Higgs production in gluon fusion at NNLO for finite top quark mass[‡]

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The evaluation of the top quark mass suppressed terms to the Higgs production cross section in gluon fusion at next-to-next-to-leading order is reported on. In the region below threshold, the Feynman diagrams are evaluated using asymptotic expansions. The result is then matched to the high-energy limit derived from k_T factorization. The result shows that the heavy-top limit used so far approximates the full result to better than 1%.

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1. Introduction

The importance of the gluon fusion mechanism for Higgs production at hadron colliders has been highlighted in the recent past by the first statistically significant exclusion limits of the combined CDF/D0 searches for the Higgs boson at the Tevatron collider [1]. This result depends crucially on the knowledge of higher order radiative corrections. If only the leading order result for the gluon fusion cross section had been taken into account in the analyses, for example, a 95% exclusion would be way out of reach, even for years. The NLO results increase the theoretical prediction by more than 100% [2, 3]. Yet, the sensitivity would still be insufficient to claim exclusion. It is only the NNLO result that allows such a claim.

In this light, it is important to ensure the validity of the theoretical prediction. The NNLO QCD result that goes into the experimental analyses has been evaluated by (more than) three different groups [4, 5, 6]. Also, various studies based on resummation have convincingly shown that we do not have to expect crucially large numerical contributions from QCD beyond NNLO (see, e.g., Refs. [7, 8]).

Concerning the electro-weak corrections, they are found to be below 6% in the relevant Higgs mass range [9]. Unfortunately, threshold effects from virtual W and Z bosons lead to spurious spikes in the range 160-190 GeV which are smoothened by the finite widths of the gauge bosons. Since there is a certain amount of freedom in this procedure (as in any other combination of all-order and fixed-order expressions), it is not completely clear to what extent this reflects in the theoretical uncertainty of this result. Knowing that the pure QCD corrections are large, one may expect that QCD effects further enhance this uncertainty. In fact, an explicit calculation of the mixed electro-weak/QCD effects (albeit in the limit $M_H \ll M_W$) confirms this [10].

Further worries may concern the use of the inclusive (M_H dependent) K-factor in the experimental analysis. However, fully exclusive NNLO calculations for gluon fusion are available and can be used to check the efficiencies [11].

In this proceedings contribution, we will report on works that have addressed another issue which has plagued the NNLO results mentioned before, namely the effects arising from a finite top quark mass.

2. Effective field theory approach

Due to their high complexity, calculations of the gluon fusion process beyond the inclusive NLO cross section were all performed in the so-called *effective field theory (*EFT*) approach*. This means that the six-flavor Lagrangian is replaced by

$$\mathscr{L}_{\text{eff}} = -\frac{1}{4v} C_1 H G_{\mu\nu} G^{\mu\nu} + \mathscr{L}_{\text{QCD}}^{(5)}, \tag{2.1}$$

where $\mathscr{L}_{\text{QCD}}^{(5)}$ is the five-flavor QCD Lagrangian (no top-quark), and $G_{\mu\nu}$ is the QCD field strength tensor. The Wilson coefficient C_1 is known to N^{4}LO [12, 13]. In the EFT approach, the loop-induced gluon-Higgs coupling is thus replaced by a tree-level coupling proportional to C_1.

Clearly, a result derived from Eq. (2.1) cannot be expected to hold beyond the top quark threshold, $M_H > 2M_t$. However, at NLO one observes that the bulk of the top quark mass dependence is

given by the LO cross section. Therefore, whenever we speak of the EFT approach in this paper, we mean the expression

$$\sigma_\infty \equiv \sigma^{(0)}(M_t) \frac{\sigma(M_t \to \infty)}{\sigma^{(0)}(M_t \to \infty)}, \tag{2.2}$$

where $\sigma^{(0)}$ is the leading order term in α_s. Even though the exact NLO result is approximated in the EFT approach to better than 1% below threshold, its validity at NNLO remained a matter of concern. At first sight, an obvious way to check it is to calculate top quark mass suppressed terms to the total cross section and ensure that they do not significantly alter the EFT result. The next section describes the corresponding calculations.

3. Top quark mass suppressed terms

Due to the absence of a gluon-Higgs vertex in the Standard Model, there is no tree-level contribution to $\sigma(gg \to H + X)$. All the corresponding Feynman diagrams contain a closed quark loop that mediates this coupling. The dominant contribution is due to a top quark loop; the bottom loop contribution amounts to only a few percent at LO. In the EFT approach, the top quark is integrated out, resulting in a direct gluon-Higgs coupling as described by the Lagrangian in Eq. (2.1), and the number of loops in the Feynman diagrams reduces by one.

Alternatively to the EFT approach, one can evaluate the Feynman diagrams approximately with the help of the method of *asymptotic expansions* (see, e.g., Ref. [14]). They allow one to obtain a systematic expansion of the relevant partonic cross sections in terms of powers and logarithms of M_H^2/M_t^2. The first term in this expansion will then agree with the result obtained from Eq. (2.1), but higher orders can be obtained in a straightforward manner by increasing the depth of intermediate Taylor expansions.

The method of asymptotic expansions expresses the Feynman diagrams under consideration in terms of products of massive vacuum and massless vertex or box integrals. The former ones are required through three loops and can be evaluated with the help of the FORM [15] program MATAD [16]. The vertex integrals are needed through two loops: in Ref. [17, 18], they were calculated using the method of Ref. [19] as implemented in Ref. [20] (the implementation is based on the FORM version of MINCER [21]). The massless boxes are only needed at the one-loop level and can be calculated by standard methods.

Note that the massless component of the $2 \to 3$ processes is given by tree-level diagrams. However, this class is the most difficult one as far as the phase space integration is concerned. In Ref. [18], these integrals were evaluated in terms of expansions around $\hat{s} = M_H^2$. As will be explained below, this approximation is fully justified in the the approach applied here.

Explicit results for all the partonic cross sections have been presented in Refs. [17, 18, 22, 23].[1]

4. Large-$\hat{s}$ region

The expansion described in Section 3 is obtained by assuming that the top quark mass is the largest mass scale of the physical system. This is, of course, not true in reality, because $\hat{s}$, the

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[1] The virtual terms were obtained through $\mathscr{O}(1/M_t^6)$ and $\mathscr{O}(1/M_t^8)$ in Ref. [17] and Ref. [22], respectively, while the real radiation contributions were obtained through $\mathscr{O}(1/M_t^6)$ in Ref. [18] and $\mathscr{O}(1/M_t^4)$ in Ref. [23].

partonic center-of-mass energy, assumes values up to the hadronic center-of-mass energy s (i.e., 1.96 TeV at the Tevatron, and – hopefully – 14 TeV at the LHC). In fact, this very same issue arises already in the EFT approach. Fortunately, however, the parton luminosity becomes very small at large $\hat{s}$. In fact, at NLO one observes that the hadronic cross section is approximated to better than 90% by neglecting contributions from $\sqrt{\hat{s}} > 2M_t$.

Including higher orders in the $1/M_t$ expansion, however, the problem becomes more severe. The reason is that the expansion of Section 3 generates terms of the form $(\hat{s}/M_t^2)^k$, leading to a power divergence at large $\hat{s}$. By coincidence, at NLO the coefficient of the $k = 1$ term vanishes (for the gg initial state). This observation was used in Ref. [24] to derive an estimate of the top mass suppressed terms at NLO.

The failure of the $1/M_t$ expansion for $\sqrt{\hat{s}} > 2M_t$ is also the reason why the so-called soft expansion around $\hat{s} = M_H^2$ for the phase space integrals mentioned in Section 3 is fully sufficient: within $M_H^2 < \hat{s} < 4M_t^2$, it is expected (and observed) to converge well, while outside this region, the $1/M_t$ expansion breaks down anyway.

At NNLO, we see no reason why the $\hat{s}/M_t^2$ term should vanish as well. In addition, the goal of Refs. [18, 25] was to derive not only an estimate of the top mass effects, but to provide a consistent quantitative approximation of these terms. This could be achieved from an additional piece of information which had recently been evaluated [26], name the true large-$\hat{s}$ limit of the partonic cross sections.

Using this information, an expression for the full partonic cross section that incorporates all known information on the NNLO cross section can be constructed as follows:

$$\hat{\sigma}_{\alpha\beta}^{(n)}(x) = \hat{\sigma}_{\alpha\beta,N}^{(n)}(x) + \sigma_0 A_{\alpha\beta}^{(n)} \left[\ln \frac{1}{x} - \sum_{k=1}^{N} \frac{1}{k}(1-x)^k \right] + (1-x)^{N+1} \left[\sigma_0 B_{\alpha\beta}^{(n)} - \hat{\sigma}_{\alpha\beta,N}^{(n)}(0) \right], \quad (4.1)$$

where $\hat{\sigma}_{\alpha\beta,N}^{(n)}(x)$ denotes the soft expansion of the N^{n}LO partonic cross section for the process $\alpha\beta \to H + X$ through order $(1-x)^N$, where $x = M_H^2/\hat{s}$. The coefficients $A_{\alpha\beta}^{(n)}$ and $B_{\alpha\beta}^{(n)}$ determine the behaviour of the partonic cross section as $x \to 0$. The leading terms at NLO and NNLO (i.e., $A_{\alpha\beta}^{(1)} = 0$, $A_{\alpha\beta}^{(2)}$, and $B_{\alpha\beta}^{(1)}$) for the gg channel were given in the form of numerical tables in Ref. [26], and for the other channels in Ref. [25].

The quality of this approach can be tested at NLO by comparing it to the exact result which is known in numerical form (see, e.g., Ref. [27]). One observes excellent agreement for the gg and the qg channel for the relevant Higgs mass range between 100 and 300 GeV, while the $q\bar{q}$ channel appears to be more problematic. This is due to the fact that the only diagram contributing to this channel vanishes at both small and large x. This leaves room for a relatively pronounced structure at threshold which cannot be described properly by our approach. However, the $q\bar{q}$ channel is down by almost three orders of magnitude relative to the gg channel.

At NNLO, the unknown constants $B_{\alpha\beta}^{(2)}$ introduced a certain amount of uncertainty to the prediction. In Ref. [25] it was estimated to be of the order of 1%, where also more detailed studies of the partonic cross sections can be found.

5. Hadronic cross section

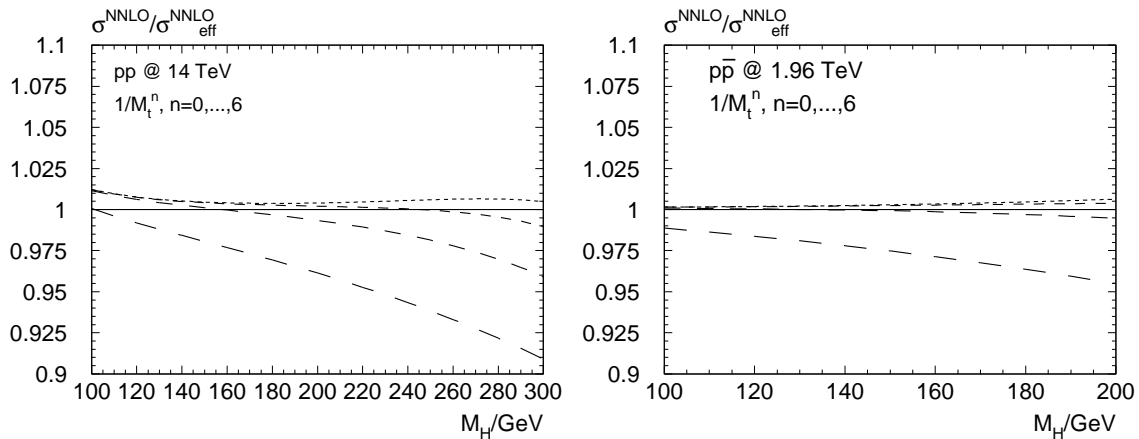The hadronic cross section is obtained by integrating the partonic expression from Eq. (4.1)

Figure 1: Ratio of the hadronic cross section as obtained from Eq. (4.1) to the EFT result (see Eqs. (2.1) and (2.2)). The various lines correspond to keeping different orders in $1/M_t$ in the numerator σ^{NNLO} (decreasing dash-length corresponds to increasing order in $1/M_t$). From Ref. [25].

over the parton densities (we use MSTW2008 [28]). The most important question is how well the EFT approximation (i.e., keeping the leading term in the $1/M_t$ expansion and factoring out the full mass dependent result at LO in α_s) describes the top quark mass effects. We therefore show in Fig. 1 the ratio of our result to the EFT approach, both for the LHC and the Tevatron. The agreement in both cases is better than 1% which is well below the current estimated theoretical uncertainty due to higher orders in α_s and PDF variations.

This is a very comforting result since meanwhile a large number of theoretical and experimental studies have been performed based on the EFT approach.

6. Conclusions

The quality of the heavy-top limit used in numerous studies and calculations for Higgs production in gluon fusion has been scrutinized by an explicit calculation of the top mass suppressed terms. The result was derived from asymptotic expansions of the relevant Feynman diagrams and the combination with the high-energy limit obtained from k_T factorization. The result justifies the use of the effective theory approach to a very high degree, at least for the inclusive cross section. It remains to be seen how this result carries over to less inclusive quantities or phase space restrictions.

Finally, let us point out that a similar, independent calculation [23, 22] was presented also by A. Pak at this conference (see these proceedings).

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