentioned for illustration.

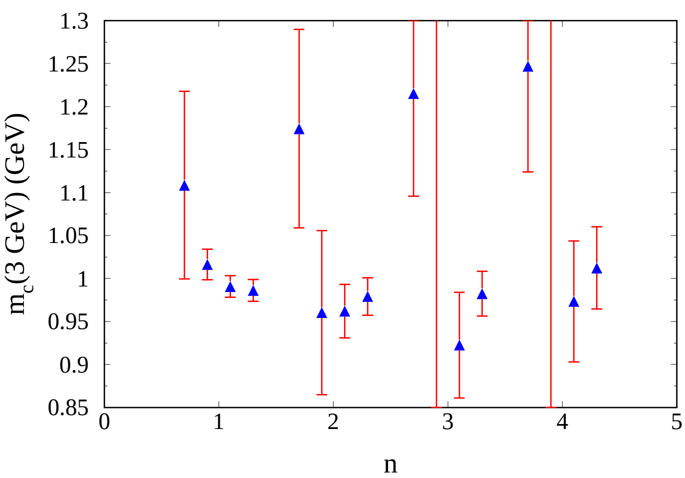

Figure 3: $m_c(3\text{ GeV})$ for $n = 1,2,3$ and 4. For each value of n the results from left to right correspond the inclusion of terms of order α_s^0, α_s^1, α_s^2 and α_s^3.

Let us start with charm. Right at the beginning it should be emphasized that the primary quantity to be determined is the running mass at the scale of 3 GeV, the scale characteristic for the production threshold and thus for the process. Furthermore, at this scale the strong coupling $\alpha_s(3\text{ GeV}) = 0.258$ is already sufficiently small such that the higher order terms in the perturbative series decrease rapidly. Last not least, for many other observable of interest, like B-meson decays into charm, or processes involving virtual charm quarks like $B \to X_s\gamma$ or $K \to \pi\nu\bar{\nu}$, the characteristic scale is of order 3 GeV or higher. Artificially running the mass first down to $\mathscr{O}(1\text{ GeV})$ and then back to a higher scale thus leads to an unnecessary inflation of the error.

The quark mass determination is affected by the theory uncertainty, resulting in particular from our ignorance of yet uncalculated higher orders, and by the error in the evaluation of the experimental moments. The former has been estimated [7] by evaluating $m_c(\mu)$ at different renormalization scales between 2 and 4 GeV (changing of course the coefficients $\bar{C}_n$ appropriately) and subsequently evolving $m_c(\mu)$ to $m_c(3\text{ GeV})$. The error estimates based on these considerations are listed in Table 1.

The stability of the result upon inclusion of higher orders is also evident from Fig. 3 where the results from different values of n are displayed separately in order α_s^i with $i = 0$, 1, 2 and 3. This argument can be made more quantitatively by rewriting eq. (4) in the form

$$\begin{aligned} m_c &= \frac{1}{2}\left(\frac{9Q_c^2}{4}\frac{C_n^{Born}}{\mathscr{M}_n^{\text{exp}}}\right)^{\frac{1}{2n}}(1+r_n^{(1)}\alpha_s+r_n^{(2)}\alpha_s^2+r_n^{(3)}\alpha_s^3) \\ &\propto 1-\begin{pmatrix}0.328\\0.524\\0.618\\0.662\end{pmatrix}\alpha_s-\begin{pmatrix}0.306\\0.409\\0.510\\0.575\end{pmatrix}\alpha_s^2-\begin{pmatrix}0.262\\0.230\\0.299\\0.396\end{pmatrix}\alpha_s^3, \end{aligned} \tag{8}$$

where the entries correspond to the moments with $n = 1$, 2, 3 and 4. Note, that the coefficients are decreasing with increasing order of α_s. Estimating the relative error through $r_n^{max}\alpha_s(3\text{ GeV})^4$ leads to 1.4 / 2.3 / 2.7 / 2.9 permille and thus to an estimate clearly smaller than the one based on the μ-dependence.

The consistency between the results for different values of n is another piece of evidence (Fig. 3 and Table 1). For the lowest three moments the variation between the maximal and the minimal value amounts to 10 MeV only. This, in addition, points to the selfconsistency of our data set. Let us illustrate this aspect by a critical discussion of the "continuum contribution", i.e. the region above 4.8 GeV, where data points are available at widely separated points only. Instead of experimental data the theory prediction for $R(s)$ has been employed for the evaluation of the contribution to the moments. If the true contribution from this region would be shifted down by, say, 10%, this would move m_c, as derived from $n = 1$, up by about 20 MeV. However, this same shift would lead to a small increase by 3 MeV for $n = 2$ and leave the results for higher n higher practically unchanged. Furthermore, theory predictions and measurements in the region from 4.8 GeV up to the bottom-meson threshold, wherever available, are in excellent agreement, as shown in Fig. 4, with deviations well within the statistical and systematical error of 3 to 5%. Last not least, the result described above is in perfect agreement with the recent lattice determination mentioned above.

Let us now discuss beauty, with m_b evaluated at $\mu = 10$ GeV. Again we first study the stability of the perturbative expansion, subsequently the consistency of the experimental input. With $\alpha_s(10\text{ GeV}) = 0.180$ the higher order corrections decrease even more rapidly. Varying the scale μ between 5 and 15 GeV leads to a shift between 2 and 6 MeV (Table 1) which is completely

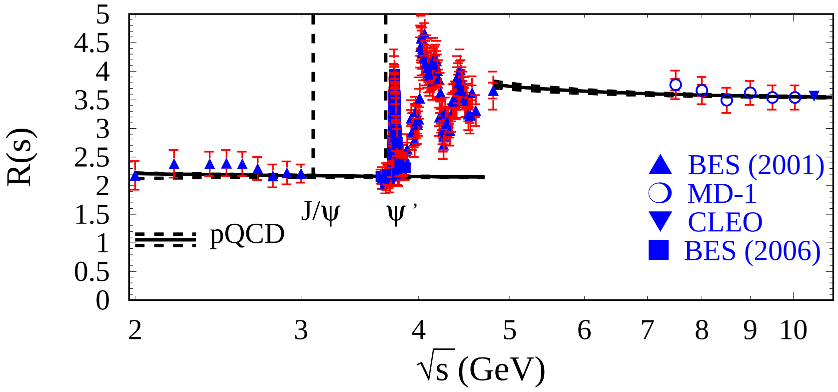

Figure 4: $R(s)$ for different energy intervals around the charm threshold region. The solid line corresponds to the theoretical prediction.

negligible. Alternatively we may consider the analogue of eq. (8) with the correction factor

$$m_b/m_b^{\rm Born} = 1 - \begin{pmatrix} 0.270 \\ 0.456 \\ 0.546 \\ 0.603 \end{pmatrix} \alpha_s - \begin{pmatrix} 0.206 \\ 0.272 \\ 0.348 \\ 0.410 \end{pmatrix} \alpha_s^2 + \begin{pmatrix} -0.064 \\ 0.048 \\ 0.051 \\ 0.012 \end{pmatrix} \alpha_s^3 . \qquad (9)$$

Taking $r_n^{max} \alpha_s^4$ for an error estimate leads to a relative error of .28 / .48 / .57 / 0.63 permille for $n = 1$, 2, 3 and 4 respectively, which is again smaller than our previous estimate. Let us now move to a critical discussion of the experimental input. The contribution from the lowest four Υ-resonances has been taken directly from PDG [22] with systematic errors of the lowest three added linearly. The analysis [13] of a recent measurement [19] of R_b in the threshold region up to 11.20 GeV has provided results consistent with the earlier analysis [7] but has lead to a significant reduction of the error in m_b.

In comparison with the charm analysis a larger contribution arises from the region where data are substituted by the theoretically predicted R_b with relative contributions of 63, 41, 26 and 17 percent for $n = 1$, 2, 3 and 4 respectively. This is particularly valid for the lowest moment. For this reason we prefer to use the result from $n = 2$, alternatively we could have also taken the one from $n = 3$. Let us now collect the arguments in favour of this approach:

i) For light and charmed quarks the prediction for R based on pQCD works extremely well already two to three GeV above threshold. No systematic shift has been observed between theory and experiment, in the case of massless quarks, starting from around 2 GeV, and for the cross section including charm at and above 5 GeV up to the bottom threshold (Fig. 4). It is thus highly unplausible that the same approach should fail for bottom production.

ii) If the true R_b in the continuum above 11.2 GeV would differ from the theory prediction by a sizable amount, the results for $n = 1$, 2 and 3 would be mutually inconsistent. Specifically, a shift of the continuum term by 5% would move m_b, derived from $n = 1$, 2 and 3 by about 64 MeV, 21 MeV and 9 MeV respectively.

To summarize: Charm and bottom quark mass determinations have made significant progress during the past years. A further reduction of the theoretical and experimental error seems difficult at present. However, independent experimental results on the R ratio would help to further consolidate the present situation. The confirmation by a recent lattice analysis with similarly small uncertainty gives additional confidence in the result for m_c.

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