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Soft and Collinear gluon corrections to Higgs production beyond two loop

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We study the large-x behaviour of cross section for the Higgs production by using resummation formula that correctly predicts threshold corrections to all orders in perturbation theory. We discuss a possible modification to resummation formula to include sub-leading large logarithms.

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The deep inelastic scattering (DIS) experiments provide rich information on the strong interaction physics which is governed by quantum chromodynamics (QCD). The predictions of QCD both in perturbative as well as non-perturbative regions are found to be in excellent agreement with the experiments. QCD provides a framework to successfully compute hadronic cross sections through factorisation theorem [1]. Using this, one can express the hard scattering cross sections in terms of perturbatively calculable partonic cross sections convoluted with non-perturbative parton distribution functions. The Higgs production cross section through gluon fusion in hadron collisions is given by

$$\sigma^H(S,m_H^2) = \frac{\pi G_B^2}{8(N^2-1)} \sum_{a,b=q,\overline{q},g} \int_x^1 dy\, \Phi_{ab}(y,\mu_F^2)\, \Delta_{ab}^H\left(\frac{x}{y},m_H^2,\mu_F^2,\mu_R^2\right) \tag{1}$$

where $x = m_H^2/S$, $N = 3$ and the factor G_B can be found from [2]. The flux $\Phi_{ab}(y,\mu_F^2)$ is given by

$$\Phi_{ab}(y,\mu_F^2) = \int_y^1 \frac{dw}{w}\, f_a(w,\mu_F^2)\, f_b\left(\frac{y}{w},\mu_F^2\right) \tag{2}$$

The partonic cross section Δ_{ab}^H contains both soft plus virtual as well as hard contributions:

$$\Delta_{ab}^H(z,m_H^2,\mu_R^2,\mu_F^2) = \Delta_{g,S}^{sv}(z,m_H^2,\mu_R^2,\mu_F^2) + \Delta_{ab}^{H,hard}(z,m_H^2,\mu_R^2,\mu_F^2) \tag{3}$$

In the above equation, coefficient functions Δ_{ab}^H are nothing but the mass factorised partonic cross sections resulting from the scattering of partons of types a,b and the parton distribution function of the parton of type a in the proton is given by $f_{a/P}$. The factorisation scale μ_F separates this perturbative region from the non-perturbative effects. The superscript *sv* means soft plus virtual and *hard* means the remaining contribution. The Δ_{ab}^H can be expanded in powers of strong coupling constant g_s as

$$\Delta_{ab}^H(z,m_H^2,\mu_R^2,\mu_F^2) = \sum_{i=0} a_s^i(\mu_R^2)\Delta_{ab}^{H,(i)}(z,\mu_F^2,m_H^2,\mu_R^2) \tag{4}$$

where $a_s(\mu_R^2) = g_s^2/16\pi^2$ with μ_R being renormalisation scale. The parton distribution functions are not calculable within perturbative theory, but their evolution with respect to the scale μ_F is completely determined by renormalisation group equations:

$$\mu_F^2 \frac{d}{d\mu_F^2} f_{a/P}(x,\mu_F^2) = \sum_b P_{ab}(x,\mu_F^2) \otimes f_{b/P}(x,\mu_F^2) \tag{5}$$

where the perturbatively calculable DGLAP splitting functions are given by

$$P_{ab}(x,\mu_F^2) = \sum_{i=0} a_s^{i+1}(\mu_F^2) P_{ab}^i(x) \tag{6}$$

The coefficient functions Δ_{ab}^H, beyond leading order in a_s, receive contributions from virtual as well as real emission processes. They are often sensitive to soft gluons through large logarithms that result from the cancellation of soft singularities arising from massless gluons in both virtual and real emission processes. They are of the form $(\log^i(1-x)/(1-x))_+$ distributions and hence dominate in the threshold region, namely $x \to 1$. We also get sub-leading contributions through the logarithms $\log^j(1-x)$ that arise from the region near collinear partons. Recall that the collinear singularities are removed using mass factorisation. Since the soft gluons dominate the cross sections in the threshold region, they need to be resummed to make reliable predictions. It can be done systematically using factorisation theorem and renormalisation group invariance. This resummation goes under the name soft-gluon exponentiation [3] and it is known to the next-to-next-to-next-to-leading logarithmic (N^3LL) accuracy for inclusive DIS [4], Drell-Yan and Higgs productions in proton-proton collisions [5, 6, 7, 8], and semi-inclusive electron-positron annihilation [9, 10]. It is not clear whether such a resummed result also contains correct sub-leading logarithmic terms. In other words, one might want to know whether resummation of sub-leading logarithms is possible. The recent works [11, 12] have shown that systematic predictions for higher-order coefficient functions fail for the sub-leading logarithms.

In the article [13], it is shown that using the known coefficient-function results of Refs. [14, 15, 16, 17, 18, 19, 20, 21, 22, 23], the non-singlet physical kernels show only a single logarithmic enhancement at least to the next-to-next-to-leading or next-to-next-to-next-to-leading order (NNLO or N^3LO) in the expansion in a_s and further it is conjectured that this behaviour will persist to all orders in a_s.

In this article we study how far the soft gluon resummed result can predict the sub-leading logarithms and how it can be improved to account for the mismatch. The soft plus virtual part $\Delta^{sv}_{ab}(z,q^2,\mu_F^2)$ receives contributions from purely virtual processes as well as processes involving at least one real emission. The former can be obtained from the form factors $\hat{F}^q$ of vector current between on-shell quark /antiquark states, while the later, denoted by Φ_S arises when at least one of the real gluons becomes soft. While the sum of these contributions is free of singularities coming from the soft gluons, there will be collinear divergences resulting from massless partons. They are removed using the DGLAP kernels, Γ_{qq} through mass factorisation. Hence we have,

$$\Delta^{sv}_{ab}(z,q^2,\mu_R^2,\mu_F^2) = \mathscr{C}\exp\left(\Psi_S(z,q^2,\mu_R^2,\mu_F^2,\varepsilon)\right)\Big|_{\varepsilon=0} \tag{7}$$

where $\Psi_S(z,q^2,\mu_R^2,\mu_F^2,\varepsilon)$ is a finite distribution, the symbol S stands for "soft". Here $\Psi_S(z,q^2,\mu_R^2,\mu_F^2,\varepsilon)$ is computed in $4+\varepsilon$ dimensions.

$$\begin{aligned}\Psi_S(z,q^2,\mu_R^2,\mu_F^2,\varepsilon) \;=\; & \left(\ln\left|\hat{F}^q(\hat{a}_s,Q^2,\mu^2,\varepsilon)\right|^2\right)\delta(1-z) + 2\,\Phi_S(\hat{a}_s,q^2,\mu^2,z,\varepsilon) \\ & -2\mathscr{C}\ln\Gamma_{qq}(\hat{a}_s,\mu^2,\mu_F^2,z,\varepsilon),\end{aligned} \tag{8}$$

In the above equation, we have introduced $\mathscr{C}$ to indicate that the normal products should be replaced by convolutions when the exponential and logarithms appearing in eqn.(8) are expanded in powers of a_s. In the above equation, $\hat{F}^q$ and Φ_S are known to all orders in ε at one loop level. At two loop level, F^q and Φ_S are known to order ε^0. The three loop results for $\hat{F}^q$ is available to order ε^0 while for Φ_S, the finite part (ε^0) is still unknown at three loop level. Γ_{qq} being the $\overline{MS}$ counter term for the collinear singularities is function of $1/\varepsilon$ alone and it is computed to three loop level. The resummed cross section Δ_{ab} correctly reproduces soft plus virtual part of the cross section in terms of $\delta(1-z)$ and $(\log^i(1-z)/(1-z))_+$ distributions. In order to check how far the above resummed result can predict sub-leading logarithms, we have expanded the exponential in powers of a_s and expanded all the convolutions to order $(1-z)^0$ including all powers of $\log(1-z)$. For the DGLAP splitting function, P_{qq} that enters in the kernel Γ_{qq}, we used the following expansion:

$$P_{qq} = 2\left(\frac{A_q(a_s)}{(1-z)_+} + B_q(a_s)\delta(1-z) + C_q(a_s)\log(1-z) + D_q(a_s)\right) + \mathscr{O}(1-z) \tag{9}$$

where A_q, B_q, C_q and D_q are known upto three loop level. Such a naive replacement in the resummation formula reproduces largest power of the $log(1-z)$ at every order in a_s, namely it predicts the coefficient of $a_s^j log^{2j-1}(1-z)$ is zero for all j. On the other hand, it fails to predict $a_s^j \log^{2j-k}(1-z)$ for $2j \leq k > 1$. In the following we modify Φ_S such that we correctly reproduce $a_s^j \log^{2j-k}(1-z)$ from the resummation formula. In this approach, we include terms $a_s^i \sum_{j=0}^{i} E_j^{(i)} \log^j(1-z)$ at ith order in a_s. We fix the coefficient $E_j^{(i)}$ by comparing with the fixed results for coefficient functions. The exponent that includes sub-leading logarithms is given by $\Phi_S \to \overline{\Phi}_S$,

$$\overline{\Phi}_S(z, q^2, \mu_R^2, \mu_F^2, \varepsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)^2}{z\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \,(i\,\varepsilon) \left[\frac{\phi^{(i)}(\varepsilon)}{1-z} + \sum_{j=0}^{i} E_j^{(i)} \log^j(1-z)\right] \tag{10}$$

In the above equation, $\hat{\phi}^{(i)}(\varepsilon)$ is given by

$$\hat{\phi}^{(i)}(\varepsilon) = \frac{1}{i\varepsilon}\left[\overline{K}^{(i)}(\varepsilon) + \overline{G}^{(i)}(\varepsilon)\right] \tag{11}$$

where the constants $\overline{K}^{(i)}, \overline{G}^{(i)}$ can be found in [8] and $E_j^{(i)}$ are determined by comparing known against fixed order result. Notice that the sum over j is controlled by the order of the perturbation denoted by the index i. This structure clearly indicates the failure of the resummation of sub-leading logarithms if we use naive approach. To summarise, we have checked explictly that the predictions of standard resummation formula does not give the correct sub-leading logarithms and a modification of the kind discussed in this article does not have predictive power.

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