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Renormalization of Non-quasipartonic Operators in QCD

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Extending the work by Bukhvostov, Frolov, Lipatov and Kuraev (BFLK) on the renormalization of quasipartonic operators we derive a complete set of two-particle renormalization group kernels that enter QCD evolution equations to twist-four accuracy. To this end we develop a new algebraic technique which essentially bypasses the calculation of usual Feynman diagrams. The kernels are presented for the renormalization of light-ray operators built of chiral fields in a particular basis such that the conformal symmetry is manifest. The results can easily be recast in momentum space, in the form of evolution equations for generalized parton distributions.

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1. Introduction

The theoretical description of higher-twist corrections in QCD is based on the Wilson Operator Product Expansion (OPE) and involves contributions of a large number of local operators. A general formalism was developed by Bukhvostov, Frolov, Lipatov and Kuraev (BFLK) [1] for the special class of so-called quasipartonic operators that are built of “plus” components of quark and gluon fields. For each twist, the set of quasipartonic operators is closed under renormalization and the renormalization group (RG) equation can be written in a Hamiltonian form that involves two-particle “interaction” kernels, cf. Fig. 1a, that can be expressed in terms of two-particle Casimir operators of the collinear subgroup $SL(2,\mathbb{R})$ of the conformal group. In this formulation symmetries of the RG equations become explicit. Moreover, the corresponding three-particle quantum-mechanical problem turns out to be completely integrable for a few important cases, and in fact reduces to a Heisenberg spin chain [2], see [3, 4] for a review and further references.

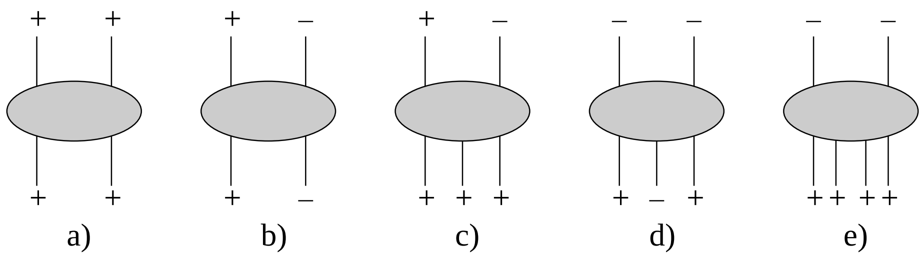

Figure 1: Schematic structure of one-loop renormalization group kernels in QCD

The goal of this study [5, 6] is to generalize the BFLK approach to the situation where not all contributing operators are quasipartonic, as it proves to be the case starting with twist four. On this way, there are two complications. First, the number of fields (“particles”) is not conserved. To one-loop accuracy, the mixing matrix of operators with a given twist has a block-triangular structure as the operators with less fields can mix with ones containing more fields but not vice versa, cf. Fig. 1c,d,e. Operators with the maximum possible number of fields for the given twist are quasipartonic. Second, operators involving “minus” and “transverse” derivatives and/or field components must be included. The problem is that transverse derivatives generally do not have good transformation properties with respect to the $SL(2,\mathbb{R})$ group so that the conformal symmetry becomes obscured.

The main new contribution of this work is the calculation of all existing $2 \to 2$ kernels of the type shown in Fig. 1b and the $2 \to 3$ kernels corresponding to operator mixing of one “partonic” and one “non-partonic” field in three-particle quasipartonic operators, Fig. 1c. To this end we suggest a new technique which bypasses the calculation of Feynman diagrams.

2. Conformal Operator Basis

In Ref. [5] we have constructed a complete basis in the sector of physical operators of arbitrary twist, i.e. operators non-vanishing on the equations of motion (EOM). The evolution kernels calculated in this basis are manifestly $SL(2,R)$ invariant which allows one to use powerful algebraic methods to calculate their renormalization [6], bypassing calculation of Feynman diagrams.

	$j=1/2$	$j=1$	$j=3/2$	$j=2$	$j=5/2$
$E=1$		ψ_+			
$E=2$	ψ_-		$D_w\psi_+$		
$E=3$		$D_{\bar w}\psi_-,\ D_{\tilde z}\psi_+$		$D_w^2\psi_+$	
$E=4$	$D_{\tilde z}\psi_-$		$D_{\bar w}^2\psi_-,\ D_wD_{\tilde z}\psi_+$		$D_w^3\psi_+$

Table 1: One-particle chiral quark operators with the lowest twist E and conformal spin j.

An essential part of our approach is going over to the spinor formalism. In this way each covariant four-vector x_μ is mapped to a hermitian matrix x, a Dirac (quark) spinor q is written in terms of two-component Weyl spinors ψ_α and $\bar\chi^{\dot\beta}$, and the gluon strength tensor $F_{\mu\nu}$ is decomposed in chiral and antichiral symmetric tensors, $f_{\alpha\beta}$ and $\bar f_{\dot\alpha\dot\beta}$, which belong to $(1,0)$ and $(0,1)$ representations of the Lorentz group.

The two independent light-cone directions, n_μ and $\tilde n_\mu$, $n^2=\tilde n^2=0$, can be parameterized in terms of auxiliary spinors

$$n_{\alpha\dot\alpha}=\lambda_\alpha\bar\lambda_{\dot\alpha}\,,\qquad \tilde n_{\alpha\dot\alpha}=\mu_\alpha\bar\mu_{\dot\alpha}\,. \tag{2.1}$$

The basis vectors in the transverse plane to $(n,\tilde n)$ can be chosen as $\mu_\alpha\bar\lambda_{\dot\alpha}$ and $\lambda_\alpha\bar\mu_{\dot\alpha}$, so that an arbitrary four-vector can be described by two real light-cone coordinates z and $\tilde z$ and two complex coordinates w, $\bar w=w^*$ in the transverse plane, $x_\mu=(x_0,x_1,x_2,x_3)\to\{z,\tilde z,w,\bar w\}$:

$$x_{\alpha\dot\alpha}=z\lambda_\alpha\bar\lambda_{\dot\alpha}+\tilde z\mu_\alpha\bar\mu_{\dot\alpha}+w\lambda_\alpha\bar\mu_{\dot\alpha}+\bar w\mu_\alpha\bar\lambda_{\dot\alpha}\,, \tag{2.2}$$

The "+" and "−" fields are defined as the projections onto λ and μ spinors, respectively: $\psi_+=\lambda^\alpha\psi_\alpha$, $\psi_-=\mu^\alpha\psi_\alpha$, $f_{++}=\lambda^\alpha\lambda^\beta f_{\alpha\beta}$, etc.

In general fields with derivatives or, equivalently, fields at arbitrary space-time positions have "bad" $SL(2,\mathbb{R})$ transformation properties. In particular under an infinitesimal special conformal transformation in the light-cone direction:

$$\psi_+(z,\tilde z,w,\bar w)\to\frac{1}{(1+z\varepsilon)^2}\left\{\psi_+\left(\frac{z}{1+\varepsilon z},\tilde z,\frac{w}{1+\varepsilon z},\frac{\bar w}{1+\varepsilon z}\right)+\varepsilon z\bar w\psi_-\Big(\dots\Big)\right\}, \tag{2.3}$$

which means that, e.g., the light-ray field $[D_{\bar w}\psi_+](z,0,0,0)$ does not transform homogeneously under $SL(2,\mathbb{R})$. The solution suggested in [5] is to allow only

$$\psi_+(z,\tilde z,w,0)\;=\;\sum_{n,k}\frac{\tilde z^k}{k!}\frac{w^n}{n!}[D_w^nD_{\tilde z}^k\psi_+](z)\,,\qquad \psi_-(z,\tilde z,0,\bar w)\;=\;\sum_{n,k}\frac{\tilde z^k}{k!}\frac{\bar w^n}{n!}[D_{\bar w}^nD_{\tilde z}^k\psi_-](z) \tag{2.4}$$

as independent one-particle operators in the basis, and eliminate another "half" of transverse derivatives using EOM, e.g. $[D_{\bar w}\psi_+](z)\equiv[D_{-+}\psi_+](z)=[D_{++}\psi_-](z)+EOM=2\partial_z\psi_-(z)+EOM$. This construction is exemplified in Table 1 where one-particle chiral quark operators are listed for the lowest few values of collinear twist E and conformal spin j. These are, in turn, the building blocks for gauge-invariant composite light-ray operators, e.g.

$$\mathbb{C}^{abc}\Big\{[0,z_1]\bar\psi_+(z_1)\Big\}^a\Big\{[0,z_2]f_{++}(z_2)\Big\}^b\Big\{[0,z_3]D_w\psi_+(z_3)\Big\}^c \tag{2.5}$$

where $[a,b]$ stands for the light-like Wilson line connecting the indicated points and $\mathbb{C}^{abc}$ is a color tensor. Notice that e.g. for twist $E=2$ one has to include ψ_- and $D_w\psi_+$ as *independent* operators. In concrete applications it is often possible to get rid of $D_w\psi_+$ using EOM and exploiting specific structure of the matrix elements of interest, e.g. if there is no transverse momentum transfer between the initial and the final state. However, after this reduction the conformal symmetry becomes obscured.

3. Embedding $SL(2,\mathbb{R})$ in the full conformal group $SO(4,2)$

The $SL(2,\mathbb{R})$-invariance of the BFLK kernels [1] and the known fact that two-particle representations of $SL(2,\mathbb{R})$ are non-degenerate, allows one to write these kernels in compact form as functions of the quadratic Casimir operator of the symmetry group. For example, for the simplest case of the two quarks with the same chirality $\mathscr{O}^{ik}(z_1,z_2)=\psi_+^i(z_1)\psi_+^k(z_2)$ (with open color indices i,k) the kernel $\mathbb{H}$ in the RG equation

$$\Big(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\frac{\alpha_s}{2\pi}\mathbb{H}\Big)\,[\mathscr{O}(z_1,z_2)]_R=0$$

can be written as

$$\mathbb{H}^{ik}_{i'k'}(\widehat{J}_{12}) \;=\; -4(t^b)_{ii'}(t^b)_{kk'}\left[\psi(\widehat{J}_{12})-\psi(1)-\frac{3}{4}\right], \tag{3.1}$$

where $\psi(J)$ is the Euler ψ-function and the operator $\widehat{J}_{12}$ in coordinate (fundamental) representation is defined formally as

$$\mathbb{C}_2^{SL(2,R)}=-\frac{\partial}{\partial z_1}\frac{\partial}{\partial z_2}(z_1-z_2)^2=\widehat{J}_{12}(\widehat{J}_{12}-1)\,.$$

As noticed in [7], the same two conditions (invariance and non-degeneracy) are fulfilled for composite operators built of "physical" fields, with respect to the full conformal group $SO(4,2)$. This implies that RG equations for the light-ray fields with arbitrary spin projections can be written in terms of the quadratic Casimir operator of $SO(4,2)$, $\mathbb{C}_2^{SO(4,2)}=\widehat{\mathbb{J}}_{12}(\widehat{\mathbb{J}}_{12}-1)$, and the functional form of this dependence must be the same for all operators in the same $SO(4,2)$ multiplet, hence $\mathbb{H}(\widehat{\mathbb{J}}_{12})$ must in fact be the same function as above (for all two chiral quark operators).

In the case of one "plus" and one "minus" quark field as in Fig. 1b we have to deal with two-dimensional (matrix) representations, i.e.

$$\mathbb{C}_2^{SO(4,2)}\begin{pmatrix}\psi_-\otimes\psi_+\\ \psi_+\otimes\psi_-\end{pmatrix}=\begin{pmatrix}\mathbb{C}_{++} & \mathbb{C}_{+-}\\ \mathbb{C}_{-+} & \mathbb{C}_{--}\end{pmatrix}\begin{pmatrix}\psi_-\otimes\psi_+\\ \psi_+\otimes\psi_-\end{pmatrix} \tag{3.2}$$

and in this particular case we find

$$\mathbb{C}_2^{SO(4,2)}=\widehat{\mathbb{J}}(\widehat{\mathbb{J}}-1),\qquad \widehat{\mathbb{J}}=-\begin{pmatrix}0 & \partial_2 z_{21}\\ \partial_1 z_{12} & 0\end{pmatrix}, \tag{3.3}$$

where $\partial_k=\partial/\partial z_k$ and $z_{12}=z_1-z_2$. Since $\mathbb{H}$ commutes with $\mathbb{C}_2^{SO(4,2)}$, they share the same eigenfunctions $\varphi_n^{\pm}(z_1,z_2)=\begin{pmatrix}1\\ \pm1\end{pmatrix}z_{12}^n$: $\mathbb{C}_2^{SO(4,2)}\varphi_n^+=(n+2)(n+1)\varphi_n^+$, $\mathbb{C}_2^{SO(4,2)}\varphi_n^-=(n+1)n\varphi_n^-$, corresponding to $J = n+1$ and $J = n$, respectively. Finally, as $\mathbb{H}(J)$ is known from the quasipartonic sector, Eq. (3.1), the result for the RG kernel acting on $(+-)$ two-quark operators can now be restored by simple algebra, see [6] for details. This technique is general and allows one to compute all kernels of the type shown Fig. 1b in terms of the standard quasipartonic kernels Fig. 1a. The complete list of kernels in coordinate representation [8] can be found in Sec. 5 of Ref. [6].

4. Particle Number Changing Kernels

It turns out that conformal symmetry is sufficient to determine the particle number changing kernels of the type shown in Fig. 1c, $\mathbb{H}_{12}^{(2\to 3)}$, in terms of quasipartonic BFLK kernels as well, although the procedure is a bit more complicated [6]. The very possibility of this connection is due to the fact that conformal (in fact, Lorentz) transformations do not preserve the number of fields. For example, applying the generator of translations in transverse direction to the light cone to the "plus" quark field one obtains, in the light-cone gauge $A_{++} = 0$,

$$i[\mathbf{P}_{\mu\bar{\lambda}}, \psi_+] = \partial_{\mu\bar{\lambda}}\psi_+ = 2\partial_+\psi_- + igA_{\mu\bar{\lambda}}\psi_+ + \text{ EOM}. \tag{4.1}$$

Eq. (4.1) is an exact operator identity which must be fulfilled by renormalized operators. Therefore, in principle, $\mathbb{H}_{12}^{(2\to 2)}$ and $\mathbb{H}_{12}^{(2\to 3)}$ kernels are related, although one has to find an effective strategy to use these relations in practical calculations. As an illustration, consider the same example as above, the $\psi_-\otimes\psi_+$ operator. We are now interested in the mixing $\psi_-\otimes\psi_+, \psi_+\otimes\psi_- \to \psi_+\otimes\psi_+\otimes\bar{f}_{++}$. Using the notation $\mathscr{O}_{++}(z_1,z_2) = \psi_+(z_1)\otimes\psi_+(z_2)\,, \mathscr{O}_{-+}(z_1,z_2) = \psi_-(z_1)\otimes\psi_+(z_2)\,, \mathscr{O}_{+-}(z_1,z_2) = \psi_+(z_1)\otimes\psi_-(z_2)\,, \mathscr{O}_f(z_1,z_2,z_3) = \psi_+(z_1)\otimes\psi_+(z_2)\otimes\bar{f}_{++}(z_3)$ and taking into account Eq. (4.1) one obtains

$$\begin{aligned}\partial_{\mu\bar{\lambda}}[\mathscr{O}^{ij}_{++}(z_1,z_2)]'_R \;=\; & 2\partial_1[\mathscr{O}^{ij}_{-+}(z_1,z_2)]'_R + 2\partial_2[\mathscr{O}^{ij}_{+-}(z_1,z_2)]'_R + ig(t^b_{ii'}\otimes I_{jj'})[A^b_{\mu\bar{\lambda}}(z_1)\mathscr{O}^{i'j'}_{++}(z_1,z_2)]'_R \\ & + ig(I_{ii'}\otimes t^b_{jj'})[\mathscr{O}^{i'j'}_{++}(z_1,z_2)A^b_{\mu\bar{\lambda}}(z_2)]'_R + \text{EOM},\end{aligned} \tag{4.2}$$

where $[\mathscr{O}]'_R = [\mathscr{O}]_R - \mathscr{O}$. This equation contains two operators $\mathscr{O}_{-+}$, $\mathscr{O}_{+-}$ which we are interested in, and the quasipartonic operators $\mathscr{O}_{++}$, $\mathscr{O}_{++}(z_1,z_2)A^b_{\mu\bar{\lambda}}(z_i)$, renormalization of which is already known. (In the light-cone gauge $(\mu\lambda)\,\bar{f}_{++} = \partial_+ A_{\mu\bar{\lambda}}$.) Note that application of the transverse derivative to the renormalized two-particle operator on the l.h.s. of (4.2) generates (because of Eq. (4.1)) both two-particle and three-particle contributions which have to be taken into account. Let $[\mathscr{O}^{\mp\pm}_k]'_R = \frac{\alpha_s}{4\pi\varepsilon}\mathbb{H}^{\mp\pm}_{\to f}\mathscr{O}_f$ be the three-particle counterterms of interest. After some algebra one obtains, schematically

$$\partial_1\mathbb{H}^{-+}_{\to f} + \partial_2\mathbb{H}^{+-}_{\to f} = \text{ known expression}. \tag{4.3}$$

This is a first-order differential equation on *two* unknown integral operators $\mathbb{H}^{-+}_{\to f}$ and $\mathbb{H}^{+-}_{\to f}$ which map functions of three variables to functions of two variables, and at first sight is not sufficient to determine them uniquely. However, $\mathbb{H}^{-+}_{\to f}$ and $\mathbb{H}^{+-}_{\to f}$ have different transformation properties with respect to the $SL(2,\mathbb{R})$ group. This implies that Eq. (4.3) is not invariant and indeed it decouples in a pair of (simple) $SL(2,\mathbb{R})$ invariant equations, one for each kernel. The final result is

$$[\mathbb{H}^{(-+)}_{\to f}\,\mathscr{O}_f](z_1,z_2) = z_{12}^2\Big\{ f^{abc}t^b\otimes t^c\int_0^1 d\alpha\int_0^{\bar\alpha} d\beta\,\beta\,\mathscr{O}_f(z_{12}^\alpha,z_2,z_{21}^\beta) + i(t^at^b)\otimes t^b\int_0^1 d\alpha\int_{\bar\alpha}^1 d\beta\,\frac{\bar\alpha\bar\beta}{\alpha}\,\mathscr{O}_f(z_{12}^\alpha,z_2,z_{21}^\beta)\Big\}, \quad (4.4)$$

where $\bar\alpha = 1-\alpha$, $z_{12}^\alpha = \bar\alpha z_1 + \alpha z_2$, etc. It coincides with the result of the direct calculation in Ref. [5]. One can check that this expression is manifestly $SL(2)$ invariant. The present approach proves to be very effective, especially for gluon operators. In total there exist 16 independent $2\to 3$ kernels of the type shown in Fig. 1c which are all calculated in Ref. [6], see Sec. 7 therein.

5. Conclusions

Combining the kernels in Fig. 1a,b,c one obtains a complete set of building blocks for the renormalization of operators that involve at most one "non-partonic" field. To the twist-4 accuracy, contributions of the type Fig. 1d,e can be dispensed off using equations of motion. Therefore, the results presented in our work are sufficient for writing down arbitrary QCD evolution equations to the twist-four accuracy and e.g. calculation of the spectrum of anomalous dimensions of arbitrary twist-four operators. The kernels are written for the renormalization of coordinate-space light-ray operators [8] built of chiral fields and are manifestly $SL(2)$ invariant. We believe that this form is most suitable in practical applications. The results can easily be recast in momentum space, in the form of evolution equations for generalized parton distributions [9] . The application of our formalism to the scale-dependence of multiparton correlation functions relevant for the single spin asymmetries is presented in [10] and to the higher-twist nucleon distribution amplitudes in [5].

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