Electro-weak responses of $^4$He using realistic nuclear interactions

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In this contribution we discuss electro-weak induced responses involving $^4$He. The wave function of the ground state is obtained accurately using an explicitly correlated basis with a realistic nuclear force. Four-body final states are expressed in a superposition of many basis functions which contain important configurations for the low-lying transition strength. Continuum state is treated properly in the complex scaling method. We discuss the electric-dipole and weak-induced strength functions of $^4$He. Also, we mention the neutrino-$^4$He reaction which is propotional to the weak-transitions.

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1. Introduction

A study of neutrino-nucleus reaction is important to the scenario of a supernova explosion. In the final stage of a core collapse supernova, $^4$He is exposed to intense flux of neutrino. The neutrino-$^4$He reaction is expected to play a significant role, and the reaction rate is proportional to the weak responses, for example, due to Gamow-Teller, dipole, spin-dipole, etc. operators. A reliable theoretical study is desired for precise evaluation of the reaction rate. $^4$He is the lightest closed shell nucleus which has several excited states above the excitation energy of 20 MeV. We have recently reported that all the observed levels below 26 MeV are well reproduced in a full four-body calculation using realistic nuclear interactions [1, 2]. It is interesting to extend this approach in order to study some responses in $^4$He.

The dipole response of $^4$He is also very interesting. In the energy region around 26 MeV, photoabsorption reaction occurs mainly through the electric dipole transition. The current experimental situation is controversial. Two groups have shown quite different cross sections [3, 4]. Because there are only few theoretical studies on the reaction starting from a realistic interaction, further study may help clarify the situation.

In this contribution, we will discuss strength functions of $^4$He related to the above mentioned electro-weak reactions. The wave function of the ground state is obtained accurately using an explicitly correlated basis and four-body final states are also expressed in a superposition of many basis functions which contain important configurations for the low-lying strength functions. We discuss the controversial of the photoabsorption cross section and discuss the strength functions induced by the weak interaction and mention the $\nu$-$^4$He reaction cross section.

2. Method

We start from the Hamiltonian of a four-nucleon system with two- and three-nucleon forces. Here we employ two types of realistic nucleon-nucleon interactions of the AV8’ [5] and G3RS [6] potentials which contain central, tensor and spin-orbit components. The AV8’ is more singular at short-range and has strong tensor component than these of the G3RS potential. A phenomenological three-body interaction [7] is used in order to get correct thresholds for the three- and four-nucleon bound states. The wave function is expressed in terms of a linear combination of the correlated Gaussian with double global vectors that enables us to obtain a precise solution of many-body equation with a realistic force [1, 2].

A nuclear response is characterized by the strength (response) function which is given in

$$S(E) = \sum_{\nu} |\langle \Psi_{\nu} | \mathcal{O} | \Psi_0 \rangle|^2 \delta(E_{\nu} - E),$$

(2.1)

where $\mathcal{O}$ is an operator which you concern and $\Psi_0$ is the ground state of $^4$He, and $\Psi_{\nu}$ is the final state with energy $E_{\nu}$. For the continuum wave function, we use a $L^2$ basis function which does not satisfy a proper boundary condition of continuum state. In order to treat it properly, we use the complex scaling method (CSM) which is a widely used method for calculating the strength function in a nuclear system [8]. In the CSM calculation, $r$ is transformed to $re^{i\theta}$. This allows us to expand the continuum wave function in our correlated bases. Theirs coefficients are determined by
diagonalizing the complex rotated Hamiltonian. The result should not depend on the scaling angle $\theta$, which can be determined practically by observing stability of $S(E)$ with respect to $\theta$.

For the continuum state, we include important configurations at around 20–40 MeV where the dipole strength is dominant. The dipole operator is an one-body operator relative to the center of mass which excites one particle in an initial state (the ground state of $^4\text{He}$). We can thus assume that the inclusion of the cluster partitions of 3+1 and 2+2 are important. Firstly it is important to include the 3N+N configuration which the coordinate between a 3N system and nucleon is excited by the dipole operator. The relative motion between the 3N and N is P-wave and it is approximated with several Gaussians. A set of parameters are taken by a geometrical progression covering from 0.1 to 10 fm. As for the parameters of the three-nucleon system, we use a set of the bases obtained by the three-body calculation with $J^T=1/2^+/1/2$ from the same Hamiltonian. As the second largest contribution, we include the 3N$^\ast$+N configuration which the 3N system in the ground state is excited by the dipole operator. We take a set of the bases obtained by the two-body calculation with $J^T=1^+0$ for the parameters of two-body subsystem in the 3N. The other relative coordinates (P- and S-wave) are approximated in several Gaussians.

3. Electric dipole strength: photoabsorption cross section

![Figure 1](image.png)

**Figure 1:** Photoabsorption cross sections calculated with the different configurations ($\theta=20^\circ$).

The photoabsorption cross section of ($^4\text{He},\gamma$) is proportional to the dipole strength function $S(E_1,E)\sigma_\gamma(E) = \frac{4\pi^2}{3c} ES(E_1,E)$ where the electric dipole operator is defined in

$$\mathcal{M}(E_1,\mu) = \sqrt{\frac{4\pi}{3}} \sum_{i=1}^A \mathcal{Y}_{1\mu}(r_i-x_A)\frac{1}{2}(1-\tau_{3i}),$$

where $x_A$ is the center of mass coordinate of the system. Fig. 1 shows the calculated photoabsorption cross sections with the different configurations. The G3RS and phenomenological three-body potentials are used. The labels, 3NN, 3N$^\ast$N, indicate the configurations that the strength are calculated with. Dashed line shows the photoabsorption cross section only with the 3N+N configuration and the full calculation is plotted by a solid line. As one can see in the figure, the peak shifts to high energy region with adding the 3N$^\ast$+N configuration. Comparing the result with three recent measurements, the result supports a sharp rise from the threshold as seen in the two of the experiments which show a peak at around 25 MeV [4, 9] but we find a large deviation from the measurement by
Shima et al. at the low energy region [3]. Our result also agrees with the other theoretical calculations starting from realistic interactions [10]. Advantage of our study is that we use “bare” realistic interactions instead of an effective interaction and the configuration space used in the calculations is different from theirs.

4. Nuclear responses by the weak interaction

In a long-wave length approximation, the nuclear current can be expanded in some multipole operators. Here we consider a neutrino inelastic scattering governed by the neutral current. There are Fermi and Gamow-Teller types which are the leading order in the weak response responsible for the allowed transitions. The Fermi type does not contribute to the reaction involving $^4$He because the total isospin of its ground state is zero. When one uses only $(0s)^4$ configuration for the ground state, the Gamow-Teller contribution becomes zero. We have however elaborated wave function for the ground state of $^4$He which contains the $D$-state probability originating from the tensor correlation, so that the Gamow-Teller operator can contribute to the reaction. The left panel of Fig. 2 displays the Gamow-Teller strength functions calculated with the G3RS and AV8’ potentials. The strength function calculated with the AV8’ potential is a little bit larger than that with the G3RS potential. That reflects difference of the $D$-state probabilities in the ground states, which are 11% and 14% for the G3RS and AV8’ potentials, respectively.

Figure 2: Left: The Gamow-Teller strength functions calculated with the AV8’ and G3RS potentials ($\theta=15^\circ$). Right: The calculated spin-dipole strength functions of $\lambda=0, 1$ and 2 ($\theta=20^\circ$). The G3RS potential is employed as a nucleon-nucleon interaction.

When the neutrino energy is 10 MeV, the contribution of the first-forbidden transitions to the reaction cross section is approximately one hundredth order of magnitude compared to the allowed transitions. If the allowed transition probabilities are small, the first-forbidden transition operators can contribute to the cross section. In the first-forbidden transition, the dipole type appears again and the spin-dipole type is interesting as an analog of the dipole transition. The spin-dipole operator excites the spin of the ground state while the dipole operator does not. With observing these strength functions, we obtain information on continuum structure of the four-nucleon system with different spin states. Here we discuss the spin-dipole operator which is given in

$$\mathcal{M}(SD, \mu) = \sum_{i=1}^{A} |\mathcal{Y}_i(r_i-x_A)\sigma_i|_{\lambda\mu} t_{3d}$$  \hspace{1cm} (4.1)
where the value of $\lambda$ can take 0, 1 and 2. The right panel of Fig. 2 plots the spin-dipole strength functions with the full configuration. The G3RS is used as a two-body interaction. The transition strength functions are very large enough to be a leading order of the contribution to the cross section compared to the Gamow-Teller strength functions in spite of the large reduction factor. As seen in the figure the peaks with $\lambda=0$ and 2 are consistent with the observed energies of 23.33 MeV and 25.28 for $J^T=0^+1$ and $2^+1$, respectively. For explaining a correspondence of the $1^+1$ states with the observed spectra (23.64 and 25.95 MeV), the dipole component should be considered as well as the spin-dipole component. As discussed in Ref.[2], the $0^+1$ and $2^+1$ states consist of more than 90% of total spin $S=1$ component while an equal weight of $S=0$ and 1 components in the $1^+1$ states.

5. Summary and future works

We have calculated the strength functions induced by the electro-weak transitions based on the four-body calculation using a realistic nuclear force. The continuum states have been expressed in many basis states including important configurations at 20–40 MeV. The configurations are constructed based on a cluster excitation.

The strength functions are obtained with a $L^2$ basis using the complex scaling method. The photoabsorption cross section which is proportional to the dipole strength agrees with some of the recent measurements. We have found that the Gamow-Teller and spin-dipole strength functions reflect the ground and continuum structures of the four-nucleon system. For the neutrino-$^4$He reaction cross section, we have to evaluate some other weak operators which are not discussed here. It is straightforward and can be calculated very soon.

References