

## NNLO QCD correction to vector boson production at hadron colliders

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We present a fully-exclusive next-to-next-to-leading order (NNLO) QCD calculation for vector boson production in hadron-hadron collisions. The calculation is implemented in a parton level Monte Carlo program, which includes  $\gamma - Z$  interference, finite-width effects, the leptonic decay of the vector bosons and the corresponding spin correlations. The code allows the user to apply arbitrary (though infrared safe) kinematical cuts on the final-states and to compute distributions in the form of bin histograms. We show some illustrative numerical results at the Tevatron and the LHC.

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Vector boson production in hadron collisions, the well known Drell-Yan (DY) process[1], has a special role for physics studies at hadron colliders. Having large production rates with relatively simple experimental signatures, this process is important for detectors calibration, it gives stringent informations on Parton Distribution Functions (PDFs), is a strong test for perturbative QCD predictions and may signals effects from physics beyond the Standard Model.

It is therefore essential to have accurate theoretical predictions for the vector-boson production cross sections and related distributions, which are predicted by perturbative QCD as an expansion in the strong coupling  $\alpha_s$ . The next-to-next-to-leading order (NNLO) QCD corrections (i.e.  $\mathcal{O}(\alpha_s^2)$ ) have been calculated analytically for the total cross section [2] and the rapidity distribution of the vector boson [3]. The fully exclusive NNLO calculation has also been performed [4, 5]. Furthermore, electroweak corrections up to  $\mathcal{O}(\alpha)$  have been computed for both  $W$  [6] and  $Z$  production [7].

The computation of higher-order QCD corrections to hard-scattering processes is a hard task. Difficulties arise from the presence of infrared singularities at intermediate stages of the calculation that prevent a straightforward implementation of numerical techniques. For the above reason, fully exclusive cross-sections in hadron collisions have been computed so far only for Higgs boson production [8, 9, 10, 11] and the Drell–Yan process [4, 5].

In this contribution we present a recent fully exclusive NNLO QCD calculation for vector boson production in hadron collisions [5]. The calculation uses the NNLO extension of subtraction formalism introduced in Ref. [10]. The method is valid in general for the production of colourless high-mass systems in hadron collisions.

We consider the hard-scattering process:

$$h_1 + h_2 \rightarrow V(q) + X, \quad (1)$$

where the colliding hadrons  $h_1$  and  $h_2$  produce the vector boson  $V$  ( $V = Z/\gamma^*, W^+$  or  $W^-$ ), with four-momentum  $q$  and high invariant mass  $\sqrt{q^2}$ , plus an inclusive final state  $X$ .

Following Ref. [10], we observe that, at LO, the transverse momentum  $q_T$  of  $V$  is exactly zero. This means that, as long as  $q_T \neq 0$ , the (N)NLO contributions are given by the (N)LO contributions to the final state  $V + jet(s)$  [12]:

$$d\hat{\sigma}_{(N)NLO}^V|_{q_T \neq 0} = d\hat{\sigma}_{(N)LO}^{V+jets}. \quad (2)$$

We compute  $d\hat{\sigma}_{NLO}^{V+jets}$  by using the subtraction method at NLO [13, 14] and we treat the remaining NNLO singularities at  $q_T = 0$  by the additional subtraction of an universal <sup>1</sup> counter-term  $d\hat{\sigma}_{(N)LO}^{CT}$  constructed by exploiting the universality of the logarithmically-enhanced contributions to the transverse momentum distribution <sup>2</sup>. Schematically we have

$$d\hat{\sigma}_{(N)NLO}^V = \mathcal{H}_{(N)NLO}^V \otimes d\hat{\sigma}_{LO}^V + \left[ d\hat{\sigma}_{(N)LO}^{V+jets} - d\hat{\sigma}_{(N)LO}^{CT} \right], \quad (3)$$

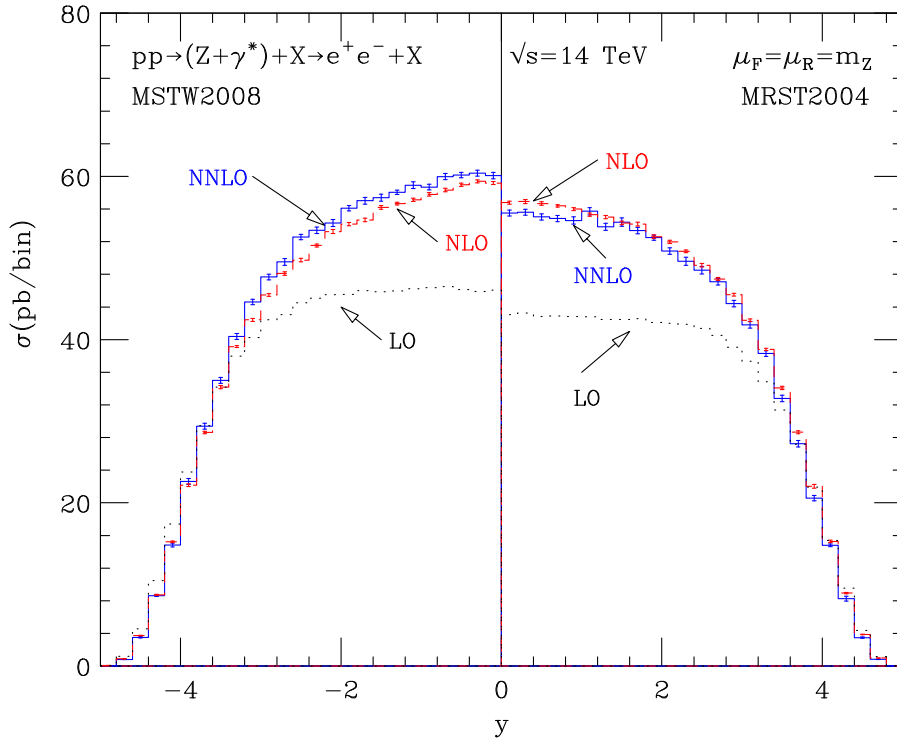
where  $\mathcal{H}_{(N)NLO}^V$  is a process-dependent coefficient function necessary to reproduce the correct normalization [16, 5].

<sup>1</sup>It depends only on the flavour of the initial-state partons involved in the LO partonic subprocess.

<sup>2</sup>For the explicit form of the counter-term see Refs.[10, 15].

We have encoded our NNLO computation in a parton level Monte Carlo event generator. The calculation includes finite-width effects, the  $\gamma - Z$  interference, the leptonic decay of the vector bosons and the corresponding spin correlations. Our numerical code is particularly suitable for the computation of distributions in the form of bin histograms.

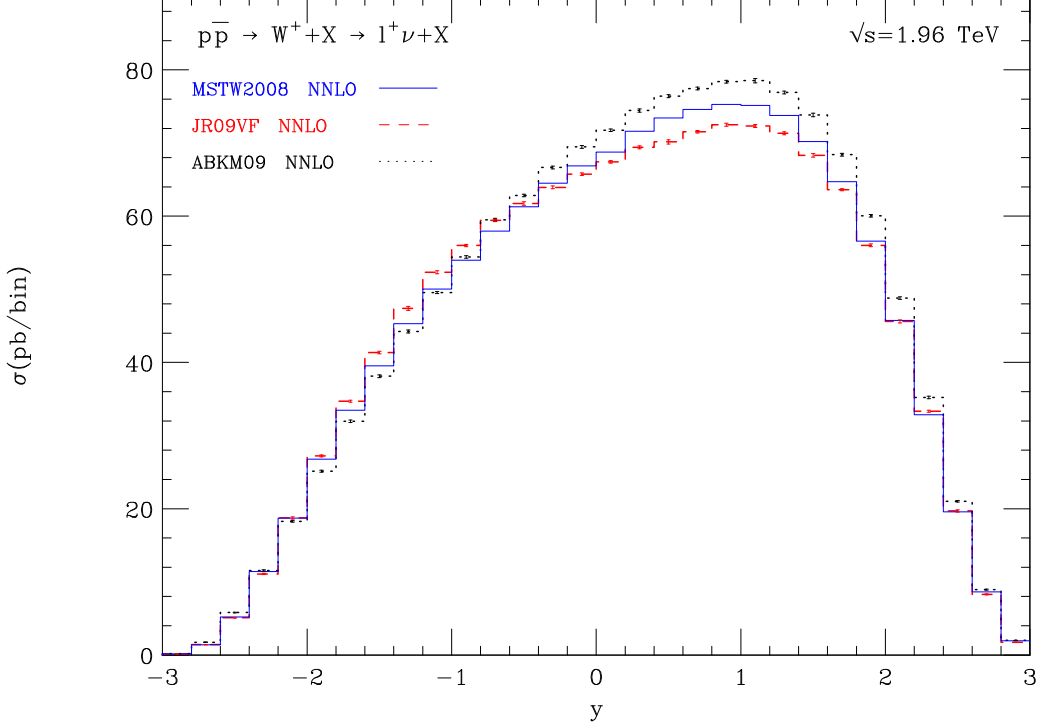
In the following we present some illustrative numerical results for  $Z$  and  $W$  production at the Tevatron and the LHC. We consider  $u, d, s, c, b$  quarks in the initial state. In the case of  $W^\pm$  production, we use the (unitarity constrained) CKM matrix elements  $V_{ud} = 0.97419$ ,  $V_{us} = 0.2257$ ,  $V_{ub} = 0.00359$ ,  $V_{cd} = 0.2256$ ,  $V_{cs} = 0.97334$ ,  $V_{cb} = 0.0415$  from the PDG 2008 [17]. We use the so called  $G_\mu$  scheme for the electroweak couplings, with the following input parameters:  $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$ ,  $m_Z = 91.1876 \text{ GeV}$ ,  $\Gamma_Z = 2.4952 \text{ GeV}$ ,  $m_W = 80.398 \text{ GeV}$  and  $\Gamma_W = 2.141 \text{ GeV}$ . As for the PDFs, we use MSTW2008 [18] as default set, evaluating  $\alpha_S$  at each corresponding order (i.e., we use  $(n+1)$ -loop  $\alpha_S$  at  $N^n\text{LO}$ , with  $n = 0, 1, 2$ ). We fix the renormalization ( $\mu_R$ ) and factorization ( $\mu_F$ ) scales to the mass of the vector boson  $m_V$ .



**Figure 1:** Rapidity distribution of an on-shell  $Z$  boson at the LHC. Results obtained with the MSTW2008 set (left panel) are compared with those obtained with the MRST2004 set (right panel).

We start the presentation of our results by considering the inclusive production of  $e^+e^-$  pairs from the decay of an on-shell  $Z$  boson at the LHC. In the left panel of Fig. 1 we show the rapidity distribution of the  $e^+e^-$  pair at LO, NLO and NNLO, computed by using the MSTW2008 PDFs [18]. The corresponding cross sections are  $\sigma_{LO} = 1.761 \pm 0.001 \text{ nb}$ ,  $\sigma_{NLO} = 2.030 \pm 0.001 \text{ nb}$  and  $\sigma_{NNLO} = 2.089 \pm 0.003 \text{ nb}$ . The total cross section is increased by about 3% in going from NLO to NNLO. In the right panel of Fig. 1 we show the results obtained by using the MRST2002 LO

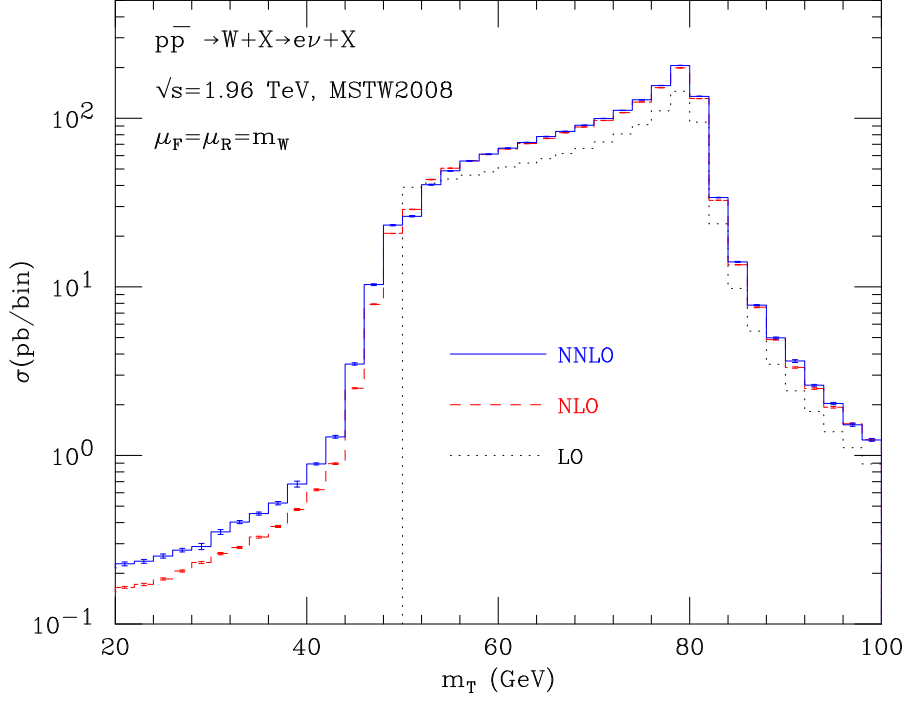
[19] and MRST2004 [20] sets of parton distribution functions. The corresponding cross sections are  $\sigma_{LO} = 1.629 \pm 0.001$  nb,  $\sigma_{NLO} = 1.992 \pm 0.001$  nb and  $\sigma_{NNLO} = 1.954 \pm 0.003$  nb. In this case the total cross section is decreased by about 2% in going from NLO to NNLO.



**Figure 2:** Rapidity distribution of an on-shell  $W^+$  boson at the Tevatron. The NNLO result obtained with the MSTW2008 set are compared with those obtained with the JR09VF and ABKM09 sets.

We next consider the production of an on-shell  $W^+$  boson at the Tevatron. In Fig. 2 we show the rapidity distribution of the  $W^+$  at NNLO, computed by using the MSTW2008 PDFs [18]. We also show, for comparison, the NNLO prediction by using the JR09VF [21] and ABKM09 [22] PDFs. The corresponding total cross sections are  $\sigma_{NNLO}^{(MSTW)} = 1.349 \pm 0.002$  nb,  $\sigma_{NNLO}^{(JRVF)} = 1.338 \pm 0.002$  nb and  $\sigma_{NNLO}^{(ABKM)} = 1.391 \pm 0.002$  nb. The differences between the three results can be reach, in the central rapidity region, the level of about 5%.

We finally consider the production of a charged lepton plus missing  $p_T$  through the decay of a  $W$  boson ( $W = W^+, W^-$ ) at the Tevatron. The charged lepton is selected to have  $p_T > 20$  GeV and  $|\eta| < 2$  and the missing  $p_T$  of the event is required to be larger than 25 GeV. The transverse mass of the event is defined as  $m_T = \sqrt{2p_T^l p_T^{\text{miss}}(1 - \cos \phi)}$ , where  $\phi$  is the angle between the the  $p_T$  of the lepton and the missing  $p_T$ . In Fig. 3 we show the transverse mass distribution at LO, NLO and NNLO: the accepted cross sections are  $\sigma_{LO} = 1.161 \pm 0.001$  nb,  $\sigma_{NLO} = 1.550 \pm 0.001$  nb and  $\sigma_{NNLO} = 1.586 \pm 0.002$  nb. Since at LO the  $W$  boson is produced with zero transverse momentum, the requirement  $p_T^{\text{miss}} > 25$  GeV sets  $m_T \geq 50$  GeV. As a consequence at LO the transverse mass distribution has a kinematical boundary at  $m_T = 50$  GeV. Around this boundary there are perturbative instabilities due to (integrable) logarithmic singularities [23]. We also note



**Figure 3:** Transverse mass distribution for  $W$  production at the Tevatron.

that, below the boundary, the NNLO corrections to the NLO result are large. This is not unexpected, since in this region of transverse masses, the  $\mathcal{O}(\alpha_S)$  result corresponds to the calculation at the first perturbative order and, therefore, our  $\mathcal{O}(\alpha_S^2)$  result is actually only a calculation at the NLO level of perturbative accuracy.

We have presented a fully exclusive NNLO QCD calculation for vector boson production in hadron-hadron collisions. Our calculation is directly implemented in a parton level event generator. This feature makes it particularly suitable for practical applications to the computation of distributions in the form of bin histograms. For illustrative purpose, we have shown some selected numerical distributions at the Tevatron and the LHC.

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