NNLO antenna subtraction with two hadronic initial states

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We discuss the extension of the antenna subtraction method to include two hadrons in the initial state (initial-initial antennae) at next-to-next-to-leading order. We sketch the construction of the subtraction terms and the required phase space transformations. We discuss the integration of the subtraction terms in detail.

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1. Introduction

Jet observables are an important tool for precision studies due to their large production cross sections at high energy colliders. Calculating perturbative higher order corrections to jet production cross sections requires a systematic procedure to extract infrared singularities from real radiation contributions. The latter arise if one or more final state particles become soft or collinear. At the next-to-leading order level (NLO), several systematic and process-independent procedures are available . The two main methods are phase space slicing [1,2] and subtraction based methods [3,4]. All subtraction methods consist in introducing terms that are subtracted from the real radiation part at each phase space point . These subtraction terms approximate the matrix element in all singular limits, and are sufficiently simple to be integrated over the corresponding phase space analytically . After this integration, the infrared divergences of the subtraction terms become explicit and the integrated subtraction terms can be added to the virtual corrections yielding an infrared finite result . Since experimental data of jet observables are reaching an accuracy of a few percent or better, accurate precision studies must rely on theoretical predictions that have the same precision . In some cases, this requires corrections at the next-to-next-to-leading order (NNLO) level in perturbative QCD .

NNLO calculations of observables with n jets in the final state require several ingredients: the two-loop correction to n-parton matrix elements, the one-loop correction to $(n+1)$-parton matrix elements, and the tree-level $(n+2)$-parton matrix elements. For most massless jet observables of phenomenological interest, the two-loop matrix elements have been computed some time ago, while the other two types of matrix elements are usually known from calculations of NLO corrections to $(n+1)$ jet production [5] . At the NNLO level, n jet observables receive contributions from the one-loop $(n+1)$-parton matrix elements, where one of the involved partons can become unresolved (soft or collinear), as well as from $(n+2)$-parton matrix elements where up to two partons can become simultaneously soft and/or collinear . In order to determine the contribution to NNLO jet observables from these configurations, two-parton subtraction terms have to be constructed . Several NNLO subtraction methods have been proposed in the literature [6–10] . Another approach used for NNLO calculations of exclusive observables is sector decomposition [11] .

In ref. [12], an NNLO subtraction method was developed for observables with partons in the final state only, the antenna subtraction method. It constructs the subtraction terms from the so-called antenna functions. The latter describe all unresolved partonic radiation between a hard pair of colour-ordered partons, the radiators. The antenna functions are derived systematically from physical matrix elements and can be integrated over their factorized phase space. At the NLO level, this formalism can handle massless partons in the initial or final states [15], as well as massive fermions in the final state [13]. For processes with initial-state hadrons, NNLO antenna subtraction terms have to be constructed for two different cases: only one radiator parton is in the initial state (initial-final antenna) or both radiator partons are in the initial state (initial-initial antenna). Recently, in ref. [14], NNLO initial-final antenna functions were derived and integrated over their factorized phase space . The case with two radiators in the initial state is however still outstanding .

In this contribution, we discuss the derivation of NNLO initial-initial antenna functions. We briefly describe the construction of the subtraction terms and the required phase space transformations and discuss how the phase space integrals for initial-initial antennae can be performed using multi-loop techniques .

2. Subtraction terms for initial-initial configurations

At NNLO, there are two types of contributions to m-jet observables that require subtraction: the tree-level $m+2$ parton matrix elements (where one or two partons can become unresolved), and the one-loop $m+1$ parton matrix elements (where one parton can become unresolved). In the tree-level double real radiation case, we can distinguish four different types of unresolved configurations depending on how the unresolved partons are colour connected to the emitting hard partons (see ref. [12] for a detailed description of the four cases). In this contribution, we focus on the case with two colour-connected unresolved partons (colour-connected). This is the only case where new ingredients are needed, namely the four-parton initial-initial antenna functions. The unintegrated ones can be obtained by crossing two partons to the initial state in the corresponding final-final antenna functions, which can be found in [12], and have then to be integrated analytically over the appropriate antenna phase space . The corresponding NNLO antenna subtraction term, to be convoluted with the appropriate parton distribution functions for the initial state partons, for a configuration with the two hard emitters in the initial state (partons i and l with momenta p_1 and p_2) can be written as:

$$
\begin{aligned}
\mathrm{d}\sigma_{NNLO}^{S,colour-connected} = \mathcal{N} \sum_{m+2} \mathrm{d}\Phi_{m+2}(k_1,\ldots,k_{m+2};p_1,p_2)\frac{1}{S_{m+2}} \\
\times \left[\sum_{jk} \left(X_{il,jk}^0 - X_{i,jk}^0 X_{Il,K}^0 - X_{l,kj}^0 X_{iL,J}^0 \right) \right. \\
\left. \times |\mathcal{M}_m(K_1,\ldots,K_L,\ldots,K_{m+2};x_1p_1,x_2p_2)|^2 J_m^{(m)}(K_1,\ldots,K_L,\ldots,K_{m+2}) \right].(2.1)
\end{aligned}
$$

The subtraction term in eq. (2.1) is constructed such that all unresolved limits of the four-parton antenna function $X_{il,jk}^0$ are subtracted, so that the resulting subtraction term is active only in its double unresolved limits, which explains the presence of the products of three-parton antennae . The subtraction terms for all the other unresolved configurations can be constructed using tree-level three-parton antenna functions. In eq. (2.1), the tree antennae $X_{il,jk}^0$, $X_{i,jk}^0$ and $X_{l,jk}^0$ depend on the original momenta p_1, p_2, k_j, k_k, whereas the rest of the antenna functions as well as the jet function J and the reduced matrix elements $\mathcal{M}_m$ depend on the redefined momenta from the phase space mapping, labeled by $I, J \ldots$. In addition to that, the reduced matrix elements depend on the momentum fractions x_1 and x_2, which we define later . The normalization factor $\mathcal{N}$ includes all QCD-independent factors as well as the dependence on the renormalized QCD coupling α_s, $\sum_{m+2}$ denotes the sum over all configurations with $m+2$ partons, $\mathrm{d}\Phi_{m+2}$ is the phase space for an $(m+2)$-parton final state in $d = 4-2\varepsilon$, and finally, S_{m+2} is a symmetry factor for identical partons in the final state . The antenna functions can be integrated analytically, provided we have a suitable factorization of the phase space . The factorization is possible through an appropriate mapping of

the original set of momenta. These mappings interpolate between the different soft and collinear limits that the subtraction term regulates. They must satisfy overall momentum conservation and keep the mapped momenta on the mass shell.

A complete factorisation of the phase space into a convolution of an m particle phase space depending on redefined momenta only, with the phase space of partons j, k, can be achieved with a Lorentz boost that maps the momentum $q = p_1 + p_2 - k_j - k_k$, with $q^2 > 0$, into the momentum $\tilde{q} = x_1 p_1 + x_2 p_2$, where $x_{1,2}$ are fixed in terms of the invariants as follows [15]:

$$
\begin{aligned}
x_1 &= \left(\frac{s_{12} - s_{j2} - s_{k2}}{s_{12}} \frac{s_{12} - s_{1j} - s_{1k} - s_{j2} - s_{k2} + s_{jk}}{s_{12} - s_{1j} - s_{1k}} \right)^{\frac{1}{2}}, \\
x_2 &= \left(\frac{s_{12} - s_{1j} - s_{1k}}{s_{12}} \frac{s_{12} - s_{1j} - s_{1k} - s_{j2} - s_{k2} + s_{jk}}{s_{12} - s_{j2} - s_{k2}} \right)^{\frac{1}{2}}.
\end{aligned}
\tag{2.2}
$$

These last two definitions guarantee the overall momentum conservation in the mapped momenta and the right soft and collinear behavior. The two momentum fractions satisfy the following limits in double unresolved configurations:

1. j and k soft: $x_1 \to 1, x_2 \to 1$,

2. j soft and $k_k = z_1 p_1$: $x_1 \to 1 - z_1, x_2 \to 1$,

3. $k_j = z_1 p_1$ and $k_k = z_2 p_2$: $x_1 \to 1 - z_1, x_2 \to 1 - z_2$,

4. $k_j + k_k = z_1 p_1$: $x_1 \to 1 - z_1, x_2 \to 1$,

and all the limits obtained from the ones above by the exchange of p_1 with p_2 and of k_j with k_k. The factorized $(m+2)$-partons phase space into an m-partons phase space and an antenna phase space is given by:

$$
\begin{aligned}
\mathrm{d}\Phi_{m+2}(k_1, \ldots, k_{m+2}; p_1, p_2) = {} & \mathrm{d}\Phi_m(K_1, \ldots, K_{j-1}, K_{j+1}, \ldots, K_{k-1}, K_{k+1}, \ldots, K_{m+2}; x_1 p_1, x_2 p_2) \\
& \times \mathscr{J} \, \delta(q^2 - x_1 x_2 s_{12}) \, \delta(2 (x_2 p_2 - x_1 p_1).q) \\
& \times [\mathrm{d}k_j] \, [\mathrm{d}k_k] \, \mathrm{d}x_1 \, \mathrm{d}x_2,
\end{aligned}
\tag{2.3}
$$

where $[\mathrm{d}k] = \mathrm{d}^d k / (2\pi)^{(d-1)} \delta^+(k^2)$, and $\mathscr{J}$ is the Jacobian factor defined by

$$
\mathscr{J} = s_{12} \left(x_1 (s_{12} - s_{1j} - s_{1k}) + x_2 (s_{12} - s_{2j} - s_{2k}) \right).
$$

The next step is to integrate the antenna functions over their factorized phase space.

3. Calculational approach for the double real radiation case $2 \to 3$

All the initial-initial antennae have the scattering kinematics $p_1 + p_2 \to k_j + k_k + q$, where q is the momentum of the outgoing particle, for example the vector boson in a vector boson plus jet process. Double real radiation antenna integrals are derived from squared matrix elements and can be represented by forward scattering diagrams as in the following figure:
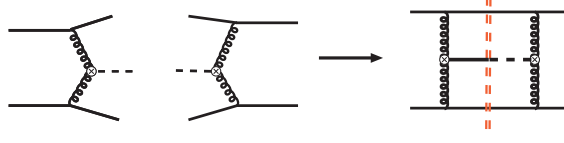
The two delta functions in eq.(2.3) can be represented as mass-shell conditions of fake particles and are shown in the previous picture as a thick solid line (representing a massive particle with mass $M = x_1 x_2 s_{12}$) and a dashed line (representing a massless particle) . This allows us to use the optical theorem to transform the initial-initial antenna phase space integrals into cut two-loop box integrals and, therefore, use the methods developed for multi-loop calculations [16, 17] . Up to 8-propagator integrals with 4 cut propagators are generated in this way . The calculation of the integrated antennae corresponds here to the evaluation of a reduced set of master integrals . We found 30 of them, obtained using integration-by-part (IBP) and Lorentz identities, following the Laporta algorithm . We then calculate this small set of integrals using the method of differential equations . The simplest master integral is the two loop box with all the internal lines cut and defined as follows

 $= I(x_1, x_2) = \int d^d q\, d^d k_j\, d^d k_k\, \delta^d\, (p_1 + p_2 - q - k_j - k_k) \times$

$$\delta^+ \left(k_j^2 \right)\, \delta^+ \left(k_k^2 \right)\, \delta^+ \left(q^2 - M^2 \right)\, \delta(2\, (x_2 p_2 - x_1 p_1).q)\,. \quad (3.1)$$

As we have discussed in section 2, the phase space integrals (and therefore the master integrals) have to be studied in four different regions of the phase space depending on the values of x_1 and x_2, namely:

- $x_1 \neq 1$, $x_2 \neq 1$, we refer to this region as the hard one

- $x_1 = 1$, $x_2 \neq 1$, and $x_1 \neq 1$, $x_2 = 1$, referred to as the collinear region

- $x_1 = 1$, $x_2 = 1$, is the soft region .

In the hard region , the solution of the system of differential equations yields two-dimensional generalized harmonic polylogarithms . The ε expansion is needed up to transcendentality 2 . In the collinear regions, additional $1/\varepsilon$ coefficients may be generated and the epsilon expansion is done up to transcendentality 3, whereas in the soft region additional $1/\varepsilon^2$ coefficients may appear and the expansion in epsilon is pushed to transcendentality 4 . We note however that the calculation of the masters in the soft and collinear regions, although needed with deeper expansions in ε, is simpler than in the hard region, and only one-dimensional harmonic polylogarithms are needed in the collinear regions . In the soft region, a direct calculation is possible giving closed form results in ε in the form of gamma functions . The boundary conditions for the differential equations are obtained, in most of the cases, by studying the master integrals in one of the collinear limits . Otherwise the soft limit is used .

In a first step towards the calculation of all the integrated initial-initial antennae for the $2 \to 3$ tree-level double real radiation case, we have focused on all the crossings of two partons from the following final-final antennae: $B_4^0(q,q',\bar{q}',\bar{q})$, $\tilde{E}_4^0(q,q',\bar{q}',g)$ and $H_4^0(q,\bar{q},q',\bar{q}')$ defined in [12], where the index 4 refers to four partons. There are 13 master integrals involved in their calculation, and the ones without irreducible scalar products are shown in Fig. 1.
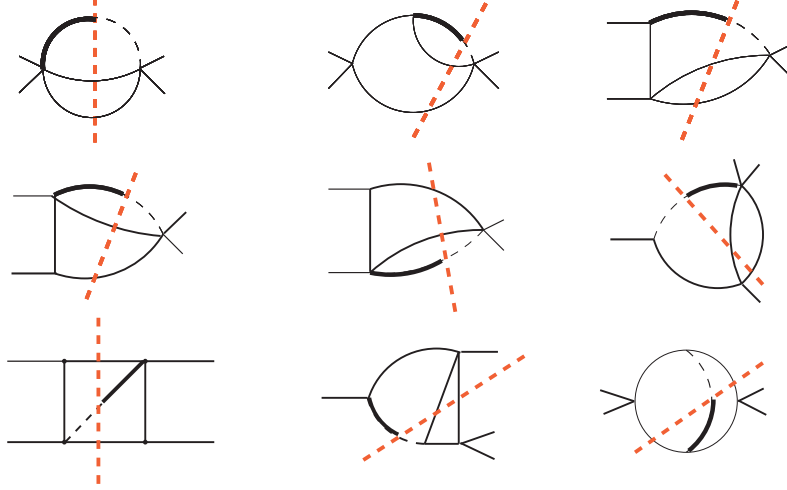

Figure 1: Master integrals for the phase space integration of the tree-level initial-initial B_4^0, H_4^0 and $\tilde{E}_4^0$ type antennae at NNLO. Thick solid and dashed lines refer to the conditions on the phase space integral implemented as auxiliary propagators. All the internal lines are massless except for the thick solid line. Only the integrals without numerators are shown in this picture.

Finally, the one-loop $2 \to 2$ antenna functions do not present any difficulty, since the needed one-loop box integrals are known analytically to all orders in ε for the final-final configuration [18]. Obtaining the initial-initial contribution using these results requires crossing two legs to the initial state and performing the necessary analytic continuation of the involved hypergeometric functions. No integrals are required in this case.

4. Outlook

In this contribution, we have discussed the extension of the antenna subtraction formalism to the initial-initial configurations, including the required phase space factorisation and mappings. We have focused on the $2 \to 3$ tree-level double real radiation contribution. In a first step towards the derivation of the complete set of integrated initial-initial antennae, we considered all the crossings of the subset of 4-parton antennae: $B_4^0(q,q',\bar{q}',\bar{q})$, $\tilde{E}_4^0(q,q',\bar{q}',g)$ and $H_4^0(q,\bar{q},q',\bar{q}')$. Completing the full set of NNLO antenna functions will allow the construction of subtraction terms needed for the evaluation of jet observables at hadron colliders.

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