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The Electroweak Model based on the Nonlinearly realized Gauge Group.
Theoretical Foundations and Phenomenological Prospects.

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A consistent strategy for the subtraction of the divergences in the nonlinearly realized Electroweak Model in the loop expansion is presented. No Higgs field enters into the perturbative spectrum. The local functional equation (LFE), encoding the invariance of the SU(2) Haar measure under local left SU(2) transformations, the Slavnov-Taylor identity, required in order to fulfill physical unitarity, and the Landau gauge equation hold in the nonlinearly realized theory. The quantization is performed in the Landau gauge for the sake of simplicity and elegance. The constraints on the admissible interactions arising from the Weak Power-Counting (WPC) are discussed. The same symmetric pattern of the couplings as in the Standard Model is shown to arise, as a consequence of the defining functional identities and the WPC. However, two independent mass invariants in the vector meson sector are possible, i.e. no tree-level Weinberg relation holds between the Z and W mass. Majorana neutrino masses can be implemented in the nonlinearly realized Electroweak Model in a way compatible with the WPC and all the symmetries of the theory.

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1. Symmetric Subtraction of Nonlinearly Realized Gauge Theories

A consistent strategy for the subtraction of nonlinearly realized gauge theories order by order in the loop expansion has been recently proposed in [1]-[7]. The discovery of the Local Functional Equation [8], encoding the invariance of the SU(2) path-integral Haar measure under local left SU(2) transformations, has provided a key tool in order to tame the divergences of this class of theories. The LFE uniquely fixes the dependence of the 1-PI amplitudes involving at least one Goldstone field (descendant amplitudes) in terms of 1-PI amplitudes with no external Goldstone legs (ancestor amplitudes). This establishes a very powerful hierarchy among 1-PI Green functions. While there is an infinite number of divergent descendant amplitudes already at one loop level, only a finite number of ancestor amplitudes exists order by order in the loop expansion if the Weak Power-Counting (WPC) condition is fulfilled [2], [3], [7], [9]. In addition to the LFE, the Slavnov-Taylor (ST) identity must be imposed in order to fulfill the requirement of Physical Unitarity [10]. It should be noted that the ST identity does not yield a hierarchy among 1-PI Green functions [7]. Thus the LFE provides an essential tool in order to carry out the consistent subtraction of nonlinearly realized gauge theories.

The WPC poses stringent constraints on the admissible terms in the tree-level vertex functional. In order to work out these constraints a convenient strategy is first to perform an invertible change of variables from the original ones to their corresponding SU(2) gauge-invariant counterparts (bleached variables). We describe the procedure for the nonlinearly realized Electroweak theory. The field content includes (leaving aside the ghosts and the Nakanishi-Lautrup fields) the $SU(2)_L$ connection $A_\mu = A_{a\mu}\frac{\tau_a}{2}$ (τ_a, $a=1,2,3$ are the Pauli matrices), the $U(1)$ connection B_μ, the fermionic left doublets collectively denoted by L and the right singlets, i.e.

$$L \in \left\{ \begin{pmatrix} l^u_{Lj} \\ l^d_{Lj} \end{pmatrix}, \begin{pmatrix} q^u_{Lj} \\ V_{jk} q^d_{Lk} \end{pmatrix}, \quad j,k = 1,2,3 \right\},$$
$$R \in \left\{ \begin{pmatrix} l^u_{Rj} \\ l^d_{Rj} \end{pmatrix}, \begin{pmatrix} q^u_{Rj} \\ q^d_{Rj} \end{pmatrix}, \quad j = 1,2,3 \right\}. \tag{1.1}$$

In the above equation the quark fields $(q^u_j, j=1,2,3) = (u,c,t)$ and $(q^d_j, j=1,2,3) = (d,s,b)$ are taken to be the mass eigenstates in the tree-level lagrangian; V_{jk} is the CKM matrix. Similarly we use for the leptons the notation $(l^u_j, j=1,2,3) = (\nu_e, \nu_\mu, \nu_\tau)$ and $(l^d_j, j=1,2,3) = (e,\mu,\tau)$. The single left doublets are denoted by L^l_j, $j=1,2,3$ for the leptons, L^q_j, $j=1,2,3$ for the quarks. Color indexes are not displayed.

One also introduces the $SU(2)$ matrix Ω

$$\Omega = \frac{1}{v}(\phi_0 + i\phi_a\tau_a)\,, \quad \Omega^\dagger\Omega = 1 \Rightarrow \phi_0^2 + \phi_a^2 = v^2\,. \tag{1.2}$$

The mass scale v gives ϕ the canonical dimension at $D=4$. We fix the direction of Spontaneous Symmetry Breaking by imposing the tree-level constraint

$$\phi_0 = \sqrt{v^2 - \phi_a^2}\,. \tag{1.3}$$

The $SU(2)$ flat connection is defined by

$$F_\mu = i\Omega\partial_\mu\Omega^\dagger\,. \tag{1.4}$$

The local $SU(2)_L$ transformations act on Ω and L on the left, A_μ and F_μ are $SU(2)_L$ connections while R and B_μ are invariant under $SU(2)_L$. Under local $U(1)_R$ transformations one has

$$\begin{aligned} &\Omega' = \Omega V^\dagger\,, && B'_\mu = B_\mu + \tfrac{1}{g'}\partial_\mu \alpha\,, \\ &A'_\mu = A_\mu\,, && L' = \exp(i\tfrac{\alpha}{2}Y_L)L\,, \\ &F'_\mu = F_\mu + i\Omega V^\dagger \partial_\mu V \Omega\,, && R' = \exp(i\tfrac{\alpha}{2}(Y_L+\tau_3))R\,. \end{aligned} \tag{1.5}$$

where $V(\alpha) = \exp(i\alpha\frac{\tau_3}{2})$. The electric charge is defined according to the Gell-Mann-Nishijima relation

$$Q = I_3 + Y\,, \tag{1.6}$$

where the hypercharge operator Y is the generator of the $U(1)_R$ transformations (1.5) and $I_3 = \frac{\tau_3}{2}$ is the third component of the weak isospin. The introduction of the matrix Ω allows to perform an invertible change of variables from the original set of fields to a new set of $SU(2)_L$-invariant ones (bleaching procedure). For that purpose we define

$$\begin{aligned} &w_\mu = w_{a\mu}\frac{\tau_a}{2} = g\Omega^\dagger A_\mu \Omega - g' B_\mu \frac{\tau_3}{2} + i\Omega^\dagger \partial_\mu \Omega\,, \\ &\tilde{L} = \Omega^\dagger L\,. \end{aligned} \tag{1.7}$$

Both w_μ and $\tilde{L}$ are $SU(2)_L$-invariant, while under $U(1)_R$ they transform as

$$w'_\mu = V w_\mu V^\dagger\,, \quad \tilde{L}' = \exp(i\frac{\alpha}{2}(\tau_3 + Y_L))\tilde{L}\,. \tag{1.8}$$

Since the bleached variables are SU(2)-invariant, the hypercharge generator coincides on them with the electric charge. Therefore any electrically neutral local monomial depending on the bleached variables and covariant derivatives w.r.t. the U(1) gauge connection B_μ is allowed on symmetry grounds.

2. The Weak Power-Counting

We further impose the validity of the WPC condition, i.e. the superficial degree of divergence of any 1-PI graph $\mathscr{G}$ with N_A gauge boson external legs and N_F fermionic legs must be bounded by

$$d(\mathscr{G}) = (D-2)n + 2 - N_A - N_F\,, \tag{2.1}$$

where D is the space-time dimension and n is the number of loops. Several comments are in order here. First of all in $D=4$ we see that the number of ancestor divergent amplitudes compatible with the bound (2.1) increases with the loop order n. Therefore we refer to formula (2.1) as the *weak* power-counting condition. Moreover from eq.(2.1) one also sees that the number of divergent ancestor amplitudes is finite at every loop order. We also notice that the UV dimension of the fermion fields in one in the nonlinearly realized theory (instead of $3/2$ as in the linearly realized models). The reason is that the gauge-invariant mass terms $l^d_{Rj}\tilde{l}^d_j$ generate upon expansion in powers of the Goldstone fields a quadrilinear vertex of the form $l^d_{Rj}\phi^2 l^d_j$ which contains two Goldstone fields. Therefore at one loop level there are logarithmically divergent graphs with four fermionic external legs [2], [3] which limit the UV dimension of massive fermions to 1.

Since the fermions have UV degree 1, massive Majorana neutrinos are allowed in the nonlinear theory. In fact the following invariant can be constructed out of the first component of $\tilde{L}$, provided that ν is Majorana

$$m\overline{\tilde{\nu}_L}\tilde{\nu}_L.$$

Moreover two mass invariants in the vector meson sector are obtained out of the charged combinations

$$w^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(w_{1\mu} \mp i w_{2\mu}) \tag{2.2}$$

and the neutral component $w_{3\mu}$. They can be parameterized as

$$M^2\left(w^+w^- + \frac{1}{2}w_3^2\right), \quad \frac{M^2\kappa}{2}w_3^2. \tag{2.3}$$

Two mass invariants are expected for the vector mesons, as a consequence of the breaking of the global $SU(2)_R$ invariance induced by the hypercharge. We remark that in the SU(2) nonlinearly realized Yang-Mills theory the WPC plus the gauge symmetry were compatible with any bilinear term with no derivatives in the bleached gauge field. The unique diagonal Stückelberg mass term was recovered by imposing an additional $SU(2)_R$ global symmetry.

3. Gauge-fixing and External sources

In order to preserve the LFE after gauge-fixing an external source $V_{a\mu}$ transforming as a $SU(2)_L$ gauge connection is introduced. The Landau gauge-fixing is implemented through the $SU(2)_L$-covariant gauge-fixing function $\mathscr{F}_a = D_\mu[V](A-V)^\mu_a$. The antifields coupled to the nonlinear BRST variation of the fields are introduced, as well as the source of the nonlinear constraint ϕ_0. Moreover the source $V_{a\mu}$ is paired with its BRST partner $\Theta_{a\mu}$ into a BRST doublet [2] and [7].

The algebra of the operators coupled with these external sources closes and consequently the functional formulation of the relevant identities can be established as discussed in [2] and [3].

The extension of the WPC to the external sources is discussed in [2].

4. The subtraction procedure

The perturbative expansion is carried out order by order in the number of loops. The LFE is solved by extending the bleaching technique to the full set of ghost fields and external sources [2] and [3]. The ST identities can be recursively studied order by order in the loop expansion. The constraints on the divergent ancestor amplitudes are derived by exploiting the nilpotency of the linearized ST operator $\mathscr{S}_0$. One has then to solve a cohomological problem in the space of bleached variables of finite dimension, due to the WPC condition [11].

Finite higher order symmetric renormalization, allowed by the WPC and the symmetries of the theory, cannot be reinserted back into the tree-level vertex functional without violating either the symmetries or the WPC. This fact implies that they cannot be interpreted as additional *bona fide* physical parameters [12] (unlike in the chiral effective field theories approach). We adopt the following *Ansatz*: minimal subtraction of properly normalized n-loop amplitudes

$$\frac{1}{\Lambda^{D-4}}\Gamma^{(n)}$$

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around $D = 4$ should be performed. This subtraction procedure is symmetric [2], [3] and [12]. In this scheme the γ_5 problem is treated in a pragmatic approach. The matrix γ_5 is replaced by a new γ_D which anti-commutes with every γ_μ . No statement is made on the analytical properties of the traces involving γ_D . Since the theory is not anomalous such traces never meet poles in $D-4$ and therefore we can evaluate at the end the traces at $D = 4$.

In this subtraction scheme the dependence on the scale Λ cannot be removed by a shift of the tree-level parameters. Hence it must be considered as an additional physical parameter setting the scale of the radiative corrections.

5. Conclusions

The electroweak model based on the nonlinearly realized $SU(2) \otimes U(1)$ gauge group can be consistently defined in the perturbative loop-wise expansion. In this formulation there is no Higgs in the perturbative series.

The present approach is based on the LFE and the WPC. There is a unique classical action giving rise to Feynman rules compatible with the WPC condition. In particular the anomalous couplings, which would be otherwise allowed on symmetry grounds, are excluded by the WPC. Two gauge bosons mass invariants are compatible with the WPC and the symmetries. Thus the tree-level Weinberg relation is not working in the nonlinear framework.

The discovery of the LFE suggests a unique *Ansatz* for the subtraction procedure which is symmetric, i.e. it respects all the identities of the theory. A linear Ward identity exists for the electric charge (despite the nonlinear realization of the gauge group). The strategy does not alter the number of tree-level parameters apart from a common mass scale of the radiative corrections.

The theoretical and phenomenological consequences of this scenario are rather intriguing. An Higgs boson could emerge as a non-perturbative mechanism, but then its physical parameters are not constrained by the radiative corrections of the low energy electroweak processes. Otherwise the energy scale for the radiative corrections Λ is a manifestation of some other high-energy physics.

Many aspects remain to be further studied. We only mention some of them here. The issue of unitarity at large energy (violation of Froissart bound) at fixed order in perturbation theory when the Higgs field is removed can provide additional insight in the role of the mass scale Λ and in the transition to the symmetric phase of the Yang-Mills theory. The electroweak model based on the nonlinearly realized gauge group satisfies Physical Unitarity as a consequence of the validity of the Slavnov-Taylor identity. Therefore violation of the Froissart bound can only occur in evaluating cross sections at finite order in perturbation theory. This requires the evaluation of a scale at each order where unitarity at large energy is substantially violated.

The phenomenological implications of the nonlinear theory in the electroweak precision fit have to be investigated. The 't Hooft gauge derived for the nonlinearly realized $SU(2)$ massive Yang-Mills theory should be extended to the $SU(2) \otimes U(1)$ nonlinearly realized model [5].

The evaluation of the radiative corrections at one loop level is currently being investigated. The phenomenological impact of the inclusion of massive Majorana neutrinos should also be addressed.

Finally the extension of the present approach to larger gauge groups (as in Grand-Unified models) could help in understanding the nonlinearly realized spontaneous symmetry breaking mechanism (selection of the identity as the preferred direction in the $SU(2)$ manifold) and the associated appearance of two independent gauge boson mass invariants.

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