Physics on huge X-ray luminosity of magnetars

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In a superhigh magnetic field, direct Urca reactions can proceed for an arbitrary proton concentration. The magnetic field is the main energy source of all the persistent and bursting emissions observed in anomalous X-ray pulsars (AXPs) and soft gamma-ray repeaters (SGRs). Employing the assumption of micro-state number, we deduce the formula of electron Fermi energy $E_F(e)$ in superhigh magnetic fields. Based on observing and introducing relevant parameters introduced, an approximate way of calculating the X-ray luminosity of a Magnetar $L_X$ in any superhigh magnetic field is introduced.

11th Symposium on Nuclei in the Cosmos
19-23 July 2010
Heidelberg, Germany.

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†A footnote may follow.
1. Introduction

AXPs and SGRs are a small group of peculiar neutron stars (NSs) that are currently believed to have superhigh magnetic fields $B \sim 10^{14} - 10^{15}$ G, and are hence identified as magnetar candidates [1], [3], [4], [5], [6], [11]. The magnetic fields of magnetars deduced from their long spin periods ($P \sim 5 - 12$ s) and spin-down rates ($P \sim 10^{-10} - 10^{-13}$ s$^{-1}$) based on magnetic dipole radiation, are so strong as to reach two orders of magnitude larger than the quantum critical threshold $B_{cr} = 4.414 \times 10^{14}$ G. Moreover, a similar estimate of a superhigh magnetic field is also derived from the observation of an absorption line in the X-ray spectrum of SGR 1806-20 [2]. Another evidence of a superhigh magnetic field is that, a strong magnetic field is required to confine a fire ball effectively when modeling giant flares of SGRs [3]. The total magnetic field induced by $3P_2$ Cooper pairs is in an anisotropic neutron superfluid at a lower interior temperature [8], [9]. Now, we briefly summarize the results of our previous job, which is relevant to our current job (studying huge X-ray luminosity of magnetars). The main points are as following:

1. Calculations show that the fossil magnetic field of a neutron star, formed during core-collapse, will be magnified 91 times by Pauli paramagnetism due to the presence of a relativistic degenerate electron gas [8].

2. The total magnetic field induced by $3P_2$ Cooper pairs is $B_{\text{tot}} = B_{\text{max}} f(\mu_B B/kT)$, $B_{\text{max}} = 2.02 \times 10^{14} \eta$ G, where $f(x)$ is the Brillouin function, $f(x) = \frac{2 \sinh(2x)}{1 + \cosh(2x)}$, when $x \ll 1$, $f(x) \approx 4x/3$; when $x \gg 1$, $f(x) \to 1$ [9].

3. The quantity $q$ depicts the probability for two neutrons being combined into a $3P_2$ Cooper pair, $q \sim 0.087$. More specifically, $\eta = \frac{m^2 c^4}{n! 3^{n} 2^{-n} \frac{R_{\text{NS,6}}^3}{\mu_B B} (\Delta E_{\text{p}})}$, here $\eta$ is the dimensionless factor describing both the macroscopic and microscopic properties of neutron stars and $R_{\text{NS,6}}$ is the radius of a neutron star in units of $10^6$ cm. When the temperature decreases to $T_c$, the value of the induced magnetic field increases and just reaches that of a magnetar, $T_c$ is the Curie Temperature of a phase transition from paramagnetism to ferromagnetism.

In this paper, the physics of huge X-ray luminosity of magnetars will be investigated.

2. Fermi sphere of electrons under ultrastrong magnetic field

As far as we know, in a weak magnetic field ($B \ll B_{cr}$), the Fermi surface of electrons is spherical, and the quantized Landau levels are insignificant, on the other hand, when in an intense magnetic field ($B \gg B_{cr}$), the behavior of an electron gas is very different from that of a weak magnetic field: the Fermi sphere becomes a Landau column and the energy levels perpendicular to the direction of the applied magnetic field are quantized. The energy of an electron with orbital quantum number $n$ ($n = 0, 1, 2, 3, \cdots$), depicts the number of Landau levels) and spin quantum number $\sigma$ ($\sigma = \pm 1$) in a superhigh magnetic field is given by

$$E_{\text{F}}^2(p, B, n, \sigma) = m^2 c^4 + p_{\text{z}}^2 c^2 + (2n + 1 + \sigma) 2m c^2 \mu_e B,$$ (2.1)
where \( p_z \) is the \( z \)-component of electron momentum, and \( \mu_e \sim 0.927 \times 10^{-20} \) ergs G\(^{-1}\) is the magnetic moment of an electron. In the light of the Pauli exclusion principle, the electrons are situated in disparate energy states in order one by one from the lowest energy state up to the Fermi energy (the highest energy) with the highest momentum \( p_F(z) \) along the magnetic field. In order to calculate electron state density \( N_{\text{phase}} \) (electron energy state number in a unit volume), we define a non-dimensional magnetic field: \( b = B/B_{cr} \) and electron momentum perpendicular to the magnetic field: \( p_\perp = m_e c \sqrt{(2n+1+\sigma)b} \). In the absence of a magnetic field, both \( dp_z \) and \( dp_\perp \) change continuously and the microscopic state number \( N_{\text{phase}} \) in a volume element of phase space \( d^3x d^3p \) is \( d^3x d^3p \) \( ph \). In the presence of a magnetic field, electrons are populated in many discrete Landau levels with \( n = 0, 1, 2, 3, \ldots \) if \( B \gg B_{cr} \), the Landau column becomes a very long and very narrow cylinder along the magnetic field, the overwhelming majority of electrons congregates in the lowest levels with \( n = 0 \) or \( n = 1, 2 \), the section radius of a Landau column is \( p_\perp \). For a given \( p_z \), there is a corresponding maximum Landau level number \( n_{\text{max}} \), whose expression is

\[
n_{\text{max}}(p_z, b, \sigma = -1) \approx n_{\text{max}}(p_z, b, \sigma = 1) = n_m(p_z, b) \tag{2.2}
\]

\[
n_{\text{max}}(p_z, b) = \text{Int}[\frac{1}{2b}[(\frac{E_F(e)}{m_e c^2})^2 - 1 - (\frac{p_\perp}{m_e c})^2 - 1]], \tag{2.3}
\]

where \( \text{Int}[x] \) denotes an integer value of the argument \( x \).

In an intense magnetic field, \( dp_\perp \) changes continuously along the \( z \)-axis direction, whereas \( dp_z \) is not continuous and must obey the relation \( p_\perp = m_e c \sqrt{(2n+1+\sigma)b} \). An envelope of the Landau circles with maximum quantum number \( n_{\text{max}}(p_z, b) \) \( (0 \leq p_z \leq p_F) \) will approximately form a sphere, i.e. Fermi sphere. For any a given electron number density with a highly degenerate state in the interior of a neutron star, the stronger the magnetic field, the larger the maximum of \( p_z \) is, hence the lower the number of states in the \( x - y \) plane according to the Pauli exclusion principle (each microscopic state is occupied by one electron only). In other words, \( n_{\text{max}}(p_z, b) \) and the number of electrons in the \( x - y \) plane decrease with the increase of \( B \), the radius of the Fermi sphere \( p_F \) is expanded which implies that the electron Fermi energy \( E_F(e) \) also increases. The higher the Fermi energy \( E_F(e) \), the more obvious the "expansion" of the Fermi sphere is, however the majority of the momentum space in the Fermi sphere is empty for not being occupied by electrons.

3. The relation between the electron Fermi energy and the magnetic field strength

By using the relation \( 2\mu_e B_{cr}/m_e c^2 = 1 \) and summing over electron energy states in a 6-dimension phase space, we can express \( N_{\text{phase}} \) as follows:

\[
N_{\text{phase}} = \frac{2\pi}{\hbar^2} \int_0^{p_F} dp_z \sum_{n=0}^{n_{\text{max}}(p_z, b)} \delta\left(\frac{p_\perp}{m_e c} - [(2n+1+\sigma)b]^{\frac{1}{2}}\right) p_\perp dp_\perp
\]

\[
= 2\pi(\frac{m_e c}{\hbar})^3 \int_0^{E_F(e)/m_e c^2} d\frac{p_\perp}{m_e c} \sum_{n=0}^{n_m(p_z, \sigma = -1, b)} g_n \int \delta\left(\frac{p_\perp}{m_e c} - (2nb)^{\frac{1}{2}}\right) dp_\perp \frac{p_\perp}{m_e c} dp_\perp
\]

\[
+ \sum_{n=0}^{n_m(p_z, \sigma = 1, b)} g_n \int \delta\left(\frac{p_\perp}{m_e c} - (2n+1)^{\frac{1}{2}}\right) dp_\perp \frac{p_\perp}{m_e c} dp_\perp, \tag{3.1}
\]
where $\delta (\frac{p_z}{m_e c} - [(2n + 1 + \sigma)b \frac{e}{c}])$ is the Dirac $\delta$-function, which is to express the quantization of Landau levels in the direction perpendicular to the applied magnetic field, $g(n)$ is the statistical weight of an energy level with quantum number $n$. According to atomic physics and nuclear physics, the higher the quantum number $n$, the larger the probability of a particle’s transition (this transition is referred to the transition from the higher energy level into lower energy level) is; the longer the lifetime of a particle is, or the wider the energy width is, then the larger the density of energy levels is and the more microscopic state number is. As to the statistical weight $g(n)$, we may take such an assumption:

$$g(n) = g_0 n^\alpha,$$  \hspace{1cm} (3.2)

where $g_0$ and $\alpha (\alpha \geq 0)$ are the statistical weight coefficient and the statistical weight index, respectively, in a given model. We then have

$$N_{\text{phase}} = \frac{\pi}{2\alpha + 3} \frac{\pi}{b(n+1)} g_0 I(\alpha) \left(\frac{m_e c}{\hbar}\right)^3 \left(\frac{E_F}{m_e c^2}\right)^{(2\alpha + 4)},$$  \hspace{1cm} (3.3)

where $I(\alpha) = \int_0^1 (1 - t^2)^{(\alpha + \frac{1}{2})}$. In the presence of a superhigh magnetic field, the energy state density of electrons $\rho_e$ can be expressed as:

$$\rho_e \approx \frac{2^{2(1-\alpha)}}{2\alpha + 3} \frac{\pi}{b(n+1)} g_0 \left(\frac{m_e c}{\hbar}\right)^3 \left(\frac{E_F}{m_e c^2}\right)^2 \left(\frac{E(e)}{m_e c^2}\right)^{1/2},$$  \hspace{1cm} (3.4)

According to the Pauli exclusion principle, electron state density $N_{\text{phase}}$ should be equal to electron number density $n_e$, $n_e = N_A \rho Y_e$, where $N_A$ is the Avogadro constant $N_A = 6.02 \times 10^{23}$, $Y_e$ is the mean electron number per baryon and $\rho$ is the matter density of a neutron star. Then we obtain the Fermi energy of electrons in ultrastrong magnetic fields

$$\frac{E_F(e)}{m_e c^2} = C \frac{Y_e}{0.05 \rho_0} \left[\frac{1}{\alpha + 1} \frac{1}{b(n+1)} \frac{1}{b(n+1)}\right],$$  \hspace{1cm} (3.5)

where $C$ is a constant relevant to a given model, the value of $C$ is determined by

$$C = (6.7 \times 10^{35}) \left(\frac{1}{\alpha + 1}\right) \left(2.44 \times 10^{-10}\right) \left(\frac{1}{\alpha + 1}\right) \left(\frac{2\alpha (2\alpha + 3)}{g_0 I(\alpha)}\right).$$  \hspace{1cm} (3.6)

Comparing eq.(3.5) with the known Fermi energy of electrons in a weak magnetic field $E_F(e) = 60(\frac{X}{\rho_0})^{0.5}$ MeV [10] gives the value of $g_0$. For simplicity, we discuss three simple models: $\alpha = 0, \alpha = 0.5$ and $\alpha = 1.0$:

1. For the model $\alpha = 0$ we have

$$E_F(e) = 60 \left[\frac{Y_e}{0.05 \rho_0}\right]^{0.5} (b > 1)$$  \hspace{1cm} (3.7)

2. For the model $\alpha = 0.5$ we have

$$E_F(e) = 60 \left[\frac{Y_e}{0.05 \rho_0}\right]^{0.5} (b > 1)$$  \hspace{1cm} (3.8)

3. For the model $\alpha = 1$ we have

$$E_F(e) = 60 \left[\frac{Y_e}{0.05 \rho_0}\right]^{0.5} (b > 1)$$  \hspace{1cm} (3.9)
where $S$ is the total magnetic field energy released into radiation energy as soft X-rays and $\gamma$-rays, $kT \sim \mu_n B \approx 10 B_{15} \text{ KeV}$. Before calculating $L_x$ of magnetars, we can make a simple evaluation as following: if all $^3P_2$ Cooper pairs are destroyed, the total magnetic field energy released is

$$E = \frac{1}{2} qN_A m(3P_2) \times 2\mu_n B \approx 1 \times 10^{47} B_{15} \frac{m(3P_2)}{0.1M_{\odot}} \text{ ergs.} \quad (3.10)$$

If the total magnetic field energy released can be transformed into radiation energy, and if the persistent X-ray fluxes observed in AXPs are powered by magnetic field energy of magnetars, magnetars may maintain over $10^4 - 10^6$ yrs for a luminosity of X-ray $\sim 10^{34} - 10^{36}$ ergs s$^{-1}$. The electron capture rate, $\Gamma$, is defined as the number of electrons captured by one proton per second, and can be computed using standard charged-current $\beta$-decay theory. The expression for $d\Gamma$ reads:

$$d\Gamma = \frac{2\pi}{\hbar} G^2_F C^2_V (1 + 3a^2)/(1 - f_\nu) \rho_\nu d\nu \delta(E_\nu + Q - E_e), \quad (3.11)$$

where $E_\nu = E_e - Q$, $T = 10^8$ K, $\rho_\nu = \frac{(E_\nu - Q)^2}{2\pi^2\hbar^2 c^2}$ and other terms appearing in eq.(3.11) have already been defined in Chapter 18 of [10]. The total magnetic field energy released is calculated by

$$L_x = \xi V(3P_2)^2 \frac{2\pi^4}{\hbar^4 V_1} G^2_F C^2_V (1 + 3a^2) \int dn_\nu d\nu_p d\nu_\rho d\nu_e \delta(E_\nu + Q - E_e) \times \delta(3\vec{K}_e - \vec{K}_\nu) S \delta(E_\nu), \quad (3.12)$$

where $S = f_e f_p (1 - f_n)(1 - f_\nu) \approx 1$, $V(3P_2)$ denotes the volume of $^3P_2$ anisotropic neutron superfluid ($V(3P_2) = \frac{4}{3}\pi R_3^3$, $R_3 = 10^5$ cm), $\xi$ is the probability of the reaction $n + (n \uparrow n \downarrow) \rightarrow n + n + n$. The value of $\xi$, though known as $\xi \ll 1$, should be calculated by condensed physics, however, $\xi$ can be estimated roughly by comparing the calculations with the observations. From eq.(3.12), the

<table>
<thead>
<tr>
<th>$B$ (G)</th>
<th>$b$ ($B_{B_e}$)</th>
<th>$E_F(e, \alpha = 0)$ (MeV)</th>
<th>$E_F(e, \alpha = 0.5)$ (MeV)</th>
<th>$E_F(e, \alpha = 1)$ (MeV)</th>
</tr>
</thead>
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<tr>
<td>2.0×10^{14}</td>
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<td>87.54</td>
<td>94.41</td>
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<td>116.23</td>
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<td>115.21</td>
<td>131.26</td>
<td>143.19</td>
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<tr>
<td>8.0×10^{14}</td>
<td>18.124</td>
<td>123.80</td>
<td>143.10</td>
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<tr>
<td>1.0×10^{15}</td>
<td>22.655</td>
<td>130.90</td>
<td>153.00</td>
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<tr>
<td>2.0×10^{15}</td>
<td>45.310</td>
<td>155.67</td>
<td>188.37</td>
<td>213.90</td>
</tr>
</tbody>
</table>

Table 1: The values of $E_F(e)$ in three simple models when $\rho_\rho = 2.8 \times 10^{14}$ g cm$^{-1}$ and $Y_e = 0.05$. The calculation results of $E_F(e)$ are shown partly in Table 1.
The physics of Magnetars
Qiu-he Peng

value of $L_x$ can be gained $\sim 10^{34} - 10^{36}$ ergs s$^{-1}$ for magnetars while the mean value of $\zeta \sim 10^{-17}$ from the observations.

After a complicated process, we gain a general formula of $L_X$ suitable to any model. The expression of $L_X$ reads:

$$L_X = \zeta^4 \frac{4}{3} \pi R_5^3 \times \frac{2\pi}{h} \frac{1}{\alpha} \times G_F C_V (1 + 3\alpha^2) \times \frac{2\pi \sqrt{2} m_n^2}{m_e^3} \times \frac{2(1-a)}{2\alpha + 3} \times \frac{\pi}{b^{\alpha+1}} \times g_0 \times \left( \frac{m_e c}{h} \right)^3 \times \frac{1}{m_e c^2} \times \frac{1}{(m_e c^2)^2 (\alpha + \frac{1}{2})} \times 2\mu_B \times (1.60 \times 10^{-6})^{2\alpha + 7.5} \times \int_{E_0}^{E_F(e)} \left( (E_n + 0.61 - E_n)^2 \right) \times \left( (E_F(e))^2 + (E_e)^2 \right)^{2+\alpha} dE_e,$$

where the relation $1$ MeV $= 1.6 \times 10^{-6}$ ergs is used. Inserting the values of following constants: $G_F = 1.4358 \times 10^{-49}$ erg cm$^3$, $C_V = 0.9737$, $a = 1.253$, $h = 1.055 \times 10^{-27}$ erg $s^{-1}$, $h = 6.63 \times 10^{-27}$ erg $s^{-1}$, $m_e = 9.109 \times 10^{-28}$ g, $m_n = 1.67 \times 10^{-24}$ g, $c = 3 \times 10^{10}$ cm $s^{-1}$, $\mu_B = 0.966 \times 10^{-21}$ erg G$^{-1}$, $R_5 = 10^5$ cm, into eq.(3.12) and using the data in Table 1 gives the value of $L_X$ in any different strong magnetic field.

4. Acknowledgments
This research is supported by Chinese National Science Foundation through grant No. 10773005, and is supported in part by XinJiang Natural Science Foundation No. 2009211B35, the key Directional Project of CAS and NNSFC under the project No.10173020, No. 10673021.

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