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Physics on huge X-ray luminosity of magnetars

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In a superhigh magnetic field, direct Urca reactions can proceed for an arbitrary proton concentration. The magnetic field is the main energy source of all the persistent and bursting emissions observed in anomalous X-ray pulsars (AXPs) and soft gamma-ray repeaters (SGRs). Employing the assumption of micro-state number, we deduce the formula of electron Fermi energy $E_F(e)$ in superhigh magnetic fields. Based on observing and introducing relevant parameters introduced, a approximate way of calculating the X-ray luminosity of a Magnetar L_X in any superhigh magnetic field is introduced.

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1. Introduction

AXPs and SGRs are a small group of peculiar neutron stars (NSs) that are currently believed to have superhigh magnetic fields B$\sim 10^{14}-10^{15}$ G, and are hence identified as magnetar candidates [1], [3], [4], [5], [6], [11]. The magnetic fields of magnetars deduced from their long spin periods ($P\sim$ 5 -12 s) and spin-down rates ($\dot{P}\sim 10^{-10}-10^{-13}$ s s^{-1}) based on magnetic dipole radiation, are so strong as to reach two orders of magnitude larger than the quantum critical threshold B_{cr}=4.414 $\times 10^{14}$ G. Moreover, a similar estimate of a superhigh magnetic field is also derived from the observation of an absorption line in the X-ray spectrum of SGR 1806-20 [2]. Another evidence of a superhigh magnetic field is that, a strong magnetic field is required to confine a fire ball effectively when modeling giant flares of SGRs [7]. For most AXPs, the observed values of $L_X\sim 10^{34}-10^{36}$ ergs s^{-1}, and the decaying pulsating tails of giant flares may have higher energy $\sim 10^{44}$ ergs, which can not be explained by the loss of rotation energy of a neutron star. It is generally believed that such huge energy released may originate from the magnetic field energy of a magnetar. For magnetars, the superhigh magnetic fields could be from the induced magnetic moments of $^3P_2$ Cooper pairs in an anisotropic neutron superfluid at a lower interior temperature [8], [9]. Now, we briefly summarize the results of our previous job, which is relevant to our current job (studying huge X-ray luminosity of magnetars). The main points are as following:

(1). Calculations show that the fossil magnetic field of a neutron star, formed during core-collapse, will be magnified 91 times by Pauli paramagnetism due to the presence of a relativistic degenerate electron gas [8].

(2). The total magnetic field induced by $^3P_2$ Cooper pairs is $B^{in}=B_{max}f(\mu_n B/kT)$, B_{max}= $2.02\times 10^{14}\eta$ G, where $f(x)$ is the Brillouin function, $f(x)=\frac{2\sinh(2x)}{1+\cosh(2x)}$, when $x\ll 1$, $f(x)\approx 4x/3$; when $x\gg 1$, $f(x)\rightarrow 1$ [9].

(3). The quantity q depicts the probability for two neutrons being combined into a $^3P_2$ Cooper pair, $q\sim 0.087$. More specifically, $\eta=\frac{m(^3P_2)}{0.1M_{sun}}R_{NS,6}^{-3}(\frac{\Delta_n(^3P_2)}{0.05MeV})^{0.5}$, here η is the dimensionless factor describing both the macroscopic and microscopic properties of neutron stars and $R_{NS,6}$ is the radius of a neutron star in units of 10^6 cm. When the temperature decreases to T_c, the value of the induced magnetic field increases and just reaches that of a magnetar, T_c is the Curie Temperature of a phase transition from paramagnetism to ferromagnetism.

In this paper, the physics of huge X-ray luminosity of magnetars will be investigated.

2. Fermi sphere of electrons under ultrastrong magnetic field

As far as we know, in a weak magnetic field ($B\ll B_{cr}$), the Fermi surface of electrons is spherical, and the quantized Landau levels are insignificant, on the other hand, when in an intense magnetic field ($B\gg B_{cr}$), the behavior of an electron gas is very different from that of a weak magnetic field: the Fermi sphere becomes a Landau column and the energy levels perpendicular to the direction of the applied magnetic field are quantized. The energy of an electron with orbital quantum number n ($n=0,1,2,3,\cdots$, depicts the number of Landau levels) and spin quantum number σ ($\sigma=\pm 1$)in a superhigh magnetic field is given by

$$E_e^2(p_z,B,n,\sigma)=m_e^2c^4+p_z^2c^2+(2n+1+\sigma)2m_ec^2\mu_eB, \tag{2.1}$$

where p_z is the z-component of electron momentum, and $\mu_e \sim 0.927\times10^{-20}$ ergs G^{-1} is the magnetic moment of an electron. In the light of the Pauli exclusion principle, the electrons are situated in disparate energy states in order one by one from the lowest energy state up to the Fermi energy (the highest energy) with the highest momentum $p_F(z)$ along the magnetic field. In order to calculate electron state density N_{phase} (electron energy state number in a unit volume), we define a non-dimensional magnetic field: $b = B/B_{cr}$ and electron momentum perpendicular to the magnetic field: $p_\perp = m_e c\sqrt{(2n+1+\sigma)b}$. In the absence of a magnetic field, both dp_z and $dp_\perp$ change continuously and the microscopic state number N_{phase} in a volume element of phase space d^3xd^3p is d^3xd^3p/h^3. In the presence of a magnetic field, electrons are populated in many discrete Landau levels with n = 0, 1, 2, 3, $\cdots$, if $B \gg B_{cr}$, the Landau column becomes a very long and very narrow cylinder along the magnetic field, the overwhelming majority of electrons congregates in the lowest levels with n= 0 or n =1, 2, the section radius of a Landau column is $p_\perp$. For a given p_z , there is a corresponding maximum Landau level number n_{max}, whose expression is

$$n_{max}(p_z,b,\sigma=-1) \approx n_{max}(p_z,b,\sigma=1) = n_m(p_z,b) \tag{2.2}$$

$$n_{max}(p_z,b) = Int[\frac{1}{2b}[(\frac{E_F(e)}{m_ec^2})^2 - 1 - (\frac{p_z}{m_ec})^2] - 1], \tag{2.3}$$

where $Int[x]$ denotes an integer value of the argument x.

In an intense magnetic field, dp_z changes continuously along the z-axis direction, whereas $dp_\perp$ is not continuous and must obey the relation $p_\perp = m_e c\sqrt{(2n+1+\sigma)b}$. An envelope of the Landau circles with maximum quantum number $n_{max}(p_z,b)$ $(0 \le p_z \le p_F)$ will approximately form a sphere, i.e. Fermi sphere. For any a given electron number density with a highly degenerate state in the interior of a neutron star, the stronger the magnetic field, the larger the maximum of p_z is, hence the lower the number of states in the $x-y$ plane according to the Pauli exclusion principle (each microscopic state is occupied by one electron only). In other words, $n_{max}(p_z,b)$ and the number of electrons in the $x-y$ plane decrease with the increase of B, the radius of the Fermi sphere p_F is expanded which implies that the electron Fermi energy $E_F(e)$ also increases. The higher the Fermi energy $E_F(e)$, the more obvious the "expansion" of the Fermi sphere is, however, the majority of the momentum space in the Fermi sphere is empty for not being occupied by electrons.

3. The relation between the electron Fermi energy and the magnetic field strength

By using the relation $2\mu_e B_{cr}/m_ec^2 = 1$ and summing over electron energy states in a 6-dimension phase space, we can express N_{phase} as follows:

$$\begin{aligned} N_{phase} &= \frac{2\pi}{h^3}\int_0^{p_F} dp_z \sum_{n=0}^{n_m(p_z,\sigma,b)} \sum_{\sigma=\pm1} \int \delta(\frac{p_\perp}{m_ec} - [(2n+1+\sigma)b]^{\frac{1}{2}})p_\perp dp_\perp \\ &= 2\pi(\frac{m_ec}{h})^3 \int_0^{\frac{E_F(e)}{m_ec^2}} d(\frac{p_z}{m_ec})[\sum_{n=0}^{n_m(p_z,\sigma=-1,b)} g_n \int \delta(\frac{p_\perp}{m_ec} - (2nb)^{\frac{1}{2}})(\frac{p_\perp}{m_ec})d(\frac{p_\perp}{m_ec}) \\ &+ \sum_{n=0}^{n_m(p_z,\sigma=1,b)} g_n \int \delta(\frac{p_\perp}{m_ec} - (2(n+1)b)^{\frac{1}{2}})(\frac{p_\perp}{m_ec})d(\frac{p_\perp}{m_ec})], \end{aligned} \tag{3.1}$$

where $\delta(\frac{p_{\perp}}{m_e c} - [(2n+1+\sigma)b]^{\frac{1}{2}})$ is the Dirac δ-function, which is to express the quantization of Landau levels in the direction perpendicular to the applied magnetic field, $g(n)$ is the statistical weight of a energy level with quantum number n. According to atomic physics and nuclear physics, the higher the quantum number n , the larger the probability of a particle's transition (this transition is referred to the transition from the higher energy level into lower energy level) is; the longer the lifetime of a particle is, or the wider the energy width is, then the larger the density of energy levels is and the more microscopic state number is. As to the statistical weight $g(n)$, we may take such an assumption:

$$g(n) = g_0 n^{\alpha}, \tag{3.2}$$

where g_0 and α ($\alpha \geq 0$) are the statistical weight coefficient and the statistical weight index, respectively, in a given model. We then have

$$N_{phase} = \frac{2^{(1-\alpha)}}{2\alpha+3}\frac{\pi}{b^{(\alpha+1)}} g_0 I(\alpha)(\frac{m_e c}{h})^3(\frac{E_F}{m_e c^2})^{(2\alpha+4)}, \tag{3.3}$$

where $I(\alpha) = \int_0^1 (1-t^2)^{(\alpha+\frac{3}{2})}$. In the presence of a superhigh magnetic field, the energy state density of electrons ρ_e can be expressed as:

$$\rho_e \approx \frac{2^{2(1-\alpha)}}{2\alpha+3}\frac{\pi}{b^{(\alpha+1)}} g_0 (\frac{m_e c}{h})^3 \frac{1}{m_e c^2}[(\frac{E_F(e)}{m_e c^2})^2 - (\frac{E(e)}{m_e c^2})^2]^{\alpha+\frac{3}{2}}. \tag{3.4}$$

According to the Pauli exclusion principle, electron state density N_{phase} should be equal to electron number density n_e, $n_e = N_A \rho Y_e$, where N_A is the Avogadro constant N_A= 6.02 $\times 10^{23}$, Y_e is the mean electron number per baryon and ρ is the matter density of a neutron star. Then we obtain the Fermi energy of electrons in ultrastrong magnetic fields

$$\frac{E_F(e)}{m_e c^2} = C[\frac{Y_e}{0.05}\frac{\rho}{\rho_0}]^{\frac{1}{2(\alpha+2)}} b^{\frac{\alpha+1}{b(\alpha+2)}}, \tag{3.5}$$

where C is a constant relevant to a given model, the value of C is determined by

$$C = (6.7\times 10^{35})^{\frac{1}{2(\alpha+2)}}(2.44\times 10^{-10})^{\frac{3}{2(\alpha+2)}}[\frac{2^{\alpha}(2\alpha+3)}{g_0 I(\alpha)}]^{\frac{1}{2(\alpha+2)}}. \tag{3.6}$$

Comparing eq.(3.5) with the known Fermi energy of electrons in a weak magnetic field $E_F(e)$= $60(\frac{\rho}{\rho_0})^{\frac{2}{3}}$ MeV [10] gives the value of g_0. For simplicity, we discuss three simple models: α= 0 ,α= 0.5 and α= 1.0:

(1) For the model α = 0 we have

$$E_F(e) = 60[\frac{Y_e}{0.05}\frac{\rho}{\rho_0}]^{\frac{1}{4}} b^{\frac{1}{4}} MeV (b>1) \tag{3.7}$$

(2) For the model α = 0.5 we have

$$E_F(e) = 60[\frac{Y_e}{0.05}\frac{\rho}{\rho_0}]^{\frac{1}{5}} b^{0.3} MeV (b>1) \tag{3.8}$$

(3) For the model α = 1 we have

$$E_F(e) = 60[\frac{Y_e}{0.05}\frac{\rho}{\rho_0}]^{\frac{1}{6}} b^{\frac{1}{3}} MeV (b>1) \tag{3.9}$$

B (G)	b (B/B_{cr})	$E_F(e,\alpha=0)$ (MeV)	$E_F(e,\alpha=0.5)$ (MeV)	$E_F(e,\alpha=1)$ (MeV)
2.0×10^{14}	4.531	87.54	94.41	99.28
4.0×10^{14}	9.062	104.10	116.23	125.09
6.0×10^{14}	13.593	115.21	131.26	143.19
8.0×10^{14}	18.124	123.80	143.10	157.60
1.0×10^{15}	22.655	130.90	153.00	169.77
2.0×10^{15}	45.310	155.67	188.37	213.90

Table 1: The values of $E_F(e)$ in three simple models when ρ=ρ_0=2.8$\times10^{14}$ g cm^{-1} and Y_e=0.05.

The calculation results of $E_F(e)$ are shown partly in Table 1.

From Table 1, we may see the effects of superhigh magnetic fields on the Fermi energy of electrons. As soon as the energy of electrons near the Fermi surface are higher than the Fermi energy of neutrons ($E_F(n)\approx 60$ MeV [10], the process $e^- + p \rightarrow n + \nu_e$ will dominate. The resulting neutrons with high energy will react with the neutrons produced in the process $n+(n\uparrow n\downarrow)\longrightarrow n+n+n$. Then the $^3P_2$ anisotropic superfluid and the superhigh magnetic field induced by the $^3P_2$ Cooper pairs will disappear. When one $^3P_2$ Cooper pair is destroyed, magnetic field energy $\sim 2\mu_n B$ would be released and transformed into thermal energy, where μ_n = 0.966 $\times10^{-23}$ erg G^{-1} is the abnormal magnetic moment of a neutron. This thermal energy released then would be transformed into radiation energy as soft X-rays and γ-rays, $kT\simeq\mu_n B\simeq 10B_{15}$ KeV. Before calculating L_x of magnetars, we can make a simple evaluation as following: if all $^3P_2$ Cooper pairs are destroyed, the total magnetic field energy released is

$$E=\frac{1}{2}qN_A m(^3P_2)\times 2\mu_n B\simeq 1\times10^{47}B_{15}\frac{m(^3P_2)}{0.1M_{sun}}ergs. \tag{3.10}$$

If the total magnetic field energy released can be transformed into radiation energy, and if the persistent X-ray fluxes observed in AXPs are powered by magnetic field energy of magnetars, magnetars may maintain over 10^4-10^6 yrs for a luminosity of X-ray $\sim 10^{34}-10^{36}$ ergs s^{-1}. The electron capture rate, Γ, is defined as the number of electrons captured by one proton per second, and can be computed using standard charged-current β-decay theory. The expression for $d\Gamma$ reads:

$$d\Gamma=\frac{2\pi}{\hbar}G_F^2C_V^2(1+3a^2)(1-f_\nu)\rho_\nu dE_\nu\delta(E_\nu+Q-E_e), \tag{3.11}$$

where $E_\nu=E_e-Q$, $T=10^8$ K, $\rho_\nu=\frac{(E_e-Q)^2}{2\pi^2\hbar^3c^3}$ and other terms appearing in eq.(3.11) have already been defined in Chapter 18 of [10]. The total magnetic field energy released is calculated by

$$\begin{aligned} L_X &= \zeta V(^3P_2)\frac{(2\pi)^4}{\hbar V_1}G_F^2C_V^2(1+3a^2)\int dn_e dn_p dn_n dn_\nu \\ &\delta(E_\nu+Q-E_e)\times\delta^3(\overrightarrow{K_f}-\overrightarrow{K_i})S\Delta E(^3P_2), \end{aligned} \tag{3.12}$$

where$S=f_e f_p(1-f_n)(1-f_\nu)\approx 1$, $V(^3P_2)$ denotes the volume of $^3P_2$ anisotropic neutron superfluid ($V(^3P_2)=\frac{4}{3}\pi R_5^3, R_5=10^5$ cm),ζ is the probability of the reaction $n+(n\uparrow n\downarrow)\longrightarrow n+n+n$. The value of ζ, though known as $\zeta \ll 1$, should be calculated by condensed physics, however, ζ can be estimated roughly by comparing the calculations with the observations. From eq.(3.12), the value of L_x can be gained $\sim 10^{34}-10^{36}$ ergs s^{-1} for magnetars while the mean value of $\zeta\sim 10^{-17}$ from the observations.

After a complicated process, we gain a general formula of L_X suitable to any model. The expression of L_X reads:

$$L_X=\zeta\frac{4}{3}\pi R_5^3\times\frac{2\pi}{\hbar}\frac{1}{2\pi^2\hbar^3c^3}\times G_F^2C_V^2(1+3a^2)\times\frac{8\pi\sqrt{2}m_n^{\frac{3}{2}}}{h^3}\times\frac{2^{2(1-\alpha)}}{2\alpha+3}$$
$$\times\frac{\pi}{b^{\alpha+1}}\times g_0\times(\frac{m_ec}{h})^3\times\frac{1}{m_ec^2}\times\frac{1}{(m_ec^2)^{2(\alpha+\frac{3}{2})}}\times 2\mu_nB\times(1.60\times10^{-6})^{2\alpha+7.5}$$
$$\int_{60}^{E_F(e)}(E_n)^{\frac{1}{2}}dE_n\int_{E_n+0.61}^{E_F(e)}(E_e+0.61-E_n)^2((E_F(e))^2+(E_e)^2)^{\frac{3}{2}+\alpha}dE_e, \tag{3.13}$$

where the relation 1MeV= 1.6$\times10^{-6}$ ergs is used. Inserting the values of following constants: G_F= 1.4358 $\times10^{-49}$ erg cm ^{3}, C_V=0.9737, a=1.253, $\hbar$=1.055 $\times10^{-27}$ erg s^{-1}, h= 6.63$\times10^{-27}$erg s^{-1}, m_e= 9.109 $\times10^{-28}$ g, m_n= 1.67 $\times10^{-24}$ g, c= 3$\times10^{10}$ cm s^{-1}, μ_n=0.966$\times10^{-23}$ erg G^{-1}, R_5= 10^5 cm, into eq.(3.12) and using the data in Table 1 gives the value of L_X in any different strong magnetic field.

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