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IR subtraction schemes: integrating the counterterms at NNLO in QCD

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We briefly review a subtraction scheme for computing radiative corrections to QCD jet cross sections that can be defined at any order in perturbation theory. Hereafter we discuss the computational methods used to evaluate analytically and numerically the integrated counterterms arising from such a subtraction scheme. Basically these methods are the Mellin-Barnes (MB) representations technique together with the harmonic summation and the sector decomposition.

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1. Introduction

In quantum chromodynamics (QCD) and more generally in any quantum field theory with massless particles one has to face among others the problem of infrared (IR) divergences when computing higher orders corrections. According to the KLN (Kinoshita-Lee-Nauenberg) theorem [1, 2] these IR singularities cancel out once one put together all the contributions at the same order which are degenerate to a fixed final state (e.g. a parton state emitting one collinear and/or soft gluon is degenerate to the state without this extra emission). This means that one has to compute the sum of several contributions which are separately IR divergent leading to a final physical and finite answer. To handle these IR singularities in a general way is non-trivial already at the next-to-leading order (NLO) accuracy, where however several solutions are known [3, 4, 5, 6, 7, 8]. In recent years a lot of effort has been devoted to the extension to the NNLO accuracy [9, 10, 11, 12, 13]. In particular the subtraction scheme for QCD jet cross sections defined in [12, 13] is the one we are interested in here. This scheme, initially defined for processes without colored particles in the initial state, has been extended to an NNLO-compatible scheme with hadronic initial states [14]. Here we will consider only the case of processes initiated by colorless particles and we briefly summarize it as follows.

In QCD the perturbative expansion for any production rate at NNLO can formally be written as

$$\sigma = \sigma^{\rm LO} + \sigma^{\rm NLO} + \sigma^{\rm NNLO} + \ldots. \tag{1.1}$$

Considering for example the $e^+e^- \to m$ jet process we have that the NNLO correction can be written as

$$\sigma^{\rm NNLO} = \int_{m+2} d\sigma^{\rm RR}_{m+2} J_{m+2} + \int_{m+1} d\sigma^{\rm RV}_{m+1} J_{m+1} + \int_m d\sigma^{\rm VV}_m J_m\,, \tag{1.2}$$

where the terms represent the doubly-real, the real-virtual and the doubly-virtual contribution respectively. Each of them is IR divergent while their sum remains finite. The restriction of the phase space to define the physical quantity is realized by the jet functions J_n. The basic steps of subtraction consist in regularizing all the integrals in Eq.(1.2) using dimensional regularization in $d = 4-2\varepsilon$; then in reshuffling the singularities between the three terms by adding and subtracting suitable counterterms in such a way that we end up with three contributions without IR singularities i.e. finite in $d = 4$ dimensions. In this way Eq.(1.2) becomes

$$\sigma^{\rm NNLO} = \int_{m+2} d\sigma^{\rm NNLO}_{m+2} + \int_{m+1} d\sigma^{\rm NNLO}_{m+1} + \int_m d\sigma^{\rm NNLO}_m. \tag{1.3}$$

Here

$$d\sigma^{\rm NNLO}_{m+2} = \left\{ d\sigma^{\rm RR}_{m+2} J_{m+2} - d\sigma^{\rm RR,A_2}_{m+2} J_m - \left[d\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - d\sigma^{\rm RR,A_{12}}_{m+2} J_m \right] \right\} \tag{1.4}$$

$$d\sigma^{\rm NNLO}_{m+1} = \left[d\sigma^{\rm RV}_{m+1} + \int_1 d\sigma^{\rm RR,A_1}_{m+2} \right] J_{m+1} - \left[d\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 d\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \tag{1.5}$$

and

$$d\sigma^{\rm NNLO}_m = \left\{ d\sigma^{\rm VV}_m + \int_2 \left[d\sigma^{\rm RR,A_2}_{m+2} - d\sigma^{\rm RR,A_{12}}_{m+2} \right] + \int_1 \left[d\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 d\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m. \tag{1.6}$$

In Eq.(1.4) above $d\sigma^{\mathrm{RR,A_1}}_{m+2}$ and $d\sigma^{\mathrm{RR,A_2}}_{m+2}$ regularize the singly- and doubly-unresolved limits of $d\sigma^{\mathrm{RR}}_{m+2}$ respectively. The last counterterm in Eq.(1.4) which is $d\sigma^{\mathrm{RR,A_{12}}}_{m+2}$ must regularize the sinlgy-unresolved limits of $d\sigma^{\mathrm{RR,A_2}}_{m+2}$ and the doubly-unresolved limits of $d\sigma^{\mathrm{RR,A_1}}_{m+2}$. Finally in Eq.(1.5) we have that the counterterms $d\sigma^{\mathrm{RV,A_1}}_{m+2}$ and $\left(\int_1 d\sigma^{\mathrm{RR,A_1}}_{m+2}\right)^{\mathrm{A_1}}$ regularize the singly-unresolved limits of $d\sigma^{\mathrm{RV}}_{m+1}$ and $\int_1 d\sigma^{\mathrm{RR,A_1}}_{m+2}$ respectively.

2. Integrating the subtraction counterterms

In order to complete the definition of the subtraction scheme, one has to compute the integral of the counterterms over the factorized one- and two-body phase spaces given in Eqs.(1.5,1.6). These integrated counterterms have to be computed in $d = 4-2\varepsilon$ dimensions and the result should be given in the form of a Laurent expansion in ε. According to the KLN theorem, the ε poles of the ε expansions of the integrated counterterms have to cancel those in the one-loop correction $d\sigma^{\mathrm{RV}}_{m}$ and the two-loop correction $d\sigma^{\mathrm{VV}}_{m}$ giving a finite contribution for the real-virtual and doubly-real cross sections. In this proceedings we will discuss the various techniques used to compute the Laurent expansions in ε for the integrated counterterms.

According to the different unresolved limits there are different mappings of the external momenta all of them *preserve momentum conservation*. The integrated singly-unresolved counterterm $\int_1 d\sigma^{\mathrm{RR,A_1}}_{m+2}$ in Eq.(1.5) (which is also the only one that is needed for a NLO computation) has been computed in [15]. As it is shown in [16, 17] the singly-unresolved integrated counterterm $\int_1 [d\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 d\sigma^{\mathrm{RR,A_1}}_{m+2}\right)^{\mathrm{A_1}}]$ in Eq.(1.6) can be reduced to three different types of basic integrals: these are the collinear, the soft and the soft-collinear integrals . Then the complete counterterm is built with them and their non trivial convolutions (see e.g. Sections 2.1 and 2.2 of Ref.[17]). As far as the doubly-unresolved counterterm $\int_2 [d\sigma^{\mathrm{RR,A_2}}_{m+2} - d\sigma^{\mathrm{RR,A_{12}}}_{m+2}]$ in Eq.(1.6) is concerned we have that the computation of the second one will be published in a forthcoming paper [18]. The computation of the first term is clearly feasible employing the same methods and is a work in progress. For $\int_2 d\sigma^{\mathrm{RR,A_{12}}}_{m+2}$ it turned out that it can be reduced to six different basic integrals divided in couples of collinear, soft and collinear-soft integrals. Analogously to the singly-unresolved case this integrated counterterm it is built up with these basic integrals and their non-trivial convolutions.

To compute the basic integrals that are involved in the definitions of the various integrated counterterms we studied different methods. A first fully numerical method for the computation of the basic integrals is based on sector decomposition (see [19] and references therein). To extract the poles in ε of these integrals a `Mathematica` package has been written implementing the sector decomposition techniques [16, 20]. This program also produces `FORTRAN` codes directly used by numerical integration programs. Another method that has been used to approach the computation of these integrals exploited integration-by-parts (IBP) identities. With this method analytical results were obtained for some of the singly-unresolved integrals [16]. However it turned out that the method based on Mellin-Barnes (MB) representations is in many cases more efficient and accurate than the numerical evaluation via sector decomposition and made the analytic results much more feasible than the techniques based on IBP [17]. Both the sector decomposition and the MB techniques have been implemented independently to cross check all the results (also for those involved in the doubly-unresolved counterterm) obtained for the Laurent expansions.

To briefly show how the MB method works we want to describe a representative example involved in the computation of the $\int_2 d\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}}$ counterterm. As anticipated a full treatment of it will appear elsewhere [18]. We begin by considering the following basic collinear integral

$$I_k(x) = x \int_0^1 d\alpha dv \frac{\alpha^{-1-\varepsilon}(1-\alpha)^{2d_0-1} v^{-\varepsilon}(1-v)^{-\varepsilon}}{[\alpha+(1-\alpha)x]^{1+\varepsilon}} \left(\frac{\alpha+(1-\alpha)xv}{2\alpha+(1-\alpha)x}\right)^k, \tag{2.1}$$

where $d_0 \geq 2$ is an arbitrary parameter, $k = -1,0,1,2$ and x is a kinematic variable in $[0,1]$. Let us now consider the particular case $d_0 = 2, k = 1$. With this choice the integral in Eq.(2.1) is split into two different contributions. Of them only the second one produces a pole in ε due to the factor $\alpha^{-1-\varepsilon}$ and we concentrate on it, which explicitly is given by

$$E(x) = x^2 \int_0^1 d\alpha \frac{\alpha^{-1-\varepsilon}(1-\alpha)^4}{[\alpha+(1-\alpha)x]^{1+\varepsilon}[2\alpha+(1-\alpha)x]}, \tag{2.2}$$

where we have neglected an overall factor of $\Gamma(2-\varepsilon)\Gamma(1-\varepsilon)/\Gamma(3-2\varepsilon)$ coming from the integration over v. Now applying the well known basic MB formula

$$\frac{1}{(a+b)^\nu} = \frac{1}{\Gamma(\nu)} \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \frac{dz}{2\pi i} a^{-\nu-z} b^z \Gamma(\nu+z)\Gamma(-z) \tag{2.3}$$

two times in Eq.(2.2) we obtain

$$E(x) = \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \frac{dz_1 dz_2}{(2\pi i)^2} 2^{z_2} x^{-\varepsilon-z_1-z_2} \frac{\Gamma(-z_1)\Gamma(-z_2)\Gamma(3-\varepsilon-z)\Gamma(1+\varepsilon+z_1)\Gamma(1+z_2)\Gamma(-\varepsilon+z)}{\Gamma(3-2\varepsilon)\Gamma(1+\varepsilon)}, \tag{2.4}$$

where $z = z_1 + z_2$. According to the definition of the complex integration path in Eq.(2.3), one has to choose the contours in such a way that the poles with a $\Gamma(\ldots+z)$ dependence are to the left of the contour and the poles with a $\Gamma(\ldots-z)$ dependence are to the right of it. Clearly if one starts with contours that do not satisfy this requirement one has to deform them taking into account the residua of the integrand while crossing every pole of it. This procedure is automatized in the `Mathematica` package `MB.m` [21]. Once this is done this package enables one to easily compute the Laurent expansion in ε where the coefficients are given as a list of MB integrals. The numerical evaluation of the integrals is also implemented in `MB.m` by use of a simple command line. Moreover according to the Cauchy theorem the integration over the complex contour can be converted into sums over the residua inside the paths. If we follow these steps for our example given in Eq.(2.4) we get

$$\begin{aligned} E(x) &= -\frac{1}{\varepsilon} + 2\log\left(\frac{x}{2}\right) - \log(2)\sum_{n=1}^{\infty} x^n \binom{n+2}{2} - \sum_{m,n=1}^{\infty} \left(\frac{x}{2}\right)^m x^n \binom{m+n+2}{2} \times \\ &\times \left[\mathrm{S}_1(\mathrm{m+n+2}) - \mathrm{S}_1(\mathrm{m+n}) + \log\left(\frac{\mathrm{x}}{2}\right)\right] + O(\varepsilon), \end{aligned} \tag{2.5}$$

where $\mathrm{S}_1(\mathrm{n}) = \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{1}{\mathrm{i}}$ denotes the usual harmonic numbers. To obtain an analytic answer the harmonic summation involved in Eq.(2.5) has to be performed. In this example, like in many other cases encountered in the complete computation of the integrated counterterms, the harmonic summation is feasible and can be performed using the `XSummer` package [22, 23] in `Form 3.0` [24]. Running a proper script we obtain for our example in Eq.(2.5) the following analytic answer:

$$\begin{aligned} E(x) \;=\; & -\frac{1}{\varepsilon} + \log(2)\left(1 - \frac{1}{(1-x)^3}\right) - \frac{x^2(3x^2-15x+14)}{2(1-x)^2(2-x)^2} \\ & + \frac{(x^6-9x^5+33x^4-78x^3+108x^2-72x+16)}{(1-x)^3(2-x)^3} + O(\varepsilon). \end{aligned} \tag{2.6}$$

Note that the limit in $x = 1$ (corresponding to the collinear limit) is well defined because we have that $\lim_{x\to 1} E = -1/\varepsilon + 53/6 - 16\log 2 + O(\varepsilon)$. This concludes the discussion of our example to show the method of MB representations.

As mentioned above, in the forthcoming paper about the counterterm $\int_2 d\sigma_{m+2}^{\mathrm{RR,A_{12}}}$ we will also involve convolutions of the basic integrals, for example the collinear one in Eq.(2.1) with itself:

$$I_k * I_l(x,y) = y\int_0^1 d\alpha dv \frac{\alpha^{-1-\varepsilon}(1-\alpha)^{2d_0-3+2\varepsilon}v^{-\varepsilon}(1-v)^{-\varepsilon}}{[\alpha+(1-\alpha)y]^{1+\varepsilon}}\left(\frac{\alpha+(1-\alpha)yv}{2\alpha+(1-\alpha)y}\right)^l I_k((1-\alpha)x). \tag{2.7}$$

For this particular integral following the steps described above we find

$$\begin{aligned} I_k * I_l(x,y) \;=\; & \frac{\delta_{k,-1}\delta_{l,-1}}{4\varepsilon^4} - \left[\frac{\delta_{k,-1}\delta_{l,-1}}{2}\ln(xy) + \frac{(1-\delta_{k,-1})\delta_{l,-1}}{2(1+k(1-\delta_{k,-1}))} + \frac{(1-\delta_{l,-1})\delta_{k,-1}}{2(1+l(1-\delta_{l,-1}))}\right]\frac{1}{\varepsilon^3} \\ & + O(\varepsilon^{-2}). \end{aligned} \tag{2.8}$$

Here the $O(\varepsilon^{-2})$ contribution is already cumbersome and will not be given here.

3. Conclusions

In this proceedings after shortly reviewing the NNLO subtraction scheme developed in [12, 13, 14] we have discussed the various methods to compute the integrated counterterms focussing on the MB representations method. Showing a simple example we described its usage discussing some new results: the computation of the integrated counterterm $\int_2 d\sigma_{m+2}^{\mathrm{RR,A_{12}}}$ is finished and the results will be reported in [18]. Our methods are clearly applicable to compute the final remaining integrated counterterm $\int_2 d\sigma_{m+2}^{\mathrm{RR,A_2}}$ which is a work in progress. We found that the method based on the MB representations can be used to obtain analytic results and in many cases is more efficient even numerically than others like sector decomposition.

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