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Recent work on PHOTOS Monte Carlo, $\gamma^* \to \pi^+\pi^-(\gamma)$ and K_{e4} decay

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PHOTOS Monte Carlo is widely used for simulating QED effects in decay of intermediate particles and resonances. It can be easily connected to other main process generators. In this paper we consider decaying processes $\gamma^* \to \pi^+\pi^-(\gamma)$ and $K^\pm \to \pi^+\pi^- e^\pm \nu\ (\gamma)$ in the framework of Scalar QED. These two processes are interesting not only for the technical aspect of PHOTOS Monte Carlo, but also for precision measurement of $\alpha_{QED}(M_Z)$, $g-2$, as well as $\pi\pi$ scattering lengths.

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1. Introduction

In high energy experiments, one of the crucial works is to compare new experiment results with predictions from the theory. If the agreement is obtained, the theory is proved to be true. Otherwise one may think that the theory calculations turned out to be wrong or the effect of new physics appeared. Monte Carlo generators, rather than analytical calculations, are required to provide theoretical results of real experiment interest. The PHOTOS Monte Carlo [1, 2] is a universal Monte Carlo algorithm that is designed for simulating QED radiative corrections in cascade decays. The program is based on exact multiphoton phase space while the matrix element is approximately taken as process independent multidimensional kernel.

Spin amplitudes are essential for design and tests of the Monte Carlo program, in particular for choice of the single emission kernels. The analysis of the spin amplitudes and tests for the algorithm in case of Z decay into pair of charged fermions, scalar particle decay into pair of fermions, spinless particle into pair of scalars and W decay were studied in Refs. [3], [4], [5] and [6, 7], respectively. In this paper we will study $\gamma^* \to \pi^+\pi^-(\gamma)$ decay. It not only provides example for studies of Lorentz and gauge group properties of spin amplitudes and cross sections, but also improves theoretical uncertainty of PHOTOS for this decay.

K_{e4} decay could give the unique information on the value of $s-$ and $p-$ wave $\pi\pi$ scattering lengths. The high statistics measurements of K_{e4} decay has been performed by NA48/2 collaboration at CERN [8]. QED corrections to this process are known to be non-negligible. They need to be taken into account with the help of Monte Carlo because their size depend on detector acceptance. In NA48 experiment, to take into account QED effects, PHOTOS Monte Carlo is used together with Coulomb correction (see Ref. [8]).

2. $\gamma^* \to \pi^+\pi^-(\gamma)$

The amplitudes of the process $e^+e^- \to \gamma^*(p) \to \pi^+(q_1)\pi^-(q_2)\gamma(k,\varepsilon)$ can be written as $M = V^\mu H_\mu$, where $V_\mu = \bar{v}(p_1,\lambda_1)\gamma_\mu u(p_2,\lambda_2)$. The $p_1,\lambda_1,p_2,\lambda_2$ are momenta and helicities of the incoming electron and positron. Let us focus on the part for virtual photon decay. Following conventions of Ref. [9], the final interaction part of the Born matrix element for such process is

$$H_0^\mu(p,q_1,q_2) = \frac{eF_{2\pi}(p^2)}{p^2}(q_1-q_2)^\mu. \tag{2.1}$$

Here $p = q_1 + q_2$. If photon is present, this part of the amplitude can be written explicitly as sum of two gauge invariant terms:

$$H^\mu = H_I^\mu + H_{II}^\mu, \tag{2.2}$$

$$H_I^\mu = \frac{e^2F_{2\pi}(p^2)}{p^2}\left((q_1-q_2)^\mu + k^\mu\frac{q_2\cdot k - q_1\cdot k}{q_2\cdot k + q_1\cdot k}\right)\left(\frac{q_1\cdot\varepsilon^*}{q_1\cdot k} - \frac{q_2\cdot\varepsilon^*}{q_2\cdot k}\right), \tag{2.3}$$

$$H_{II}^\mu = \frac{2e^2F_{2\pi}(p^2)}{p^2}\left(\frac{k^\mu(q_1\cdot\varepsilon^* + q_2\cdot\varepsilon^*)}{q_2\cdot k + q_1\cdot k} - \varepsilon^{*\mu}\right). \tag{2.4}$$

One can easily see that Eq.(2.3) has a typical form for amplitudes of QED exclusive exponentiation [10], that is Born-like -expression multiplied by an eikonal factor $\left(\frac{q_1\cdot\varepsilon^*}{q_1\cdot k} - \frac{q_2\cdot\varepsilon^*}{q_2\cdot k}\right)$. The expression in front of the factor indeed approaches the Born one in soft photon and collinear photon limit.

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If one takes separation (2.2) for the calculation of two parts of spin amplitudes, then after spin average, the expression for the cross section takes the form:

$$\sum_{\lambda,\varepsilon} |M|^2 = \sum_{\lambda,\varepsilon} |M_I|^2 + \sum_{\lambda,\varepsilon} |M_{II}|^2 + 2\sum_{\lambda,\varepsilon} M_I M_{II}^* . \tag{2.5}$$

We should stress that Eq.(2.5) can have its first term even closer to Born-times-eikonal-factor form. For that purpose it is enough to adjust normalization of the first part of Eq.(2.5) to Born amplitude times eikonal factor by replacing $|M_I|^2$ with

$$|M_I'|^2 = |M_I|^2 \frac{|\vec{q}_1 - \vec{q}_2|^2_{\text{Born}}}{|\vec{q}_1 - \vec{q}_2 + \vec{k}\frac{q_2\cdot k - q_1\cdot k}{q_2\cdot k + q_1\cdot k}|^2}. \tag{2.6}$$

Then adjustment to the remaining parts of Eq.(2.5) is necessary . Since $\sum_{\lambda,\varepsilon} |M_I'|^2$ is the expression used in `PHOTOS` Monte Carlo in Ref. [5], such a modification is of interest. In the next step, we will perform our numerical investigations with respect to Ref. [5] which is a reference for us.

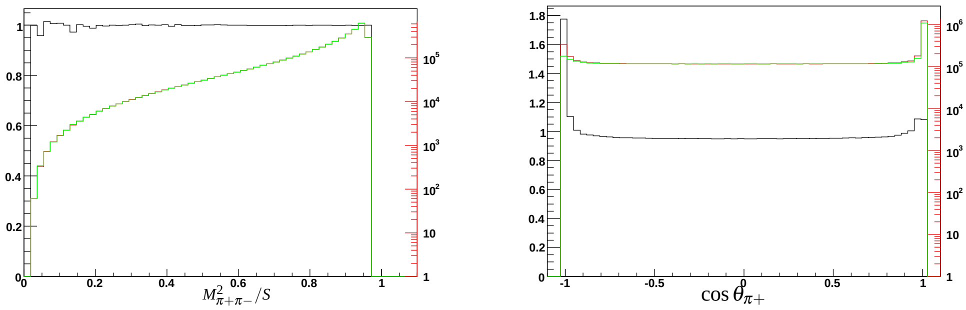

Figure 1: Comparison of results using $\sum_{\lambda,\varepsilon} |M_I'|^2$(green line) with that using matrix element taken from Ref. [5] (red line). Black line represents their ratio.

We will show results at 2 GeV center of mass energy. Comparison of result from $\sum_{\lambda,\varepsilon} |M_I'|^2$ with result from `PHOTOS` with matrix element taken from Ref. [5] is shown in Fig.1. One can see that agreement is excellent all over the phase space for the case when distributions are averaged over the orientation of the whole event with respect to incoming beams (or spin state of the virtual photon). Differences appear in distribution sensitive to initial state spin orientation, see the right side of Fig. 1. On this plot angular distribution of π^+ momentum with respect to the beam line are shown. Regions of phase space giving near zero contribution at the Born level are becoming more populated if approximation for the photon radiation matrix element [5] is used.

If instead of $\sum_{\lambda,\varepsilon} |M_I'|^2$ one would directly use $\sum_{\lambda,\varepsilon} |M_I|^2$, that is when normalization of Born-like factor is not performed, difference with respect to formulas in Ref. [5] is much larger, see the left side of Fig. 2. Finally let us compare result of complete scalar QED matrix element with that of matrix element taken from Ref. [5], see the right side of Fig. 2. At high photon energy region, there is clear surplus of events with respect to formula in Ref. [5]. That contribution should not be understood as bremsstrahlung, but rather as genuine process. Anyway in that region of phase space scalar QED is not expected to work well.

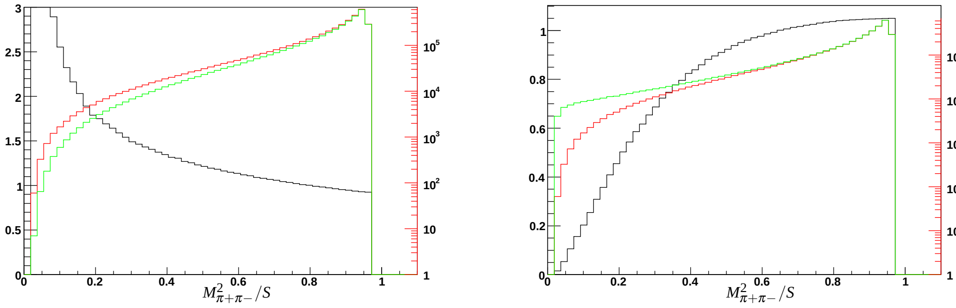

Figure 2: Comparison of results using $\sum_{\lambda,\varepsilon} |M_I|^2$ (left, green line) and complete matrix element (right, green line) with that using matrix element taken from Ref. [5] (red line). Black line represents their ratio.

3. $K^\pm \to \pi^+\pi^- e^\pm \nu(\gamma)$

Following approximations explained in Ref. [11], neglecting diagrams with photons emission from hadronic or weak blocks, one can calculate the virtual photon corrections to K_{e4} decay $K^\pm(p) \to \pi^+(q_+) + \pi^-(q_-) + e^\pm(p_e) + \nu(p_\nu)$. Contribution of virtual diagrams reads

$$\begin{aligned} \frac{d\Gamma_{\text{virt}}}{d\Gamma_{\text{Born}}} &= \frac{\alpha}{\pi}\left[\ln\frac{m}{\lambda}\left(4 + \frac{L_-}{\beta_-} - \frac{L_+}{\beta_+} - 2\rho - \frac{1+\beta^2}{\beta}L_\beta + 2\ln\frac{p_e\cdot q_+}{p_e\cdot q_-}\right)\right. \\ &\left. +\pi^2\frac{1+\beta^2}{2\beta} + \rho^2 + \frac{1}{2}\rho + 2\rho\ln\frac{m}{2E_e} + K_v\right], \end{aligned} \tag{3.1}$$

$$\begin{aligned} \rho &= \ln\frac{2E_e}{m_e}, \qquad \beta = \sqrt{1-\frac{4m^2}{s_\pi}}, \qquad L_\beta = \ln\frac{1+\beta}{1-\beta}, \\ s_\pi &= (q_+ + q_-)^2, \qquad \beta_\pm = \sqrt{1-\frac{m^2}{E_\pm^2}}, \qquad L_\pm = \ln\frac{1+\beta_\pm}{1-\beta_\pm}. \end{aligned} \tag{3.2}$$

where m is the charged pion mass, λ is photon mass used as infrared regulator. K_v depends on masses of particles and kinematics.

The soft photon contribution can be easily obtained by integrating out solid angle of photon momentum k and over its energy ω up to a limit $\omega < \Delta\varepsilon$,

$$\frac{d\Gamma_{\text{soft}}}{d\Gamma_{\text{Born}}} = \frac{\alpha}{\pi}\left[\ln\left(\frac{2\Delta\varepsilon}{\lambda}\right)\left(-4 - \frac{L_-}{\beta_-} + \frac{L_+}{\beta_+} + 2\rho + \frac{1+\beta^2}{\beta}L_\beta - 2\ln\frac{2p_e\cdot q_+}{2p_e\cdot q_-}\right) + \rho - \rho^2 + K_s\right] \tag{3.3}$$

Function K_s is dependent on masses of particles and kinematics.

The contribution of soft and virtual photons can be easily combined. It reads

$$\begin{aligned} \frac{d\Gamma_{\text{Born+virt+soft}}}{d\Gamma_{\text{Born}}} &= 1 + \sigma P_\delta + \frac{\pi\alpha(1+\beta^2)}{2\beta} + \frac{\alpha}{\pi}K_{vs}, \\ P_\delta &= 2\ln\frac{\Delta\varepsilon}{E_e} + \frac{3}{2}, \quad \sigma = \frac{\alpha}{2\pi}(2\rho - 1), \end{aligned} \tag{3.4}$$

the expression of K_{vs} depends not only on masses of particles and kinematics, but also on soft photon energy cutoff $\Delta\varepsilon$.

Photon emission from $e^\pm$ will give non negligible collinear contribution. This part can be calculated with the help of collinear-photon-approximation. The remaining part is calculated by soft-photon-approximation. Finally hard (real) photon bremsstrahlung for photons of energy above $\Delta\varepsilon$ reads

$$\begin{aligned}\frac{d\Gamma_{\text{Hard}}}{d\Gamma_{\text{Born}}} &= -\sigma P_\delta + \frac{\alpha}{2\pi}\left(3-\frac{2}{3}\pi^2\right) \\ &+\frac{\alpha}{\pi}\ln\left(\frac{\Delta\varepsilon}{E_e}\right)\left(3+\frac{L_-}{\beta_-}-\frac{L_+}{\beta_+}-\frac{1+\beta^2}{\beta}L_\beta+2\ln\frac{2p_e\cdot q_+}{2p_e\cdot q_-}\right). \qquad (3.5)\end{aligned}$$

Real and virtual photons contribution is combined, it gives,

$$\frac{d\Gamma_{\text{Born+virt+real}}}{d\Gamma_{\text{Born}}} = 1+\frac{\pi\alpha(1+\beta^2)}{2\beta}+\frac{\alpha}{\pi}K. \qquad (3.6)$$

Complicated, but numerically small function K is dependent on masses of particles and kinematics of this process. Note that the result does not depend on the large logarithm $\ln\frac{2E_e}{m_e}$, and soft photon energy cut $\Delta\varepsilon$.

As one can see, Eqs. (3.4),(3.5) and (3.6) are obtained with the help of approximations. Effectively it was assumed that matrix element at Born level can be always factorized out and photonic corrections can be calculated independently. Further corrections are assumed to be negligible and not affecting the nature of hard interaction. This may be good as starting point, but cannot be left without future discussion/improvements [1].

Our formulas are based on the same scheme of calculation as explained in Ref. [11] and in principle they should coincide numerically. Some differences in both analytical and numerical results are nonetheless present. The exact expressions for K_s, K_v, K_{vs} and K, as well as differences between our analytical results and these in Ref.[11] will not be listed here for the limit of paper length. They will be present elsewhere.

Let's switch our attention to numerical tests. In the left side of Fig. 3 we show that dominant part of Eq.(3.6) represents Coulomb correction. The difference is much smaller than the effect of Coulomb correction itself, see the right side of Fig.3 where results for 1000 000 Born level events are placed in the histograms. We may conclude that our numerical implementation of Eq.(3.6) works well since its dominant part represents Coulomb correction.

We have done numerical tests with `PHOTOS` and found the distribution for soft photons from `PHOTOS` and from Eq.(3.3) is identical. We continued the test using hard photon expression Eq. (3.5) and found again excellent agreement in the soft photon region as expected. For harder photon energy regions `PHOTOS` and Eq.(3.5) remain in agreement (better than 10%) even at the end of the spectrum. We can conclude that agreement of the hard photon expression with `PHOTOS` is good, as expected. Though differences especially in harder photon energy ranges can be seen.

4. Summary

We have presented the new tests of `PHOTOS` Monte Carlo, where the exact matrix element of $\gamma^* \to \pi^+\pi^-\gamma$ is implemented and its numerical result is compared with the kernel of `PHOTOS`.

[1]We are grateful to Prof. J. Gasser for stressing this point.

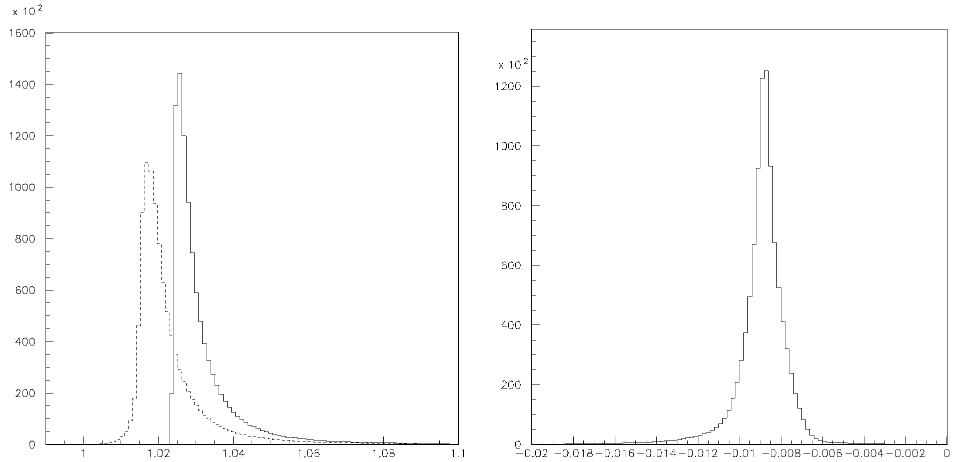

Figure 3: Coulomb correction from Ref. [8] (left, solid), radiative correction in Eq. (3.6) (left, dashed), and the difference (right) calculated event by event

QED radiative correction to process $K^\pm \to e^\pm \nu \pi^+\pi^-(\gamma)$ is also studied. Reasonable numerical agreement of analytical results and simulations including Coulomb correction and PHOTOS Monte Carlo was found. Since several assumptions are employed in both approaches, further work is necessary. Our result is of practical interest for experiments. They confirm that at least on technical level the Monte Carlo program works well; as expected.

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