

**WE-Heraeus Summer School
on Nuclear Astrophysics in the Cosmos**

12-17 July 2010

**Element production
stellar evolution
explosive nucleosynthesis**

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Centre for Stellar and Planetary Astrophysics - Monash University - Australia

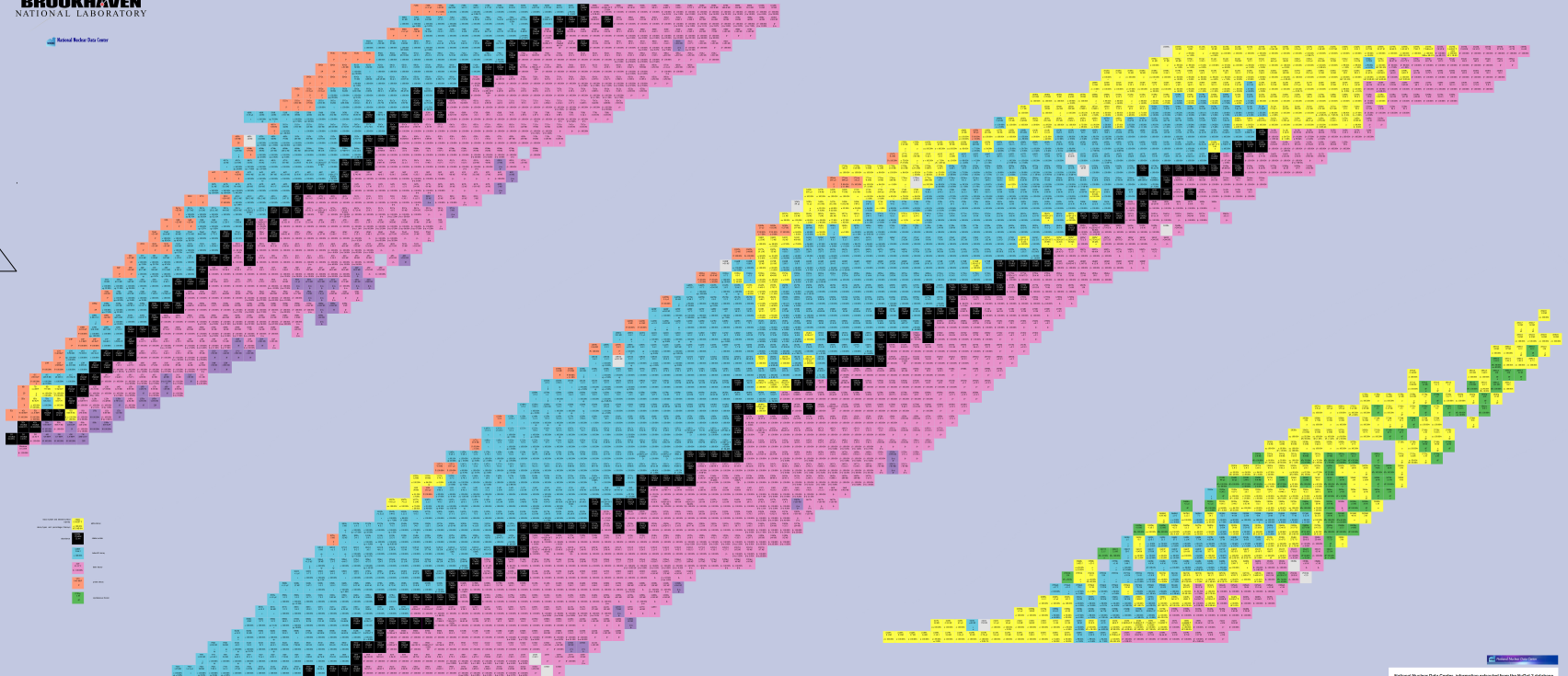
Email: alessandro.chieffi@iasf-roma.inaf.it

TABLE OF NUCLIDES

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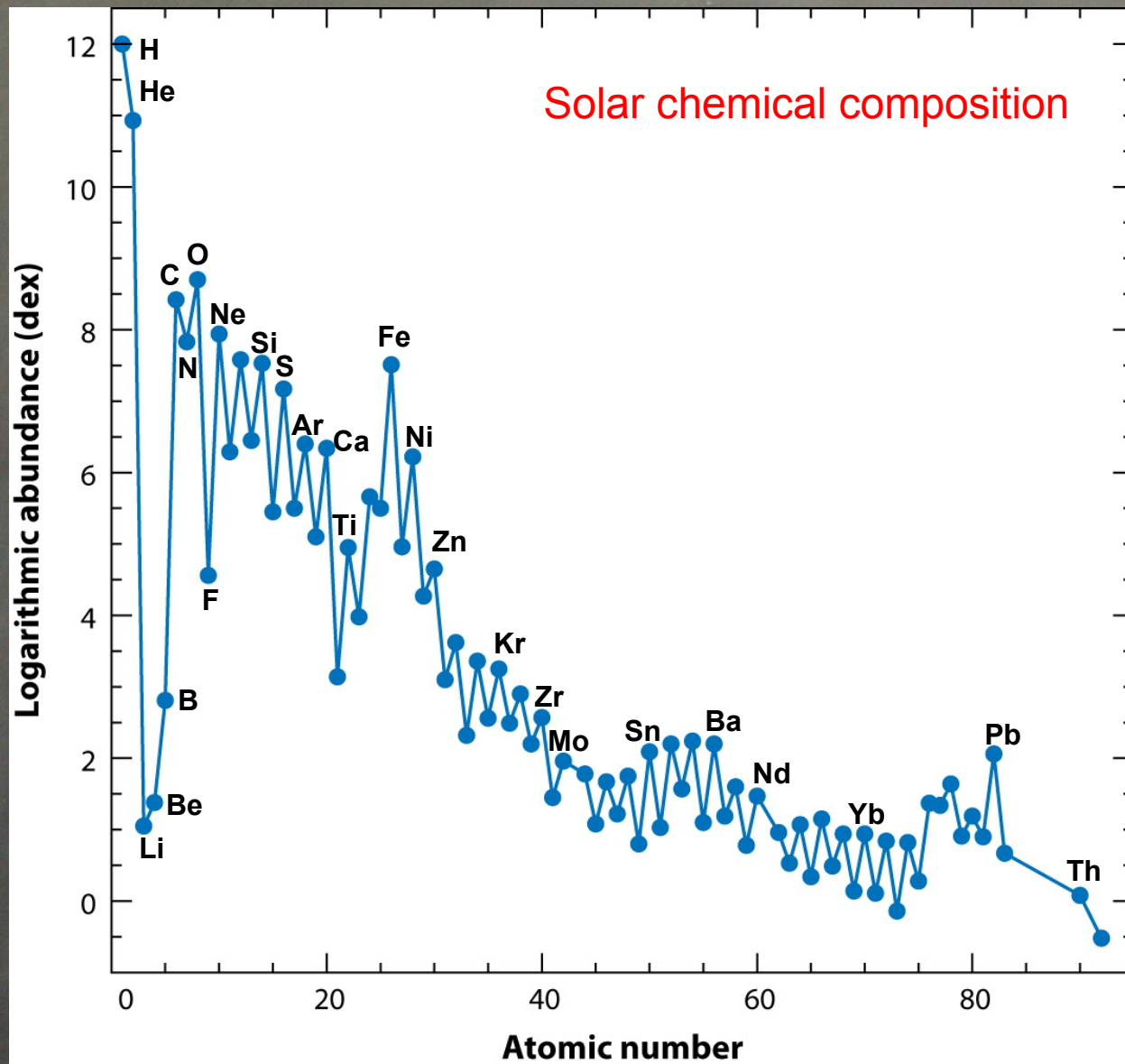
Nuclear Data Center

↑
protons



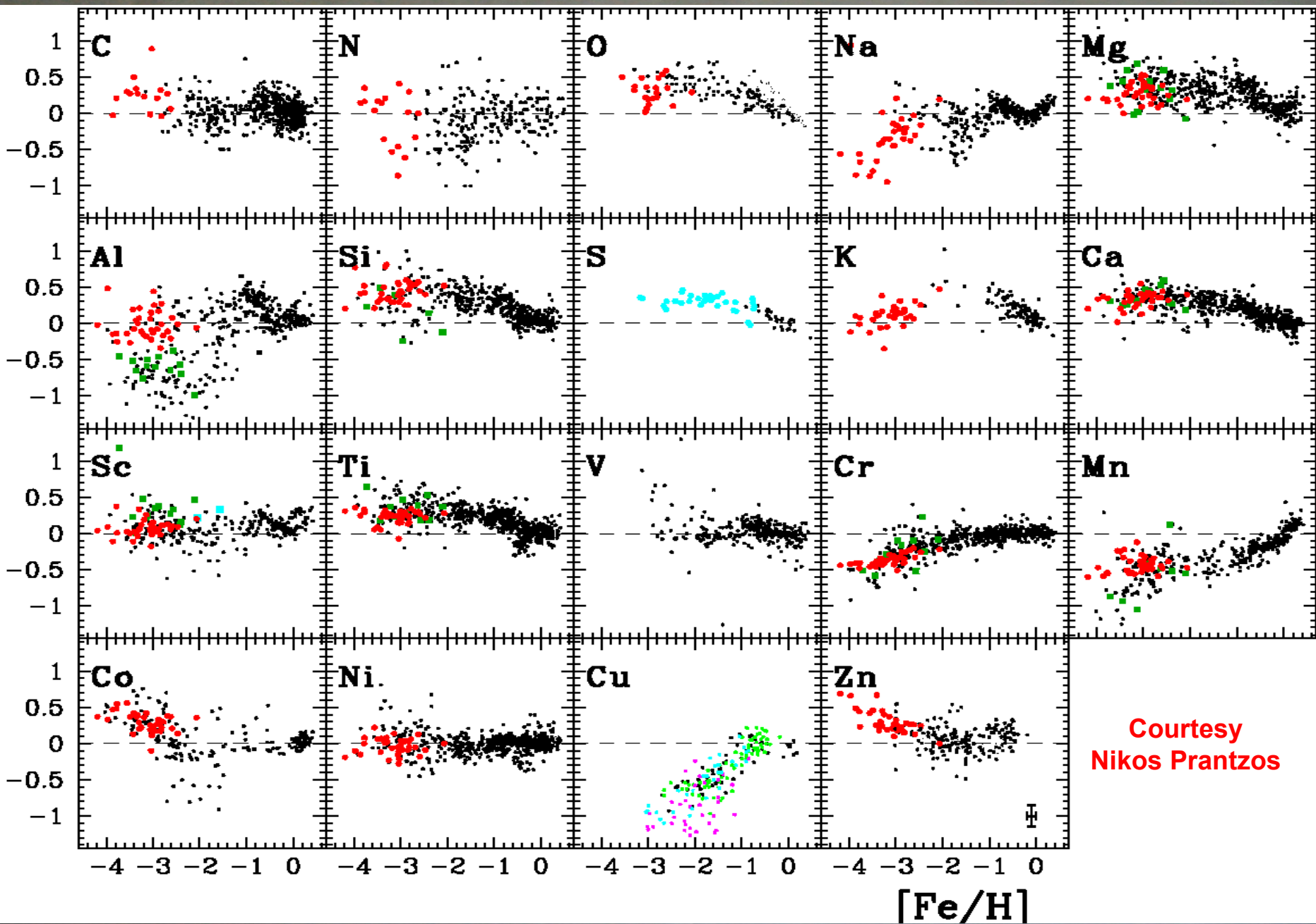
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neutrons

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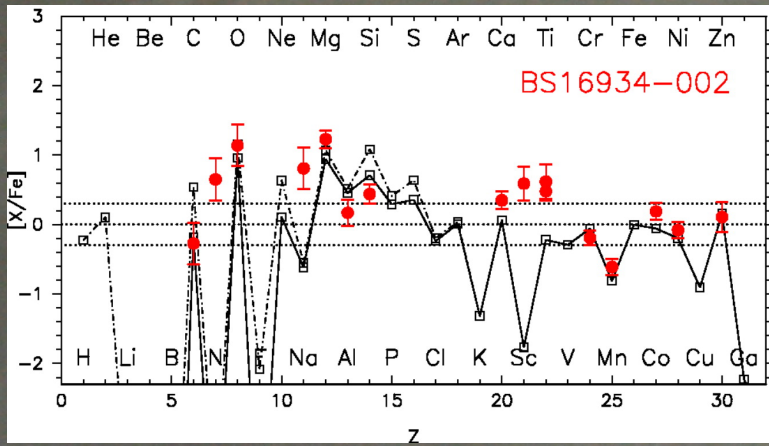


Asplund M, et al. 2009.

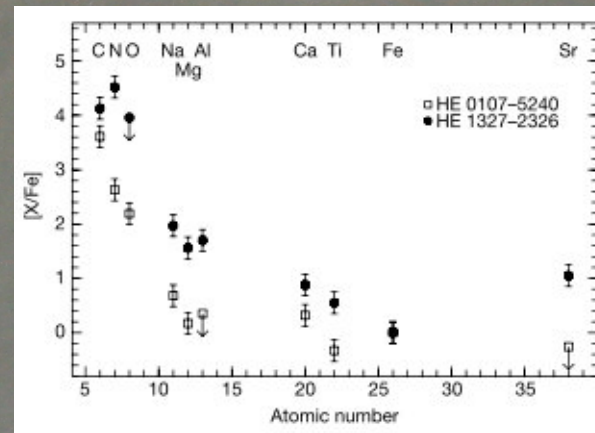
Annu. Rev. Astron. Astrophys. 47:481–522



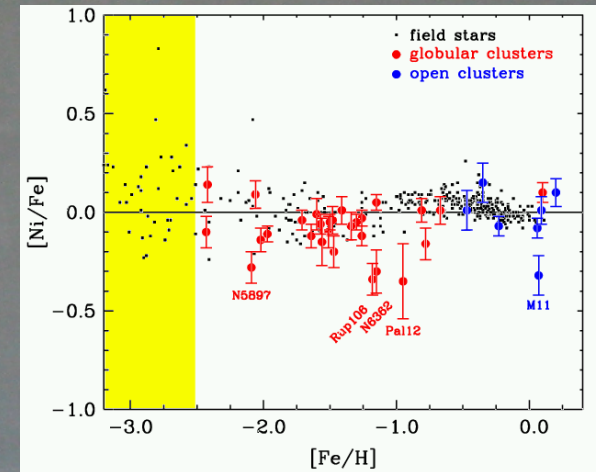
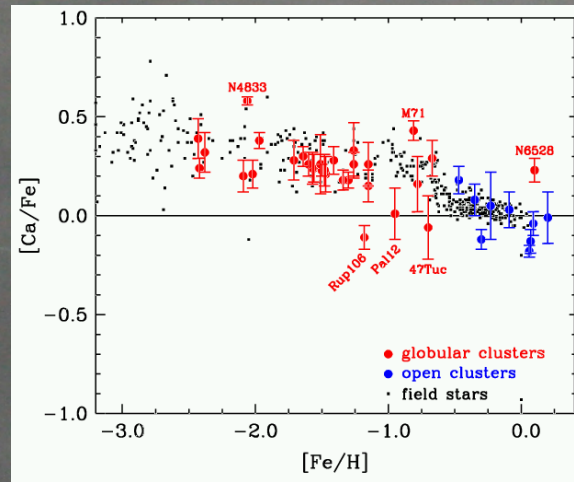
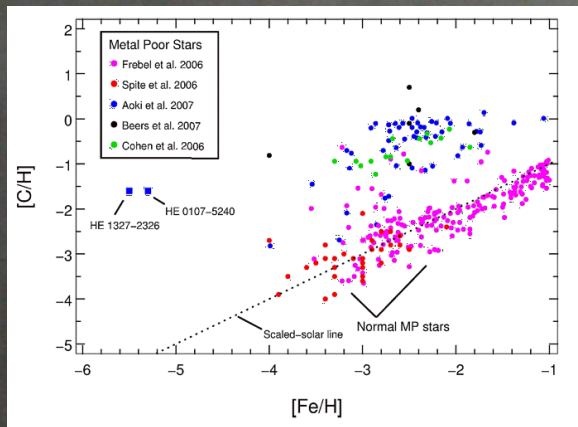
Courtesy
Nikos Prantzos



Aoki et al. 2007 ApJ 660,747

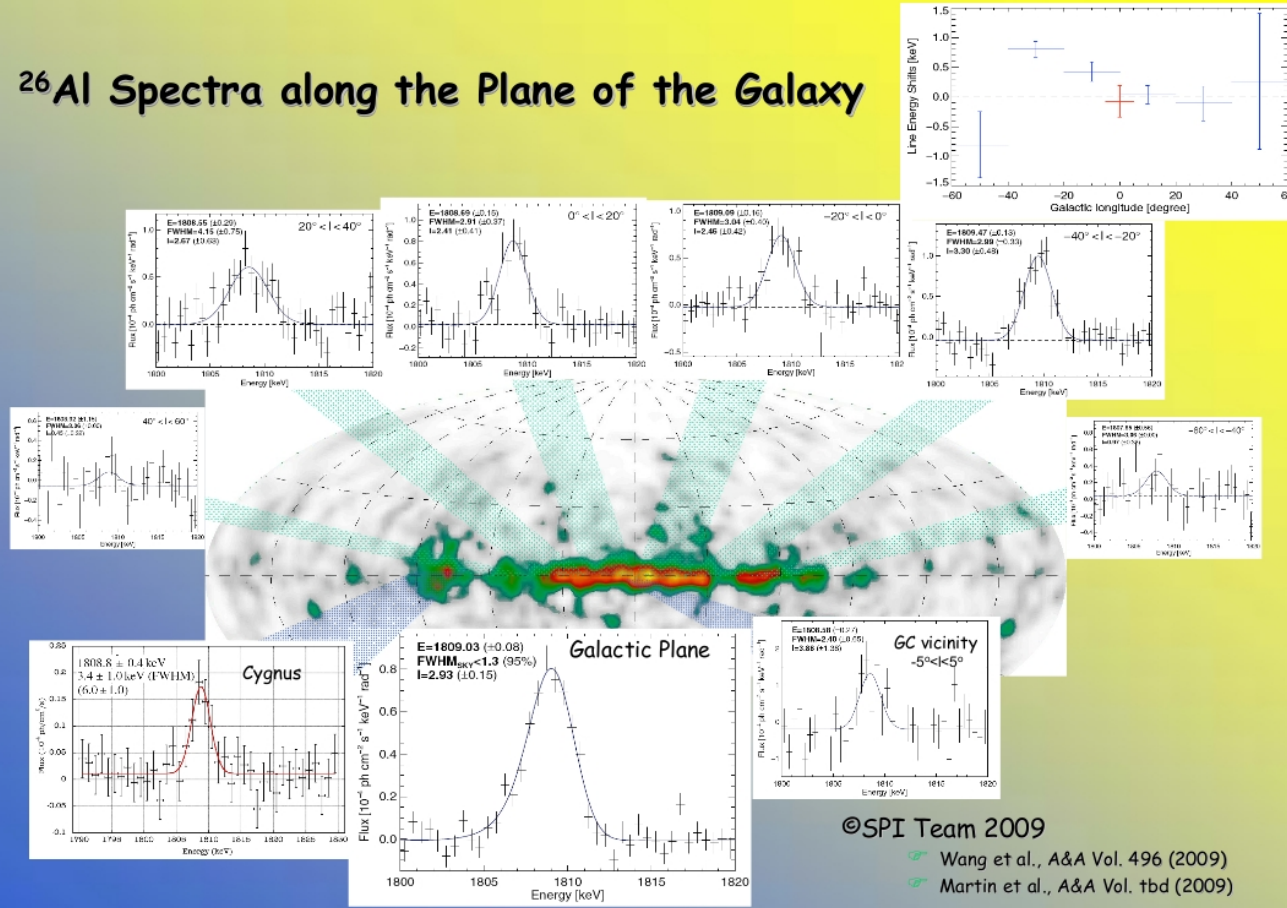


Frebel et al. 2005 Nature 434,871



Courtesy: C. Sneden

^{26}Al Spectra along the Plane of the Galaxy



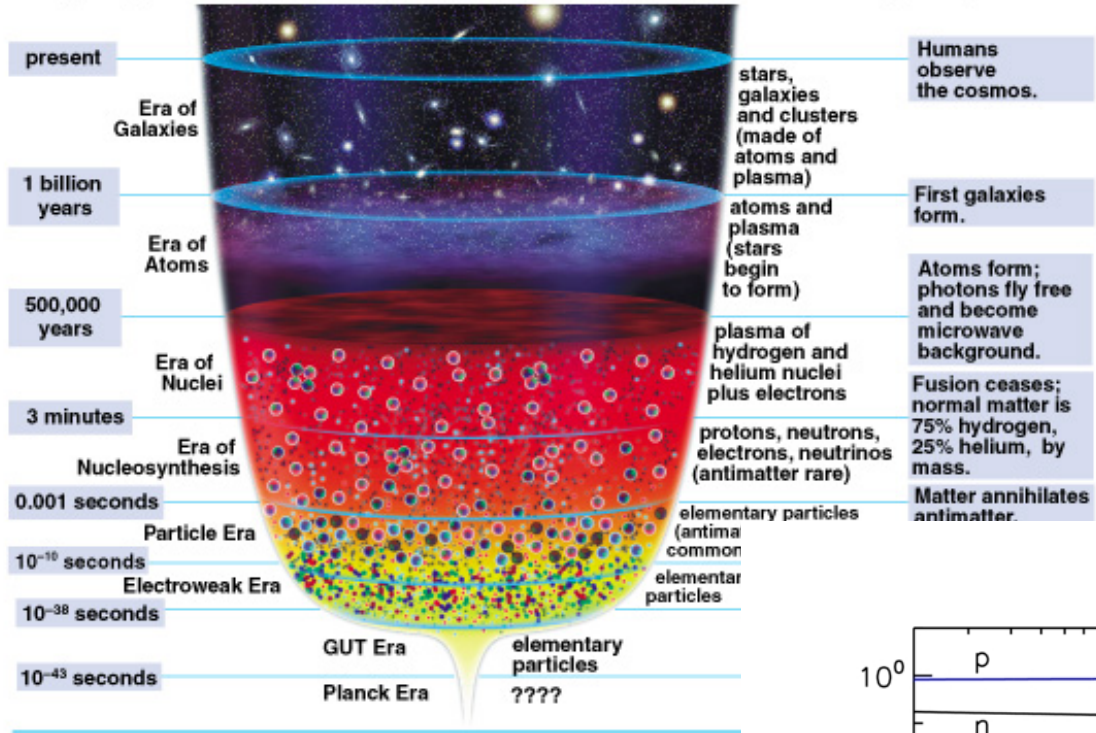
©SPI Team 2009
 Wang et al., A&A Vol. 496 (2009)
 Martin et al., A&A Vol. tbd (2009)

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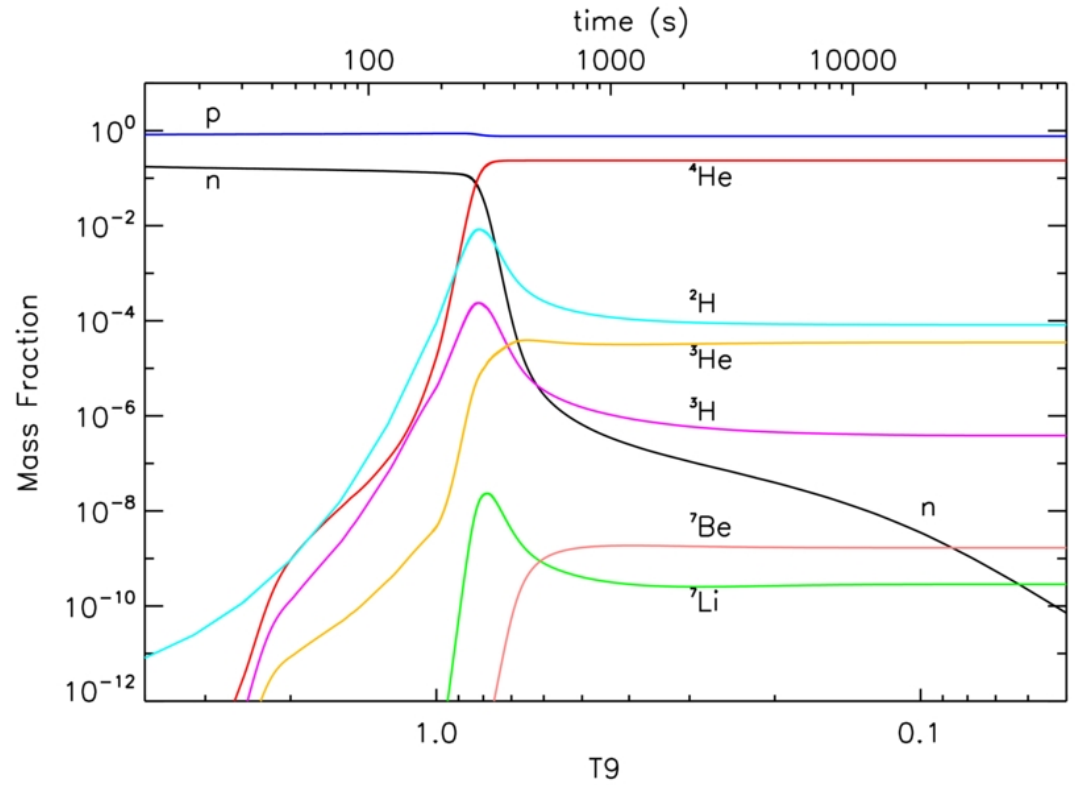
^{60}Fe ^{44}Ti ^{56}Ni

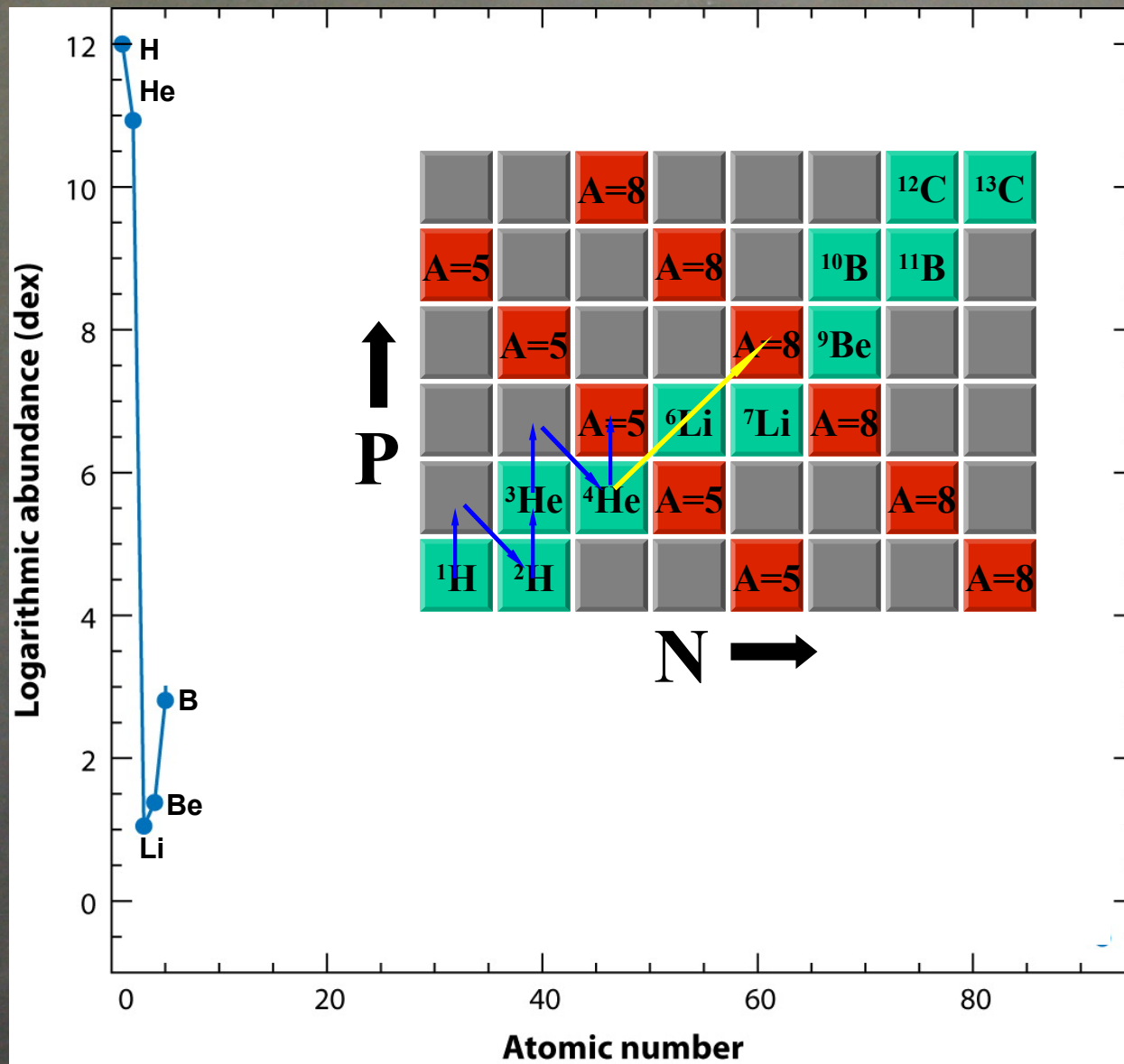
Time Since Big Bang

Major Events Since Big Bang



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Asplund M, et al. 2009.

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Evidence

The **Coulomb barrier** prevents an easy fusion between charged particles: only a combination of **high temperatures**, **high densities** and **long timescales** may lead to a substantial amount of fusion.

Even the fusion of the lightest nuclei, protons, requires

$T > \text{several } 10^6 \text{ K}$

$\rho > \text{several grams / cm}^3$

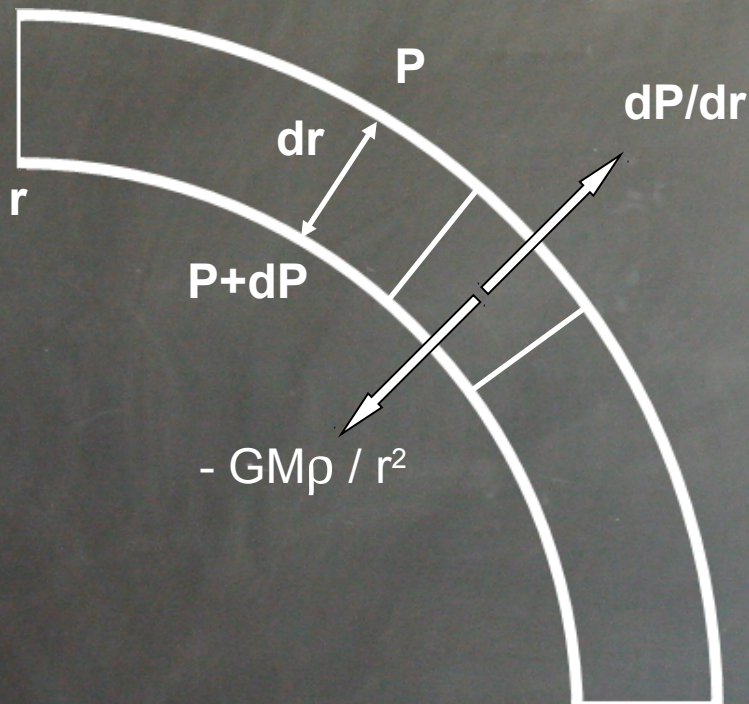
to burn a significant amount of nuclei on a timescale shorter than the age of the Universe

These conditions are met only in stars

A star is formed by a gas cloud that contracts under its own gravity and whose luminosity is produced in its interior

In many cases the contraction occurs on “long” timescales because matter naturally settles on a quasi equilibrium configuration in which the various forces acting on each element of matter tend to counterbalance each other:

$r+dr$



Hydrostatic equilibrium:

$$\frac{dP}{dr} = - \frac{GM \rho}{r^2}$$

Equation of continuity:

Mass conservation: $\frac{dM}{dr} = 4 \pi r^2 \rho$

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \quad \leftarrow \text{Hydrostatic equilibrium}$$

$$\int_0^M \frac{dP}{dr} \frac{r}{\rho} dm = -\int_0^M \frac{Gm}{r} dm$$

$$-\int_0^M \frac{Gm}{r} dm = \Omega$$

Gravitational potential energy

Ω may be regarded as the total amount of gravitational energy liberated in the contraction from "infinity" to the present configuration.

$$\int_0^M \frac{dP}{dr} \frac{r}{\rho} dm = \int_0^M \frac{dP}{dm} \frac{dm}{dr} \frac{r}{\rho} dm = \int_0^M \frac{dP}{dm} 4\pi r^2 \rho \frac{r}{\rho} dm = \int_0^M \frac{dP}{dm} 4\pi r^3 dm$$

This can be integrated by parts

$$(P 4\pi r^3)_0^M - \int_0^M P 4\pi \frac{dr^3}{dm} dm = -\int_0^M P 4\pi 3r^2 \frac{dr}{dm} dm = -\int_0^M P \frac{3}{\rho} dm$$

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \quad \leftarrow \text{Hydrostatic equilibrium}$$

$$\int_0^M \frac{dP}{dr} \frac{r}{\rho} dm = -\int_0^M \frac{Gm}{r} dm$$

$$-\int_0^M 3 \frac{P}{\rho} dm = -\int_0^M \frac{Gm}{r} dm = \Omega$$

Gravitational potential energy

Ω may be regarded as the total amount of gravitational energy liberated in the contraction from "infinity" to the present configuration.

At this point we need an equation of state, i.e. a relation between pressure and density

Let us firstly consider a perfect gas; in this case we can write:

$$\left. \begin{aligned} E &= \frac{3}{2} NKT \\ P &= NKT \end{aligned} \right\} P = \frac{2}{3} E \quad \Rightarrow \quad \frac{P}{\rho} = \frac{2}{3} u$$

Where u represents the energy per unit mass

$$-\int_0^M 3 \frac{2}{3} u dm = \Omega \quad \Rightarrow \quad -2 \int_0^M u dm = \Omega \quad \Rightarrow \quad -2 E_i = \Omega$$

$\int_0^M u dm = E_i$ <= total internal energy

Virial Theorem (perfect gas)

Virial theorem (perfect gas)

$$2 E_i + \Omega = 0$$

What does it mean?

$$\Delta E_i = -\frac{1}{2} \Delta \Omega$$

$\Delta \Omega$ is negative! hence a contraction implies necessarily an increase of the internal energy E_i . However only 50% of the energy gained by the gravitational field remains locked in the star, the other 50% must be lost!

The requirement that some energy must be lost in a contraction introduces the idea that the contraction requires some finite timescale to occur, it cannot occur instantaneously. Since energy is basically lost through photons from the surface, this timescale is dictated by the efficiency of the outward photon flux. In other words no additional contraction may occur until the energy losses required by the virial theorem have been effectively lost!

What about the total energy of the system?

$$E_{TOT} = E_i + \Omega \quad E_{TOT} = -\frac{1}{2} \Omega + \Omega \quad E_{TOT} = \frac{1}{2} \Omega \quad \Delta E_{TOT} = \frac{1}{2} \Delta \Omega$$

Once again, Ω is negative! Hence a contraction ($\Delta \Omega < 0$) implies a reduction of the total energy.

The system is more bound!

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \quad \leftarrow \text{Hydrostatic equilibrium}$$

$$\int_0^M \frac{dP}{dr} \frac{r}{\rho} dm = -\int_0^M \frac{Gm}{r} dm$$

$$-\int_0^M 3 \frac{P}{\rho} dm = -\int_0^M \frac{Gm}{r} dm = \Omega$$

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Virial Theorem (perfect gas)

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \quad \leftarrow \quad \text{Hydrostatic equilibrium}$$

$$-\int_0^M 3\frac{P}{\rho} dm = -\int_0^M \frac{Gm}{r} dm = \Omega$$

Gravitational potential energy

Ω may be regarded as the total amount of gravitational energy liberated in the contraction from “infinity” to the present configuration.

Generalized Virial Theorem

$$-\int_0^M 3(\gamma - 1)U dm = \Omega$$

$$3(\gamma - 1)E_i + \Omega = 0$$

$$\gamma = \frac{4}{3}$$

is very “special” because in this case

$$E_i + \Omega = 0 \quad \Delta E_i = -\Delta \Omega$$

All the energy gained by the gravitational field is stored in the star (as internal energy) and no energy is lost outward.

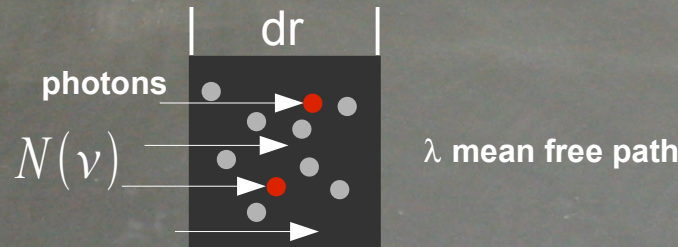
$$\Delta E_{TOT} = \Delta E_i + \Delta \Omega = -\Delta \Omega + \Delta \Omega = 0 \quad \text{For } \gamma=4/3 \text{ a contraction does not increase the binding energy but leaves } E_{TOT} \text{ constant!}$$

Since the contraction does not require the ejection of any energy no “delay” is necessary for a further contraction to occur. This is an unstable situation that leads to the collapse of the structure

The second basic equation necessary to describe a stellar structure is the one that controls the energy transport through the star.

Let us firstly assume that the energy is transported by radiation only:

The momentum (dq) transferred by a flux N of photons of frequency ν per unit time is given by:



Mean number of interactions

$$\lambda = \frac{1}{\kappa \rho}$$

Opacity coefficient

$$dq = N(\nu) \frac{h\nu}{c} \frac{dr}{\lambda} dS dt = \frac{L \kappa \rho}{4\pi R^2 c} dr dS dt$$

Momentum per photon

But, the momentum transferred may be also expressed as the variation of the radiation pressure:

$$dq = -dP_r dS dt = -\frac{1}{3} a dT^4 dS dt = -\frac{4}{3} a T^3 dT dS dt$$

By equating the two:

$$\frac{4}{3} a T^3 \frac{dT}{dr} = -\frac{L \kappa \rho}{4\pi R^2 c} \longrightarrow \frac{dT}{dr} = -\frac{3}{16\pi a c} \frac{\kappa \rho L}{R^2 T^3}$$

2nd basic equation

Associated continuity equation:

$$\frac{dL}{dM} = \epsilon = \epsilon_{nuc} + \epsilon_{grav} - \epsilon_\nu$$

SUMMARIZING

The set of equations that describe the structure of a star is given by:

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \quad \text{Hydrostatic equilibrium}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho \quad \text{Mass conservation}$$

$$\frac{dT}{dr} = -\frac{3}{16ac\pi} \frac{\kappa\rho L}{r^2 T^3} \quad \text{Energy transport (radiative case)}$$

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon \quad \text{Energy conservation}$$

Equation Of State, i.e. $P(\rho, T, c.c.)$

+

Opacity coefficient, i.e. $\kappa(\rho, T, c.c.)$

Energy generation coefficient, i.e. $\epsilon(\rho, T, c.c.)$

The solution of this system of equations is very difficult and requires COMPUTERS!

but...

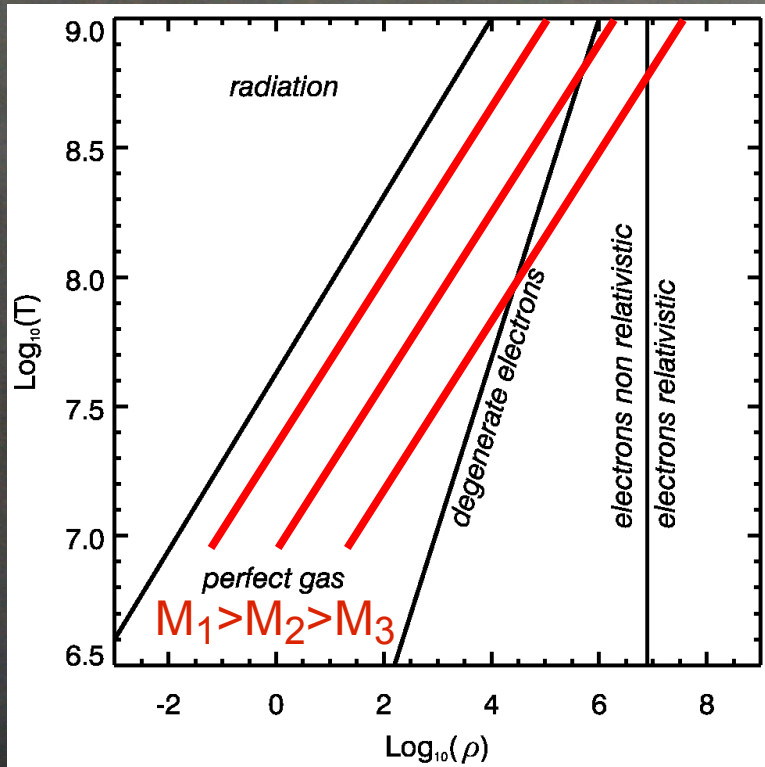
...we can try to be clever!

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \Rightarrow \frac{P}{R} \propto \frac{M\rho}{R^2} \Rightarrow P \propto \frac{M\rho}{R}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho \Rightarrow \frac{M}{R} \propto R^2 \rho \Rightarrow R \propto \left(\frac{M}{\rho}\right)^{\frac{1}{3}}$$

$$\left. \begin{array}{l} P \propto M^{\frac{2}{3}} \rho^{\frac{4}{3}} \\ P \propto M^3 \rho^{\frac{4}{3}} \end{array} \right\} \begin{array}{l} \text{Perfect gas} \\ P \propto \rho T \quad \frac{T^3}{\rho} \propto M^2 \\ \text{Pure radiation} \\ P \propto T^4 \quad \frac{T^3}{\rho} \propto M^{0.5} \end{array}$$

$$\log(T) = K(M) + \frac{1}{3} \log(\rho)$$



Interesting!

Just the hydrostatic equilibrium + perfect gas imply that the centre of a star must evolve along a straight line in the $\text{Log}(T_c)$ - $\text{Log}(\rho_c)$ plane.

What else?

The constant k scales inversely with the mass, so that the density increases as the mass decreases (for each fixed T)

We found that stars naturally separate in two basic groups: stars less massive than a critical value enter the region of electron degeneracy while the more massive ones don't!

$$\left. \begin{aligned} \frac{dP}{dr} = -\frac{Gm\rho}{r^2} &\Rightarrow \frac{P}{R} \propto \frac{M\rho}{R^2} \Rightarrow P \propto \frac{M\rho}{R} \\ \frac{dM}{dr} = 4\pi r^2 \rho &\Rightarrow \frac{M}{R} \propto R^2 \rho \Rightarrow R \propto \left(\frac{M}{\rho}\right)^{\frac{1}{3}} \\ \frac{dT}{dr} = -\frac{3}{16ac\pi} \frac{k\rho L}{r^2 T^3} &\Rightarrow \frac{T}{R} \propto \frac{\rho L}{R^2 T^3} \Rightarrow T^4 R^4 \propto ML \end{aligned} \right\} \begin{array}{l} P \propto M^{\frac{2}{3}} \rho^{\frac{4}{3}} \\ \text{Perfect gas } P \propto \rho T \\ TR \propto M \\ \text{Pure radiation } P \propto T^4 \\ TR \propto M^{0.5} \end{array}$$

$$\begin{array}{l} \text{Perfect gas } L \propto M^3 \\ \text{Pure radiation } L \propto M \end{array}$$

What about the surface temperature of the star?

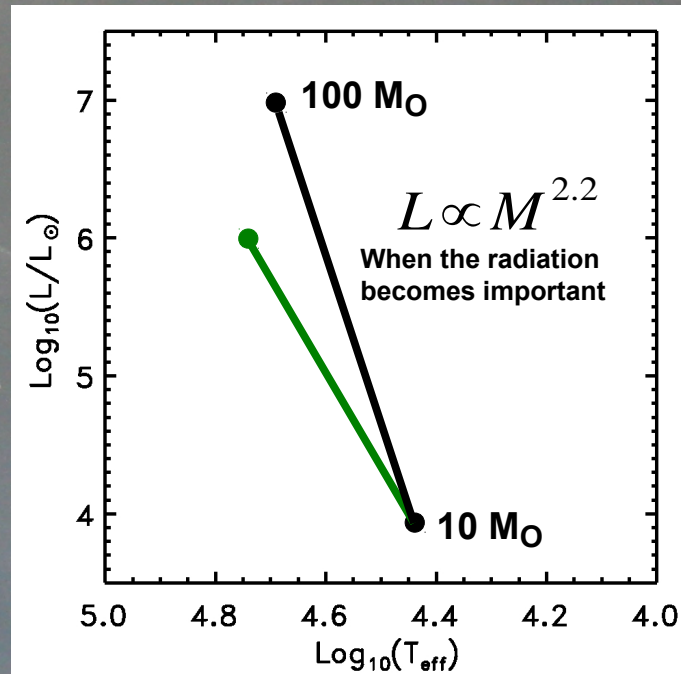
If we assume a black body: $L = 4\pi R^2 \sigma T_{eff}^4$

and also that

the central temperature is roughly independent on the mass :

$$R \propto M$$

$$M^3 \propto R^2 T_{eff}^4 \Rightarrow T_{eff}^4 \propto M \Rightarrow T_{eff} \propto M^{\frac{1}{4}}$$



$$\left. \begin{aligned}
 \frac{dP}{dr} &= -\frac{Gm\rho}{r^2} \Rightarrow \frac{P}{R} \propto \frac{M\rho}{R^2} \Rightarrow P \propto \frac{M\rho}{R} \\
 \frac{dM}{dr} &= 4\pi r^2 \rho \Rightarrow \frac{M}{R} \propto R^2 \rho \Rightarrow R \propto \left(\frac{M}{\rho}\right)^{\frac{1}{3}}
 \end{aligned} \right\} P \propto M^{\frac{2}{3}} \rho^{\frac{4}{3}}$$

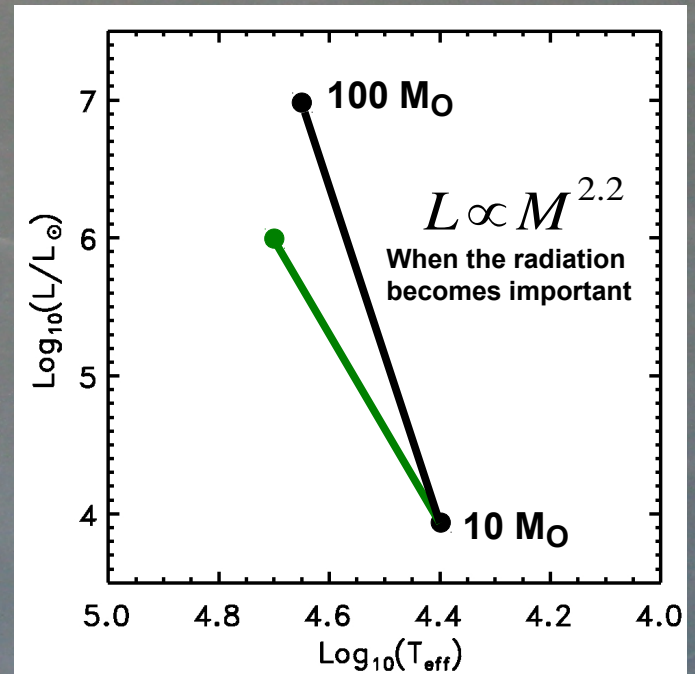
Perfect gas
 $P \propto \rho T$
 $TR \propto M$

$$\frac{dT}{dr} = -\frac{3}{16ac\pi} \frac{k\rho L}{r^2 T^3} \Rightarrow \frac{T}{R} \propto \frac{\rho L}{R^2 T^3} \Rightarrow T^4 R^4 \propto ML \Rightarrow L \propto M^3$$

What about the lifetime of the stars?

$$\tau \propto \frac{E}{L} \quad \tau \propto \frac{qM}{L} \quad \tau \propto \frac{qM}{M^3} \approx \frac{1}{M^2}$$

When radiation contributes significantly to the EOS $\Rightarrow \tau \propto \frac{qM}{M^{2.2}} \approx \frac{1}{M^{1.2}}$



We learned a lot of things up to now (without really solving any equation!)

If the EOS is dominated by a perfect gas:

hydrostatic equilibrium is “stable” because $\gamma > 4/3$

the evolution of the core follows a straight line in the $\text{Log}(T_c)\text{-Log}(\rho_c)$ plane

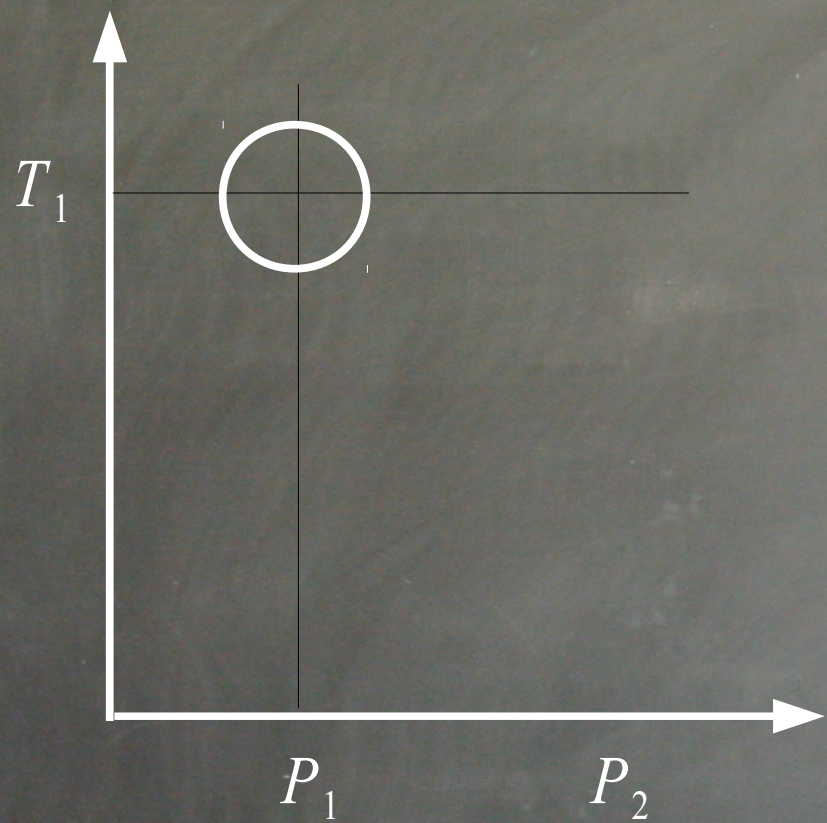
Star less massive than a critical value enter the region where degenerate electrons count

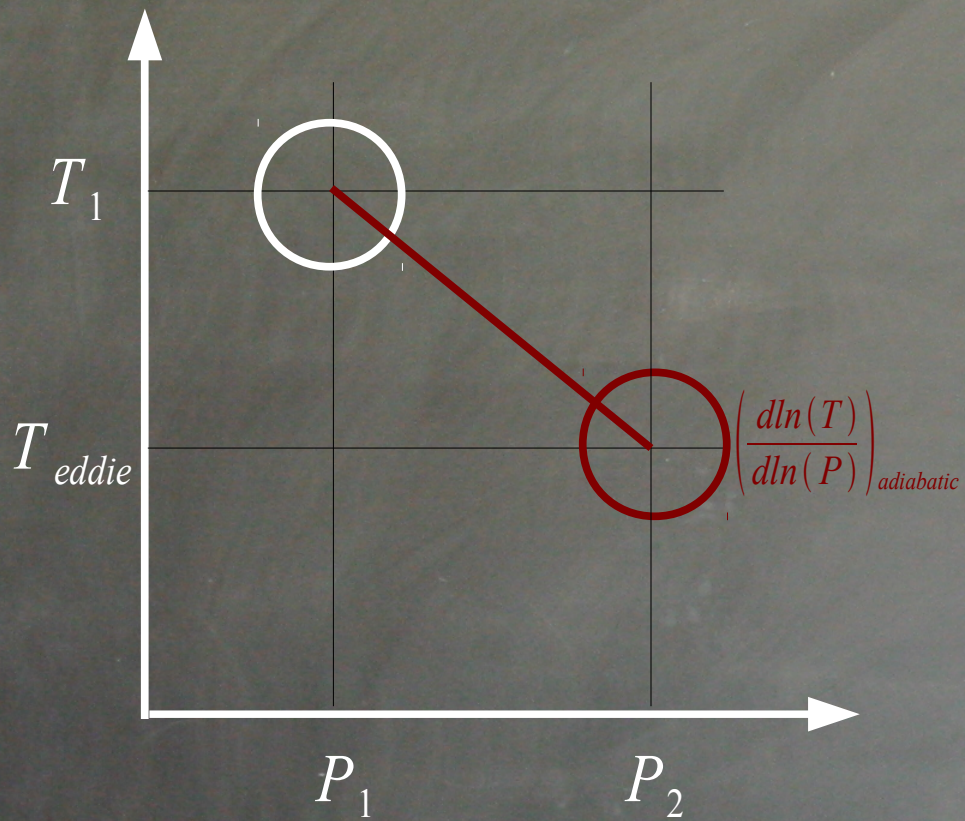
Star more massive than a critical value do not enter the region where degenerate electrons count (at least until the central temperature does not exceed a few billions of K)

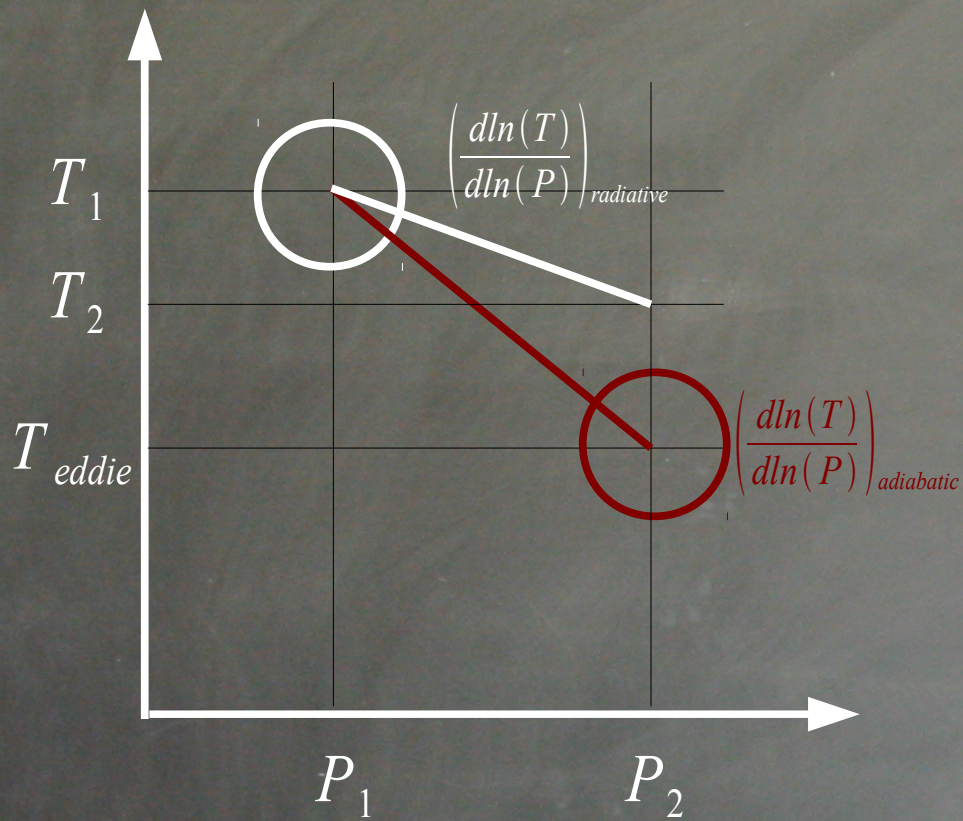
The energy losses from the surface (L) scale as M^3 (perfect gas) or as M (pure radiation)

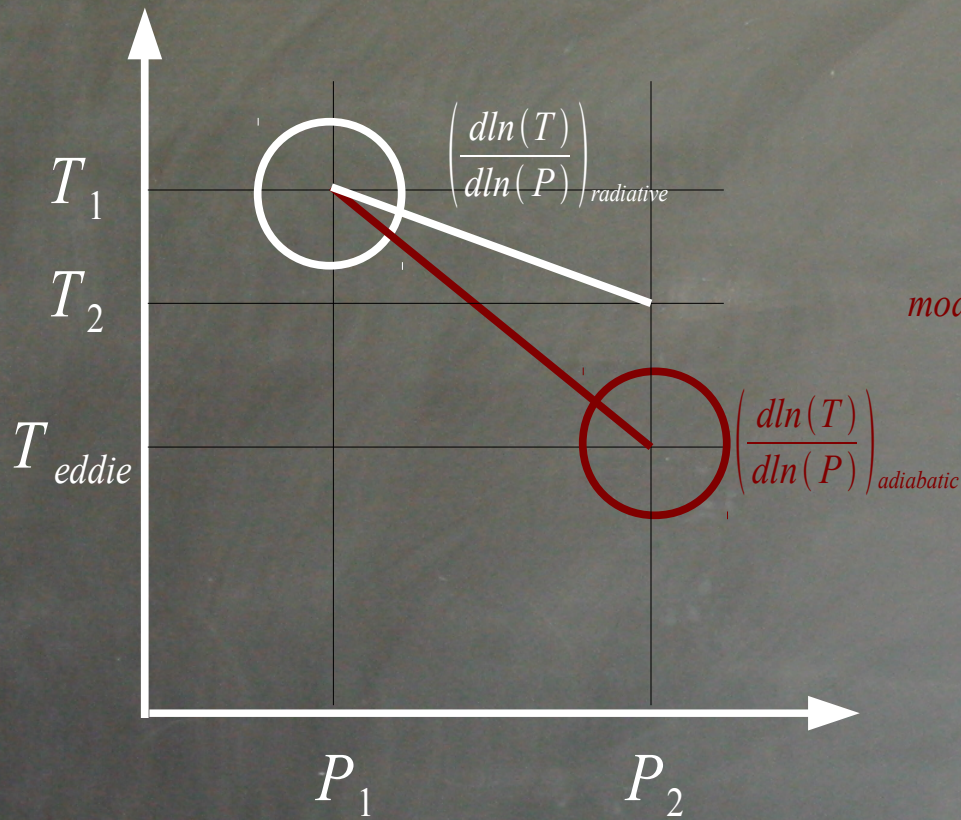
The lifetime of a star (t) scales as M^{-2}

Unfortunately this is not enough ... it's time to introduce CONVECTION









$$\text{mod} \left(\frac{d\ln(T)}{d\ln(P)} \right)_{\text{adiabatic}} > \text{mod} \left(\frac{d\ln(T)}{d\ln(P)} \right)_{\text{radiative}}$$

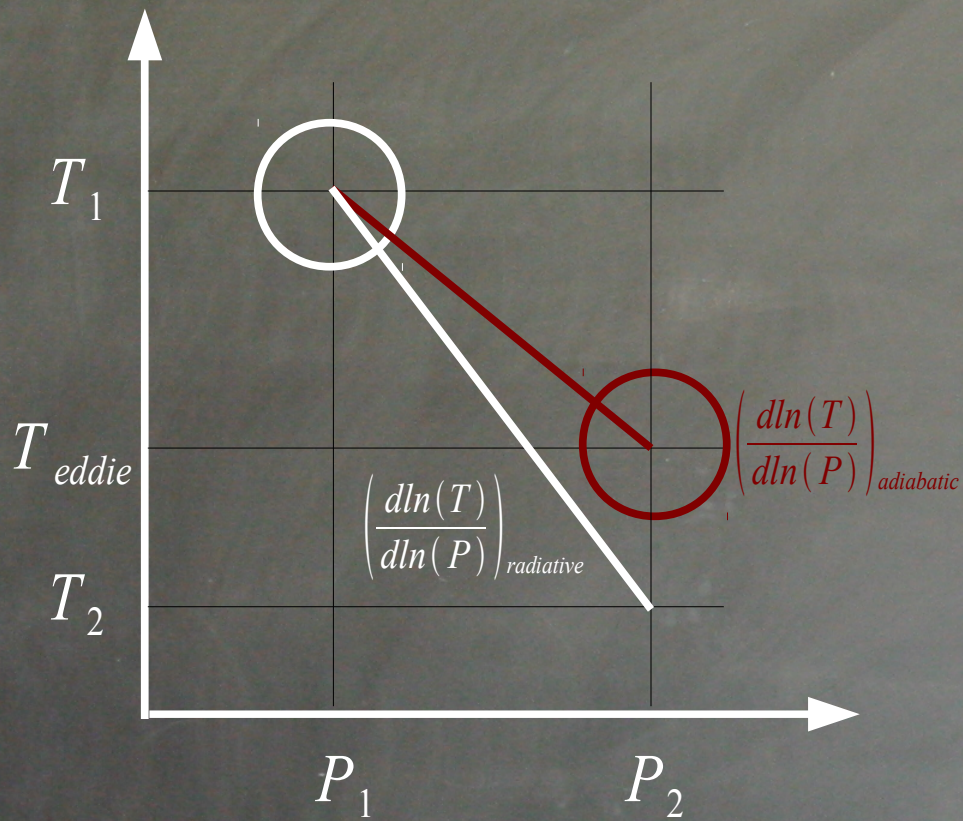
$$P \propto \rho T = \text{const}$$

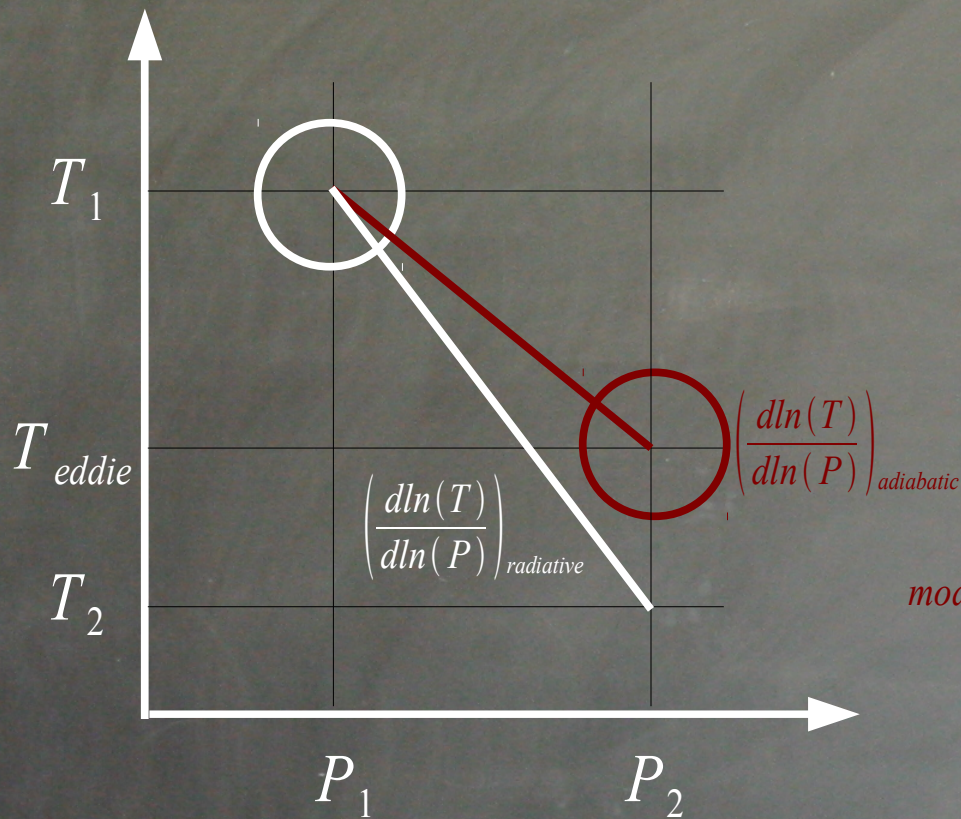
$$T_{\text{eddie}} < T_2$$

$$\rho_{\text{eddie}} > \rho_2$$

The buoyancy force $f \approx -\frac{g}{\rho} \frac{\delta \rho}{\delta r} \Delta r$

is negative and pushes back the eddie





$$\text{mod} \left(\frac{d \ln(T)}{d \ln(P)} \right)_{\text{adiabatic}} < \text{mod} \left(\frac{d \ln(T)}{d \ln(P)} \right)_{\text{radiative}}$$

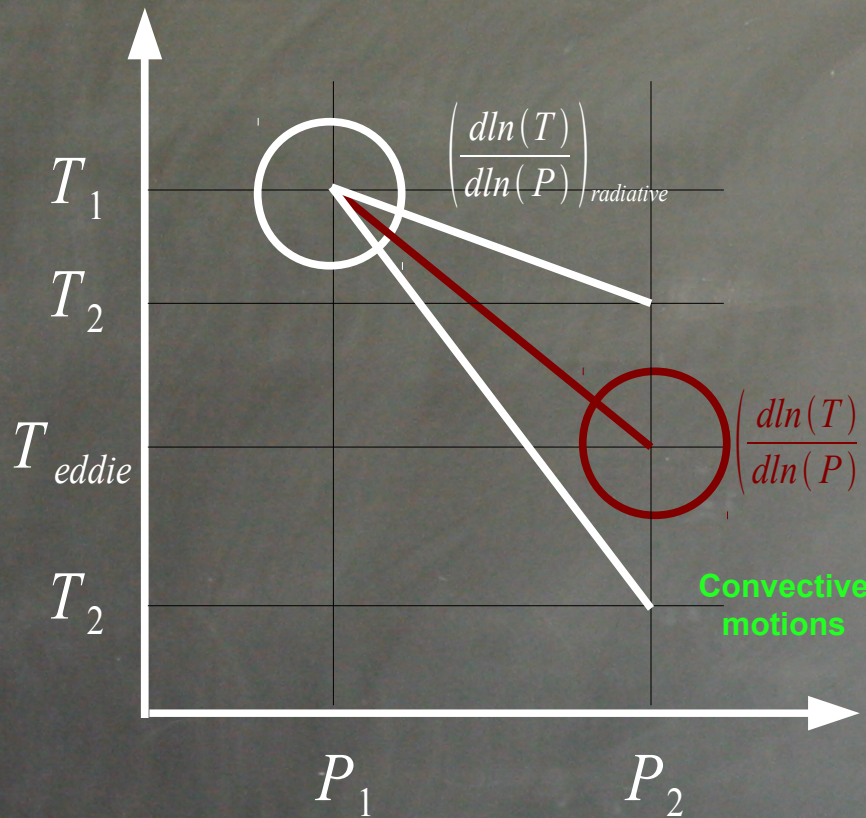
$$T_{\text{eddie}} > T_2$$

$$\rho_{\text{eddie}} < \rho_2$$

**The eddie is accelerated outward.
Large scale motions activate.**

The buoyancy force $f \approx -\frac{g}{\rho} \frac{\delta \rho}{\delta r} \Delta r$ is now positive

Schwarzschild criterion!



$$\text{mod} \left(\frac{d \ln(T)}{d \ln(P)} \right)_{\text{adiabatic}} > \text{mod} \left(\frac{d \ln(T)}{d \ln(P)} \right)_{\text{radiative}}$$

$$P \propto \rho T = \text{const}$$

$$T_{\text{eddie}} < T_2$$

$$\rho_{\text{eddie}} > \rho_2$$

$$\text{mod} \left(\frac{d \ln(T)}{d \ln(P)} \right)_{\text{adiabatic}} < \text{mod} \left(\frac{d \ln(T)}{d \ln(P)} \right)_{\text{radiative}}$$

$$T_{\text{eddie}} > T_2$$

$$\rho_{\text{eddie}} < \rho_2$$

Both the temperature gradient and the mass extension of the convective regions are very difficult to compute properly and still constitute one of the major uncertainties in the stellar modelling.

Which are the basic consequences of the growth of convective motions?

Matter is mixed.

1st side effect: new fuel pulled inward – products of burning pushed outward

2nd side effect: change of the mean molecular weight in the whole convective region

The temperature gradient can't become steeper than the adiabatic one in most of the interior of a star; only in the outer region it can raise towards the radiative one because of the inefficiency of the eddies in carrying the energy.

At this point we are ready to follow the evolution of a star, but...

...first a “stupid” question...

Why should a star “evolve”?

because...

...stars lose energy (e.g. from the surface: the Luminosity)
that must be replaced in order to maintain the hydrostatic equilibrium!

Energy may be gained by either:

contraction (energy is extracted from the gravitational field)

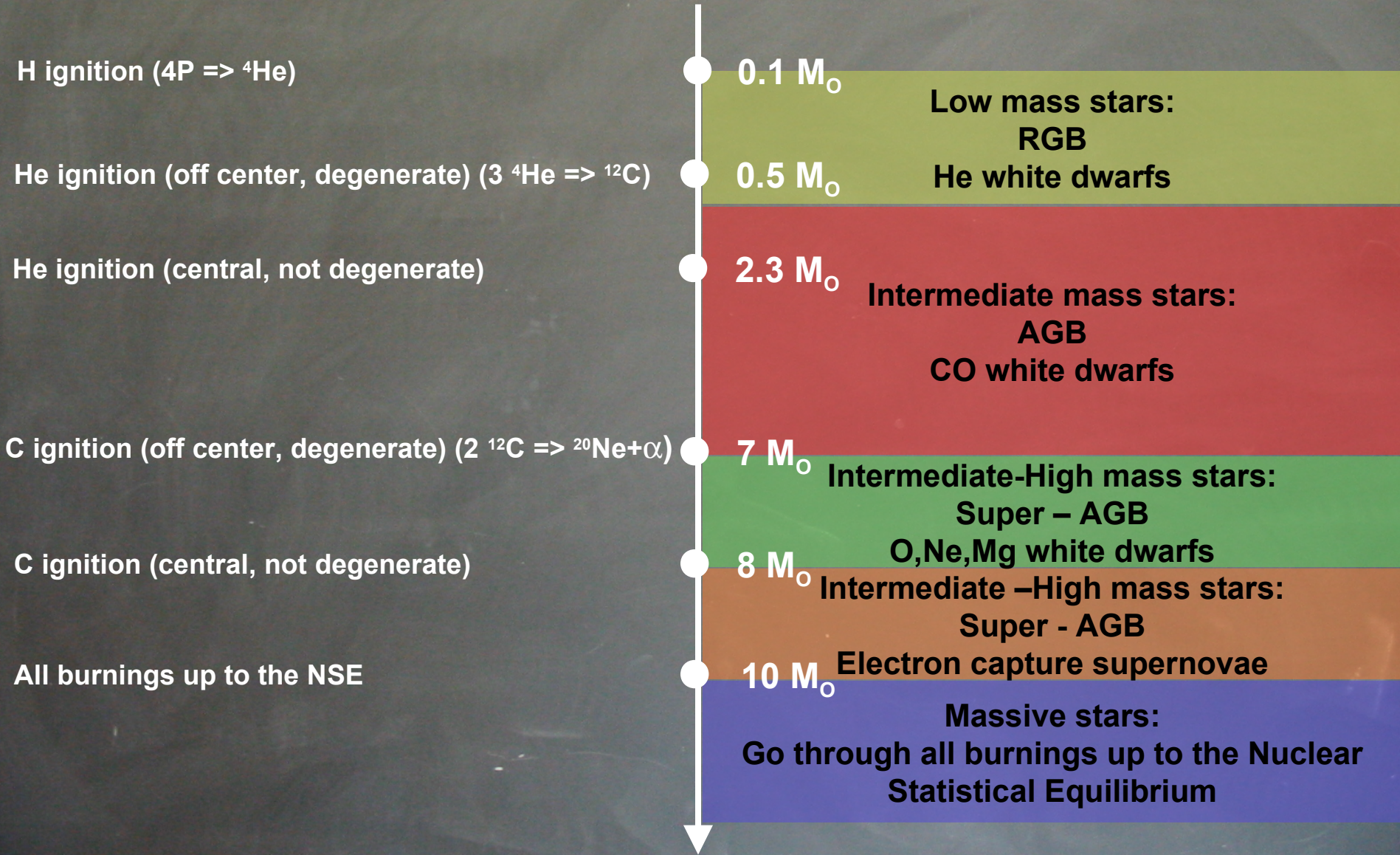
Side effect => the interior heats (Virial theorem)

and / or

Nuclear reactions (energy is extracted from the fusion of nuclei)

Side effect => mean molecular weight increases (P decreases)

Critical masses:



Hydrostatic evolution

H-burning

He-burning

C-burning

Ne-burning

O-burning

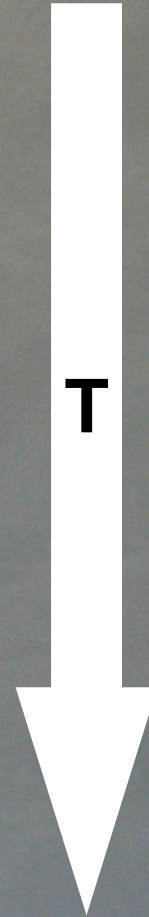
Si-burning

Core collapse

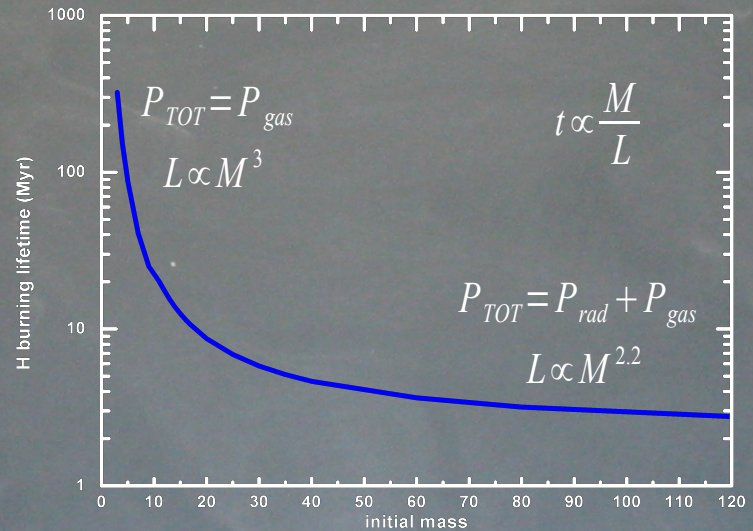
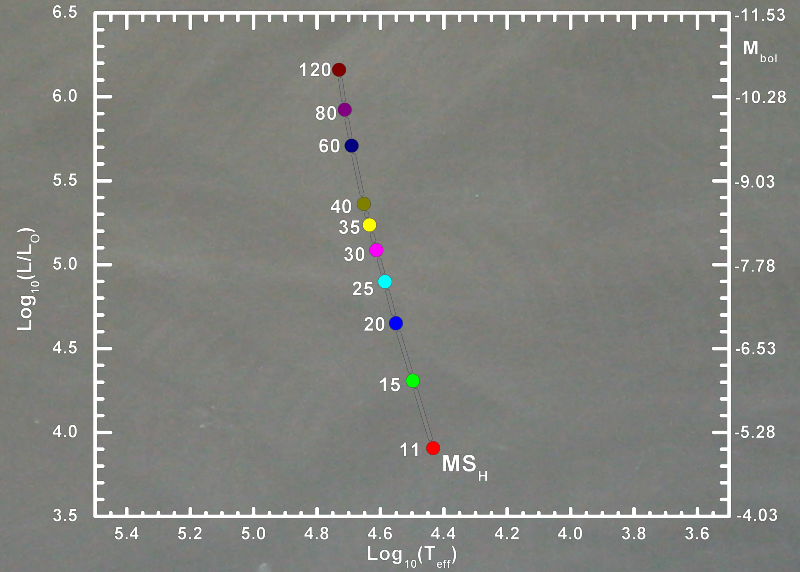
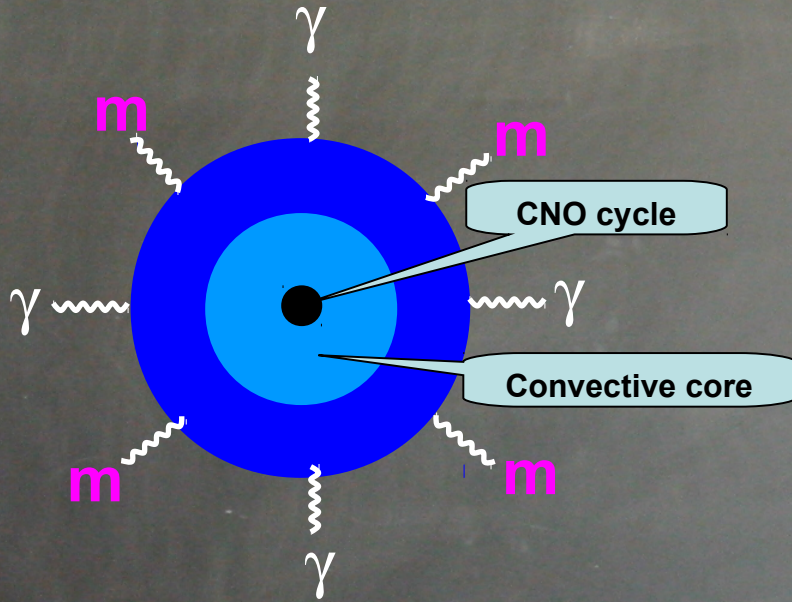
Hydrodynamic evolution

Bounce at nuclear densities
Formation of the shock wave

Explosive nucleosynthesis
yields



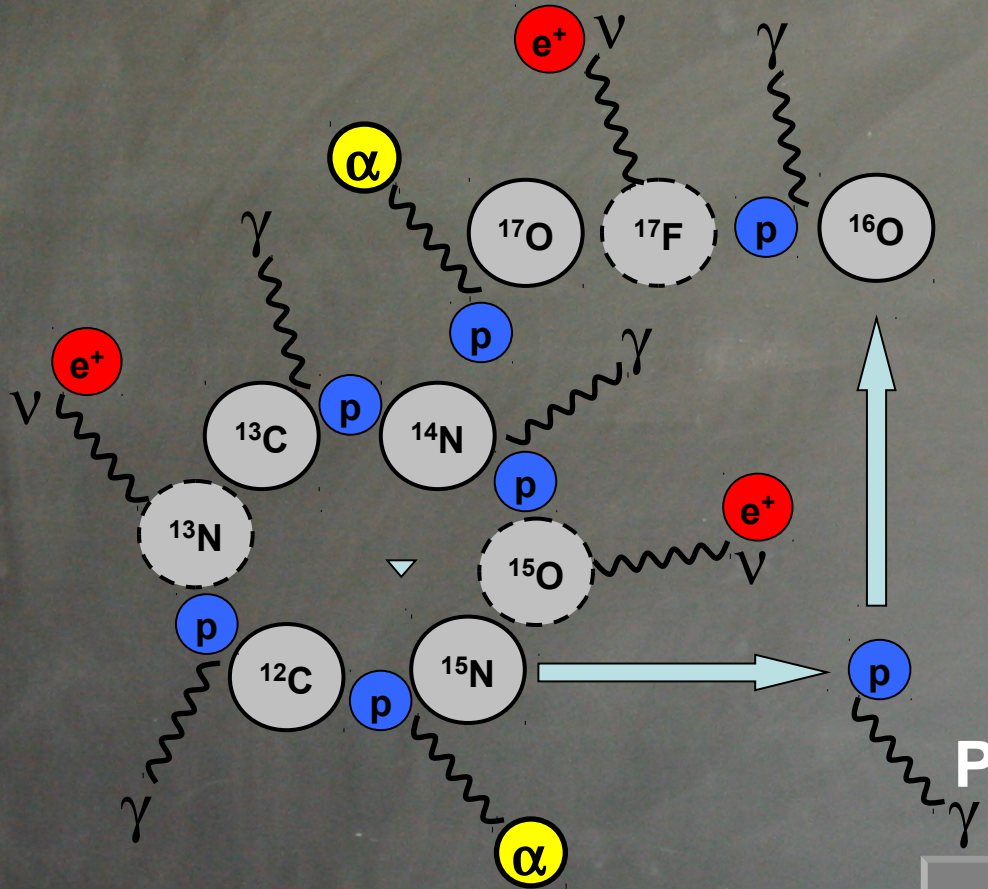
H – burning: luminosity and lifetime



Energy budget



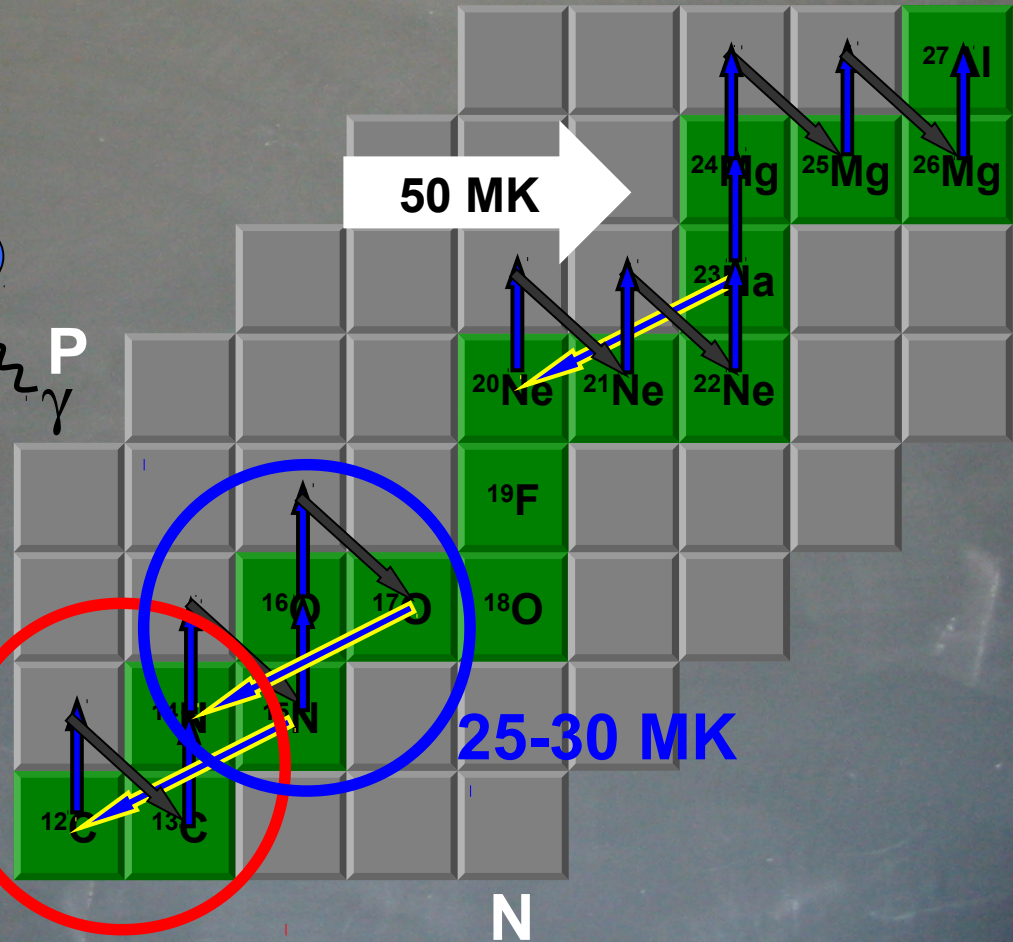
$$6.44 \cdot 10^{18} \text{ erg g}^{-1}$$



20 MK

25-30 MK

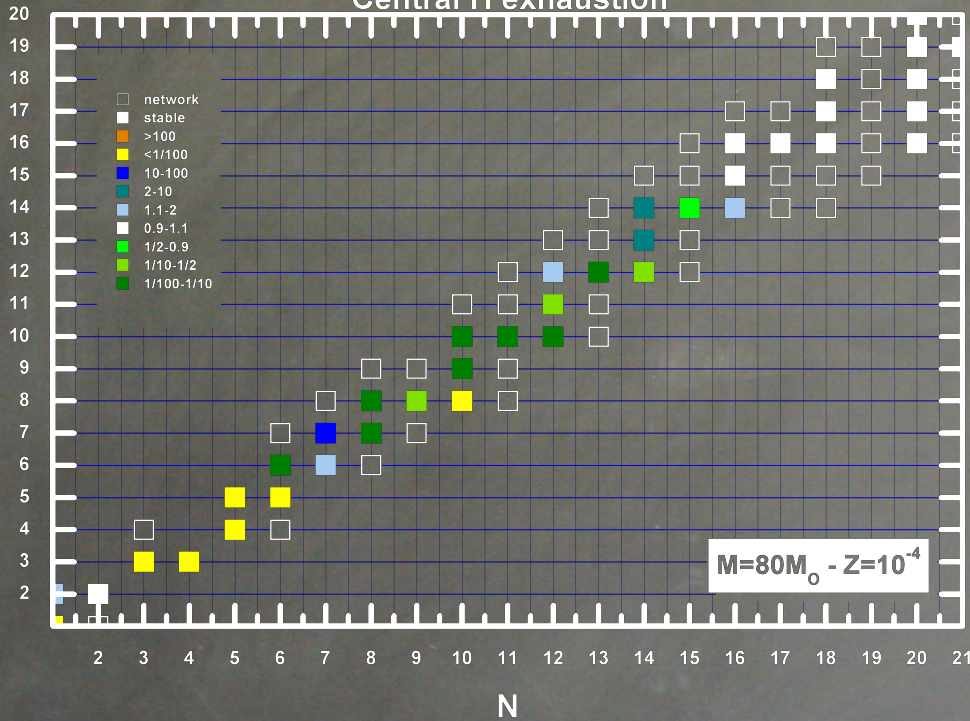
50 MK



Trailer time!

H burning movie

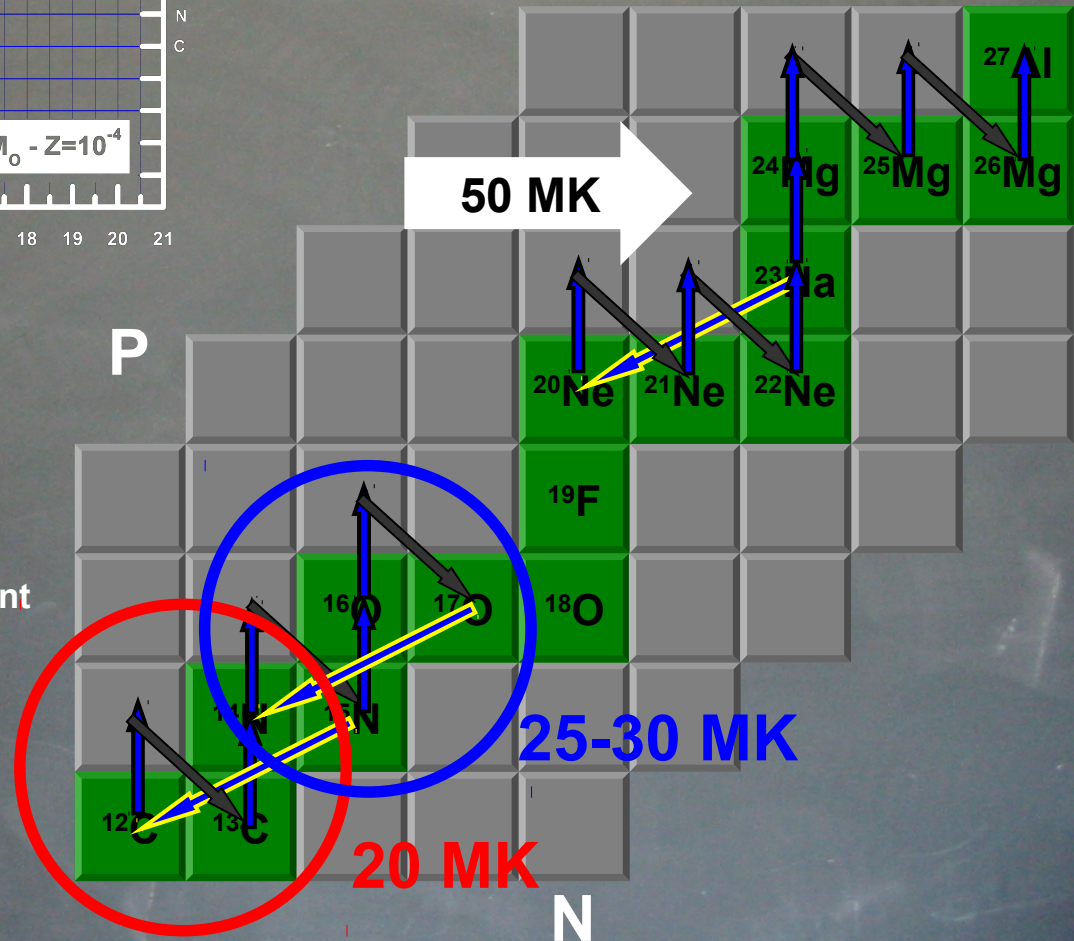
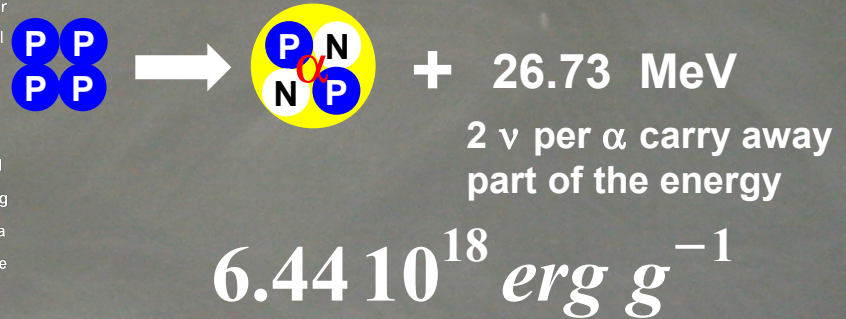
Central H exhaustion



Basic effects of the H burning on the elemental abundances:

- H converted in He (strong neutronization)
- O & C converted in N (becomes the most abundant element after H and He!)
- Redistribution among Ne – Na – Mg – Al
- F destroyed

Energy budget



H – burning: mass loss

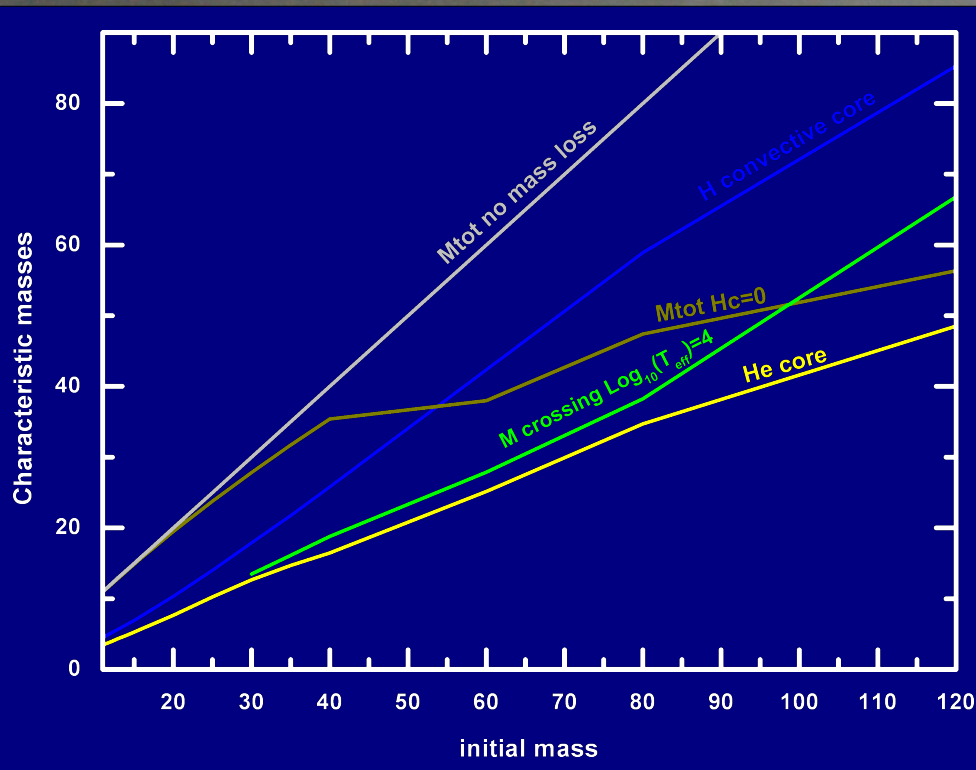
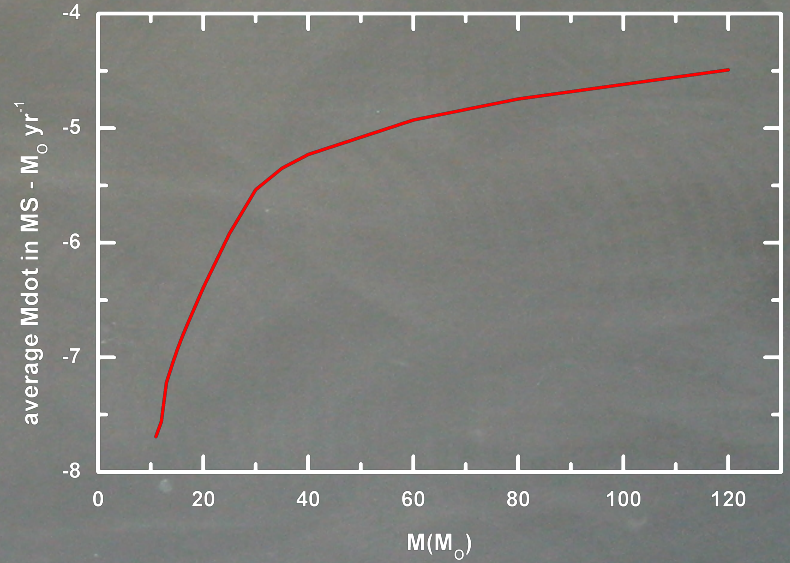
Astron. Astrophys. 362, 295–309 (2000)

ASTRONOMY
AND
ASTROPHYSICS

New theoretical mass-loss rates of O and B stars

J.S. Vink¹, A. de Koter², and H.J.G.L.M. Lamers^{1,3}

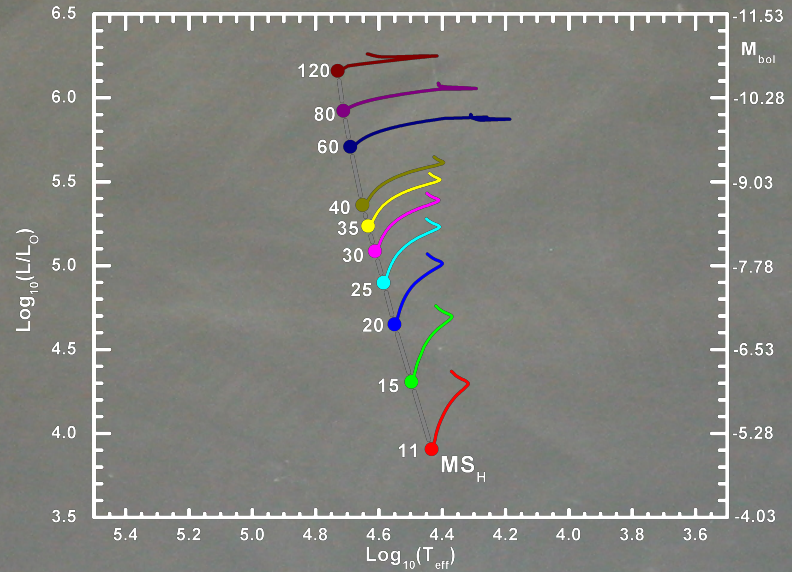
$$\dot{M} \text{ (O B stars)} \propto \frac{L^{2.2}}{M^{1.3}} \quad \& \quad (T_{\text{eff}}, V_{\text{inf}} / V_{\text{esc}})$$



H rich mantle

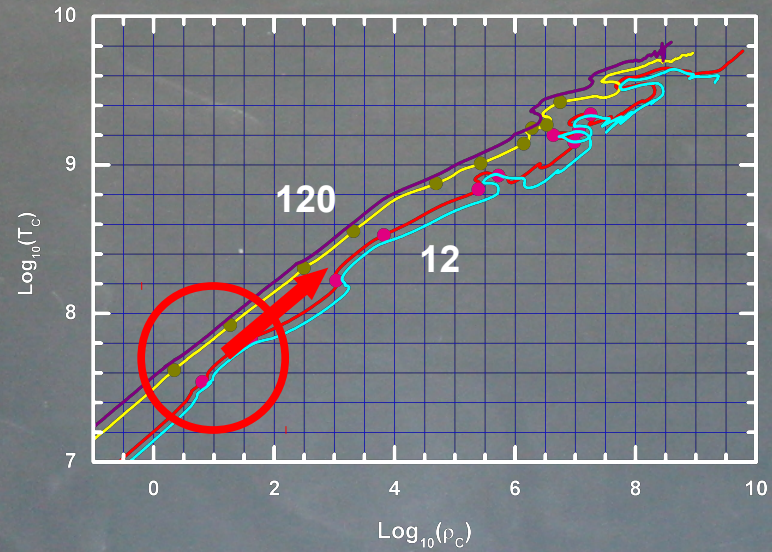
He core

$H \rightarrow He$
 $C, O, F \downarrow - N \uparrow$
 Ne, Na, Mg, Al, Si

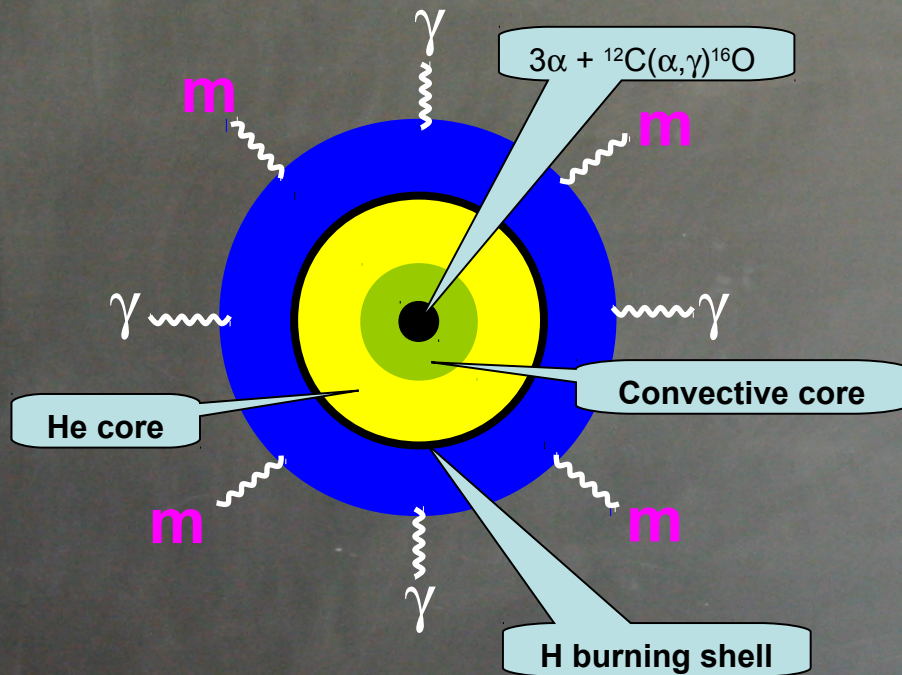


He core

$H \rightarrow He$
 $C, O, F \downarrow - N \uparrow$
 Ne, Na, Mg, Al, Si



The central He burning

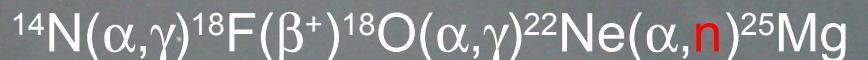


All stars form a convective core

The physical evolution of the star requires the inclusion of just 2 processes:



The chemical evolution of the star requires the inclusion of many, many processes because an efficient n producing chain activates:



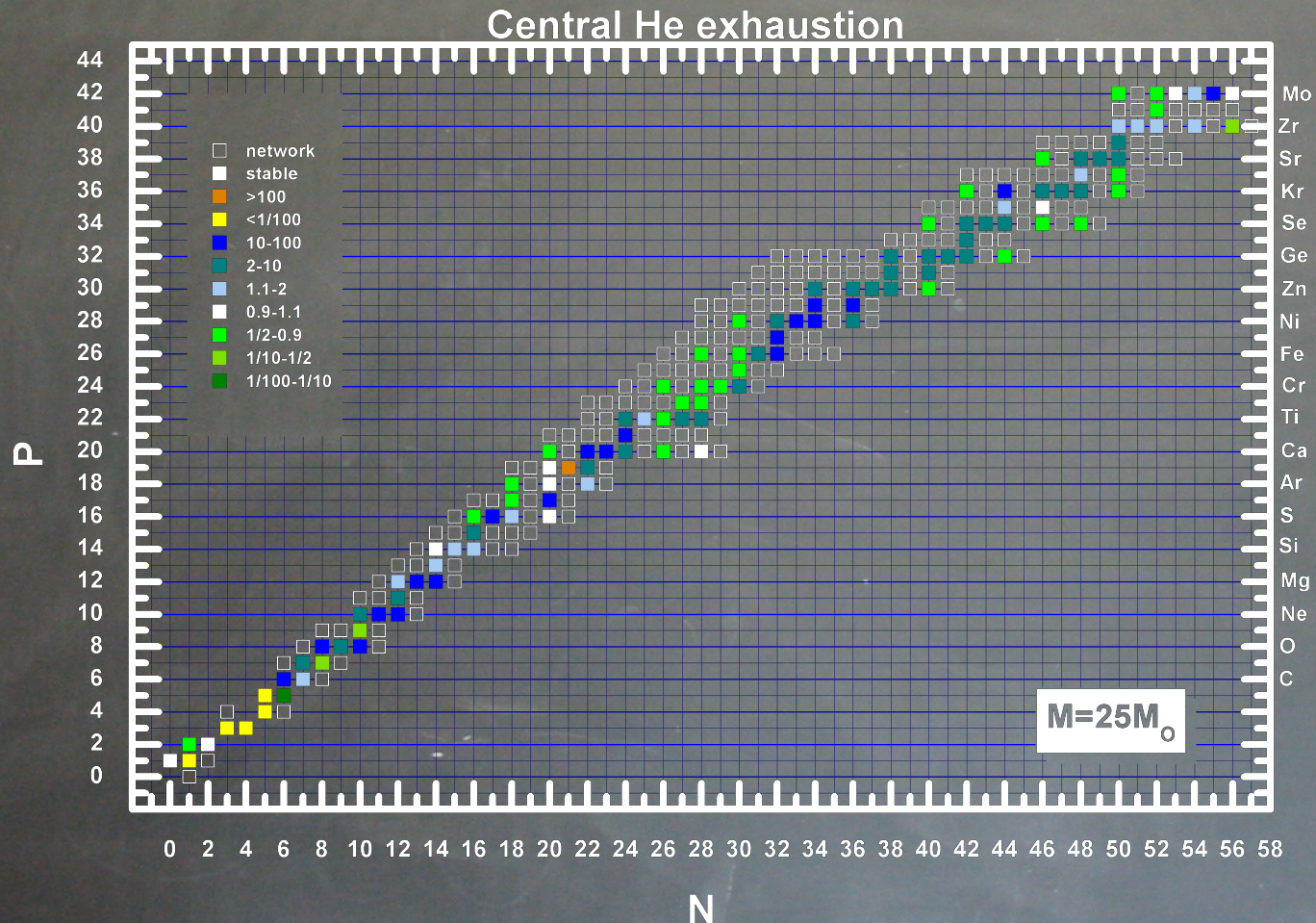
Note that the ${}^{14}\text{N}$ abundance is roughly equal to the initial abundance of the sum of the CNO nuclei that are more than 70% of the initial metallicity of the star!

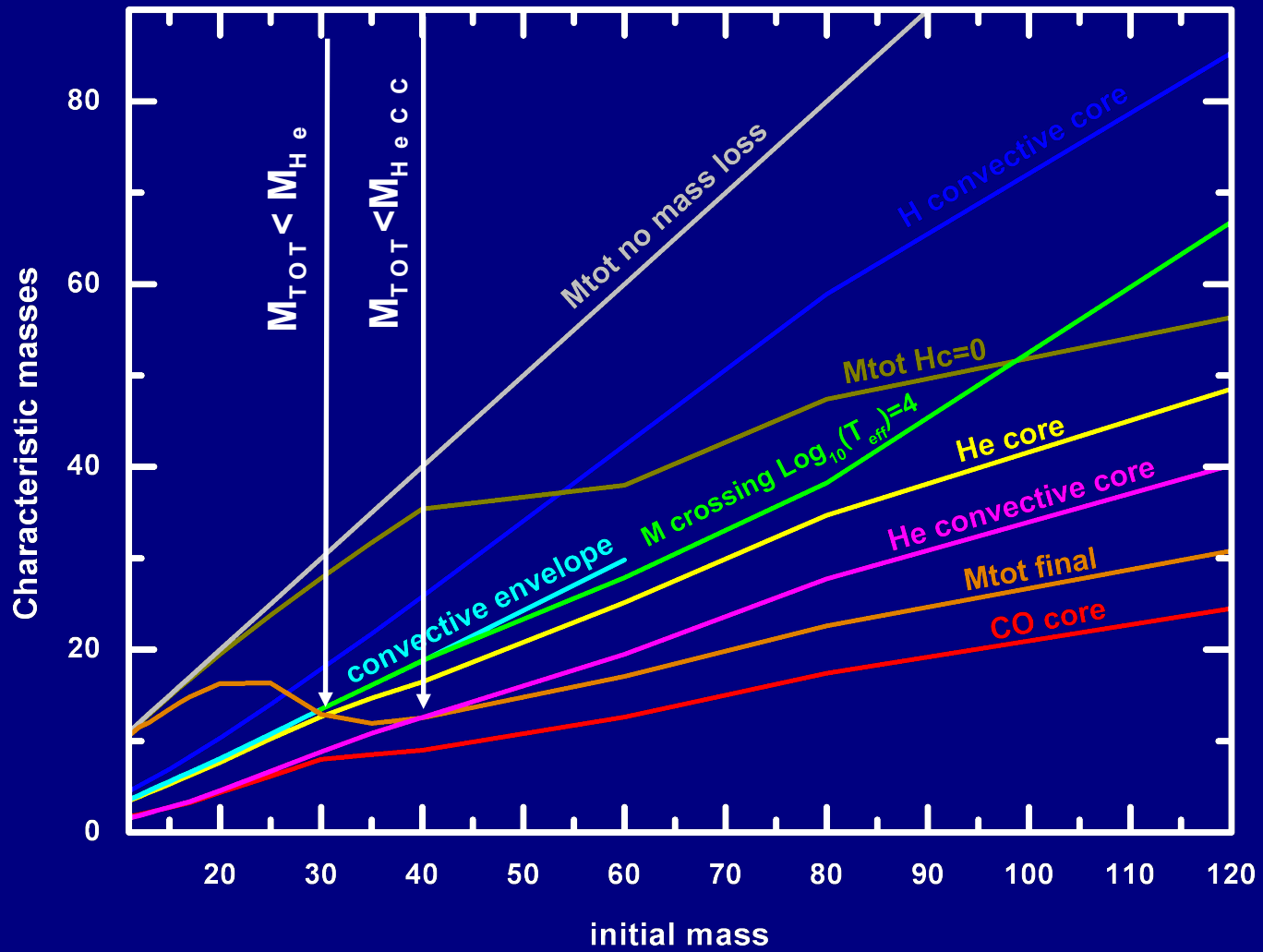
Trailer time!

He burning movie

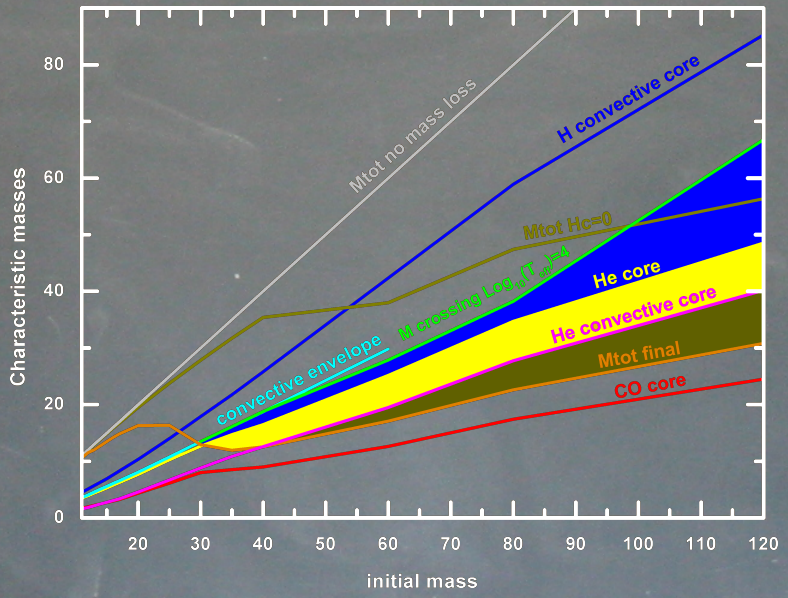
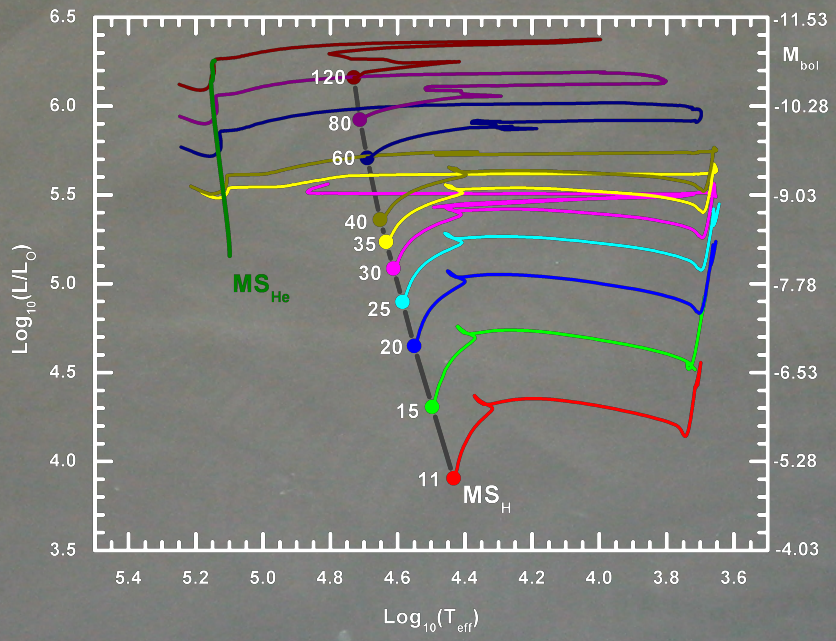
The central He burning

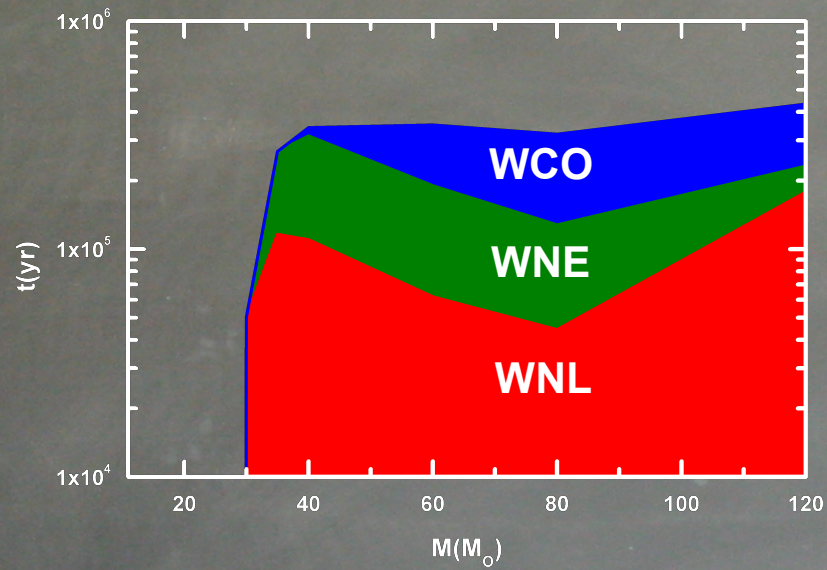
The n emitted by the $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ process are mainly captured by the Fe peak nuclei so that nuclei up to $A=90$ (S weak-component) are produced





$\text{Log}_{10}(T_{\text{eff}}) > 4.0$





Summarizing:

RSG	$< 30 M_{\odot}$	RSG		
WNL	$< 40 M_{\odot}$	WNL	$< 60 M_{\odot}$	WNL
WNE		WNE		WNE
		WCO		WCO

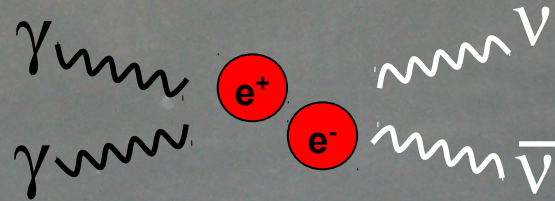
Let's enter the advanced burnings...

The central temperature at the central He exhaustion is of the order of $4 \cdot 10^8$ K and at roughly $8 \cdot 10^8$ K the next fuel, carbon, starts burning.

But in the mean time....

...as the temperature increases, the peak of the Planck distribution moves towards higher energies and the number of photons having energy equal to 0.511 MeV (i.e. the mass of the electrons) increases dangerously.

When this happens, $\gamma+\gamma$ begin to efficiently produce electrons-positron pairs.



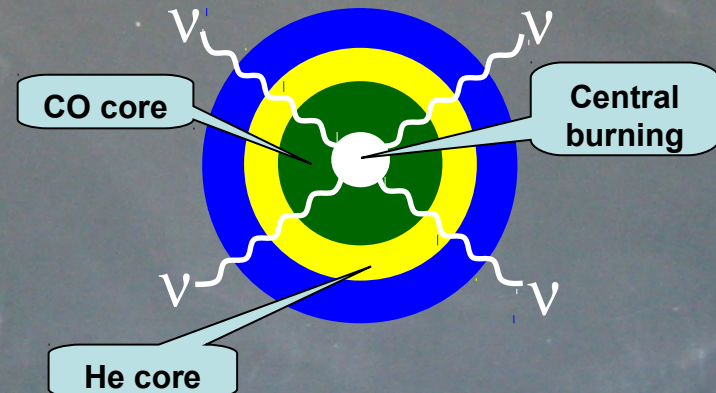
A blackbody radiation has its maximum wavelength at : $\lambda_{MAX} = 2.898 \cdot 10^{-3} T^{-1}$ [m / K]

BUT:

$$E = h\nu = hc/\lambda = hcT/(2.898 \cdot 10^{-3}) = 4.3 \cdot 10^{-10} T \text{ [MeV]}$$

HENCE:

$$T = E / 4.3 \cdot 10^{-10} = 0.5 / 4.3 \cdot 10^{-10} \Rightarrow 1.15 \cdot 10^9 \text{ K}$$

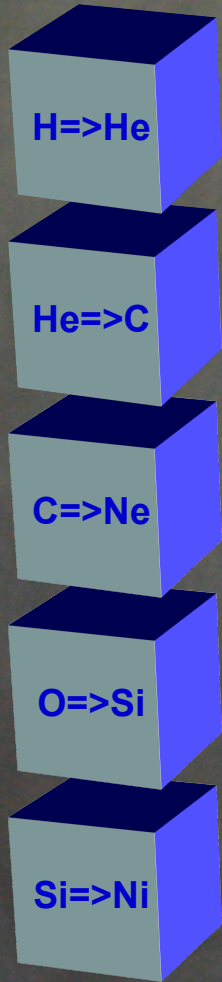


Energy budget

$$t = E / L$$

H=>He	$6.44 \cdot 10^{18} \text{ erg gr}^{-1}$	$t = 1.06 \cdot 10^{11} (L/L_0)^{-1} \text{ [yr / } M_{\odot}]$
He=>C	$5.84 \cdot 10^{17} \text{ erg gr}^{-1}$	$t = 9.64 \cdot 10^9 (L/L_0)^{-1} \text{ [yr / } M_{\odot}]$
C=>Ne	$1.85 \cdot 10^{17} \text{ erg gr}^{-1}$	$t = 3.05 \cdot 10^9 (L/L_0)^{-1} \text{ [yr / } M_{\odot}]$
O=>Si	$2.89 \cdot 10^{17} \text{ erg gr}^{-1}$	$t = 4.77 \cdot 10^9 (L/L_0)^{-1} \text{ [yr / } M_{\odot}]$
Si=>Ni	$1.88 \cdot 10^{17} \text{ erg gr}^{-1}$	$t = 3.10 \cdot 10^9 (L/L_0)^{-1} \text{ [yr / } M_{\odot}]$

$$M=80 M_{\odot} \quad t = E / 10^6$$



$6.44 \cdot 10^{18} \text{ erg gr}^{-1}$

$5.84 \cdot 10^{17} \text{ erg gr}^{-1}$

$1.85 \cdot 10^{17} \text{ erg gr}^{-1}$

$2.89 \cdot 10^{17} \text{ erg gr}^{-1}$

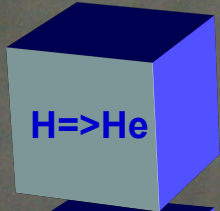
$1.88 \cdot 10^{17} \text{ erg gr}^{-1}$

$L \Rightarrow$ total luminosity: L_{γ}

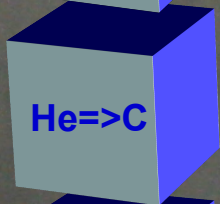
Mcc	Estimated lifetime	Real lifetime	Revised lifetime	L_{TOT}
60	$6 \cdot 10^6$	$3.2 \cdot 10^6$		10^6
20	$2 \cdot 10^5$	$3.3 \cdot 10^5$		10^6
1.5	$4.5 \cdot 10^3$	$4.7 \cdot 10^2$		10^6
1	$4.8 \cdot 10^3$	$4.6 \cdot 10^{-2}$		10^6
1	$3.1 \cdot 10^3$	$4.3 \cdot 10^{-3}$		10^6

$L \Rightarrow$ total luminosity: $L_\gamma + L_\nu$

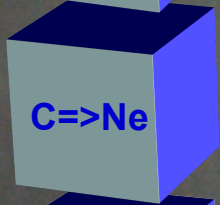
$M=80 M_\odot \quad t = E / 10^6$



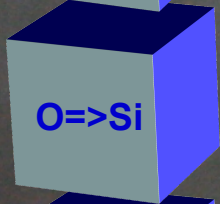
$6.44 \cdot 10^{18} \text{ erg gr}^{-1}$



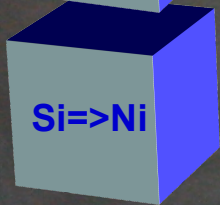
$5.84 \cdot 10^{17} \text{ erg gr}^{-1}$



$1.85 \cdot 10^{17} \text{ erg gr}^{-1}$



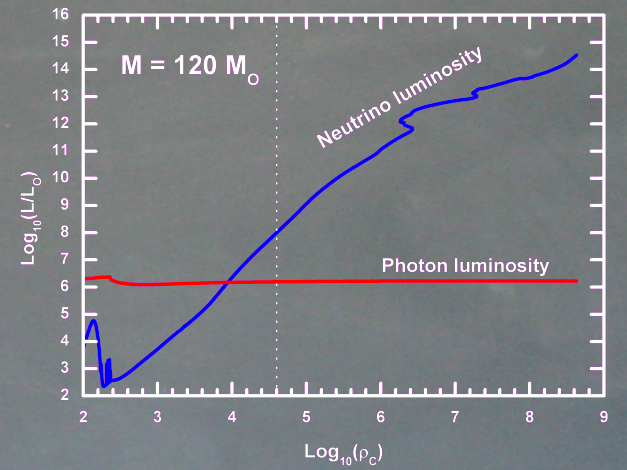
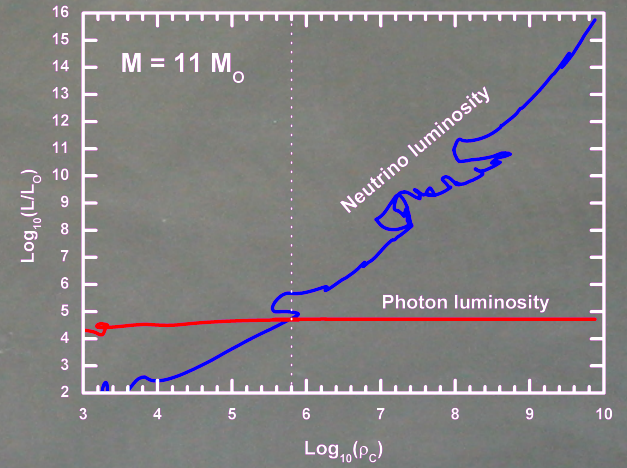
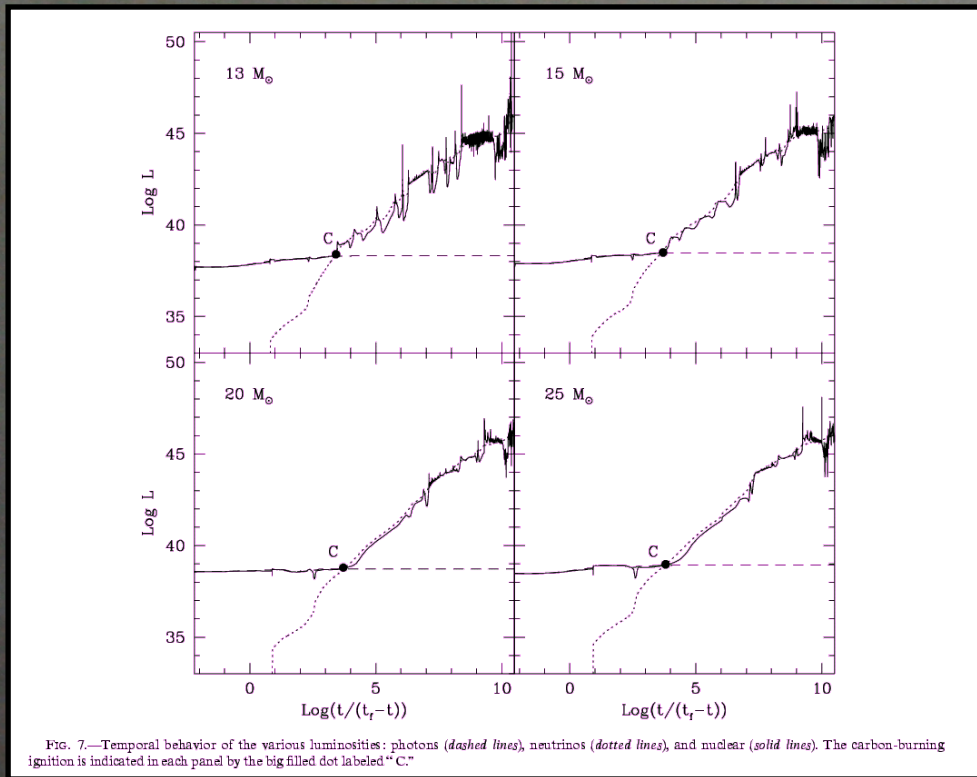
$2.89 \cdot 10^{17} \text{ erg gr}^{-1}$



$1.88 \cdot 10^{17} \text{ erg gr}^{-1}$

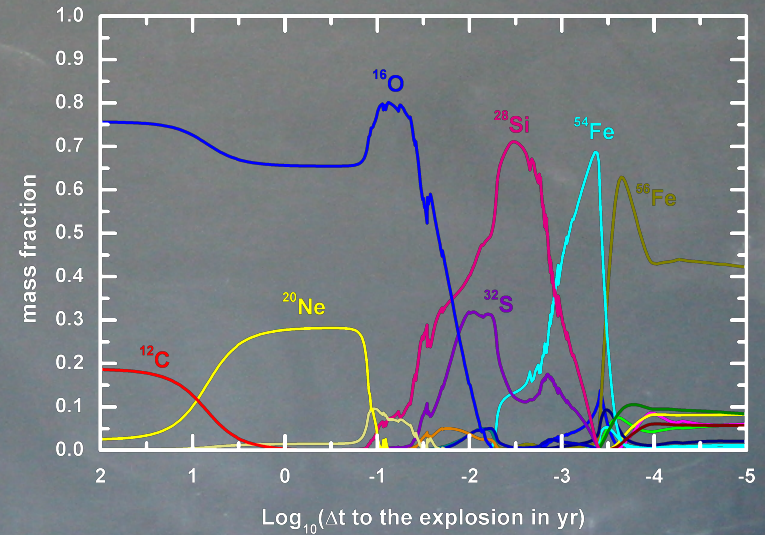
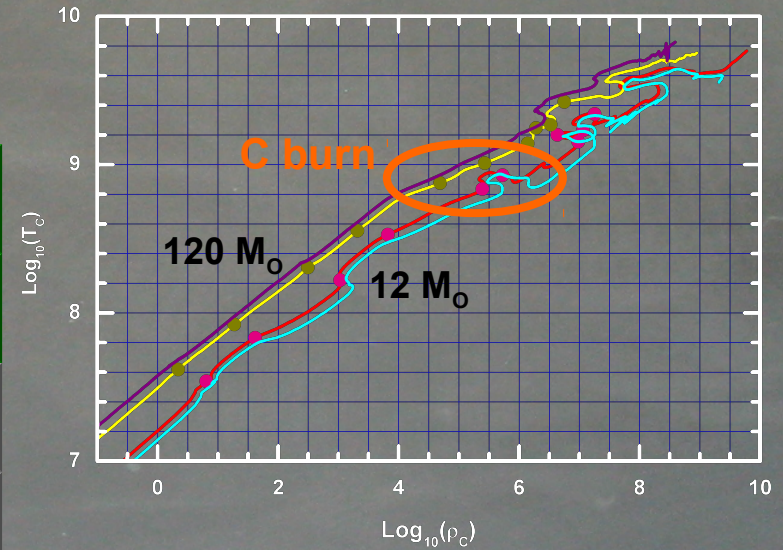
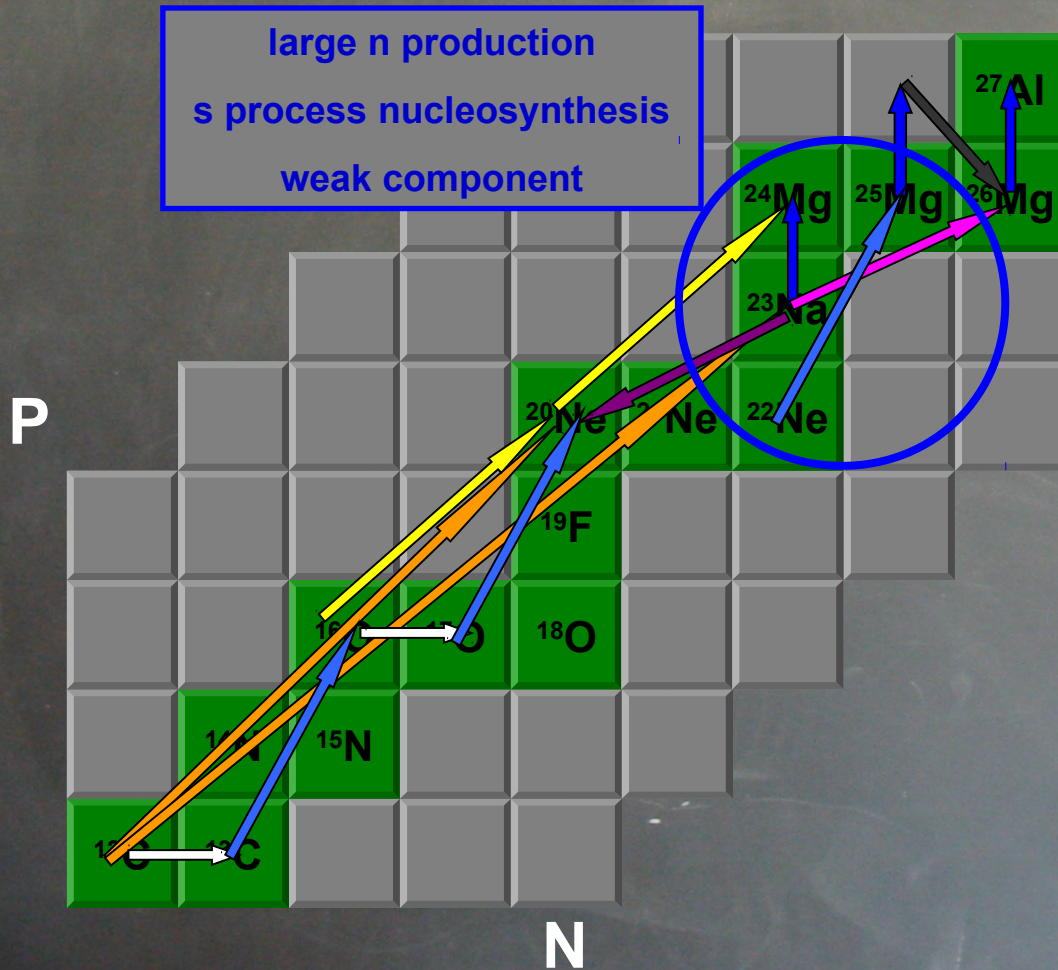
Mcc	Estimated lifetime	Real lifetime	Revised lifetime	L_{TOT}
60	$6 \cdot 10^6$	$3.2 \cdot 10^6$		10^6
20	$2 \cdot 10^5$	$3.3 \cdot 10^5$		10^6
1.5	$4.5 \cdot 10^3$	$4.7 \cdot 10^2$	$4.5 \cdot 10^2$	10^7
1	$4.8 \cdot 10^3$	$4.6 \cdot 10^{-2}$	$4.8 \cdot 10^{-2}$	10^{11}
1	$3.1 \cdot 10^3$	$4.3 \cdot 10^{-3}$	$3.1 \cdot 10^{-3}$	10^{12}

All the advanced phases are really neutrino dominated...



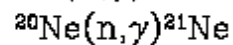
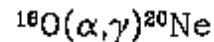
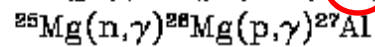
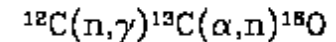
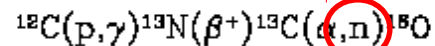
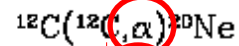
C burning

Typical temperature: 0.8-1.0 BK

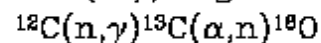
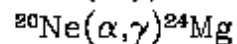
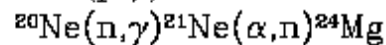
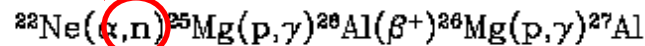
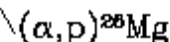
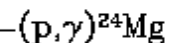
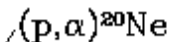
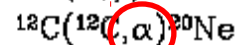
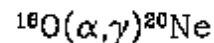


Just the main processes in ...

Panel a)



Panel b)

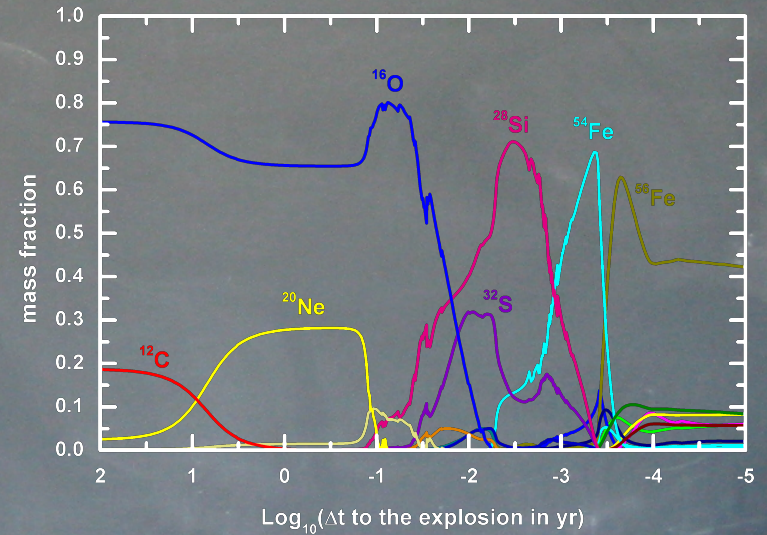
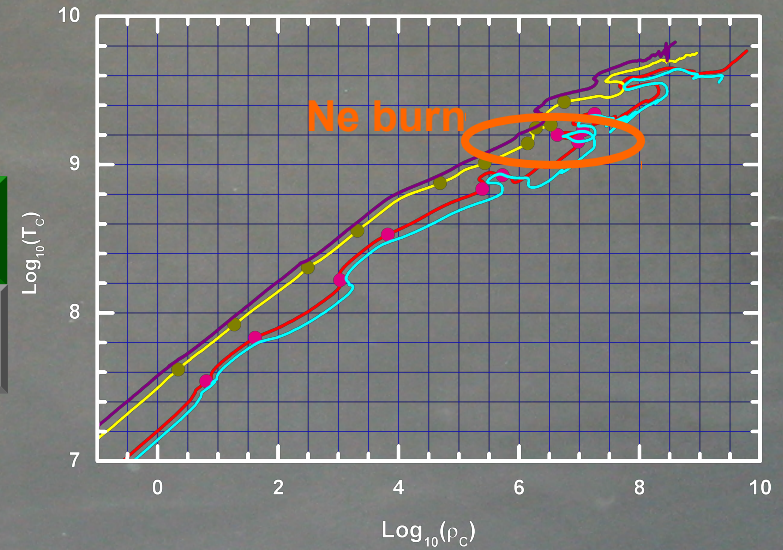
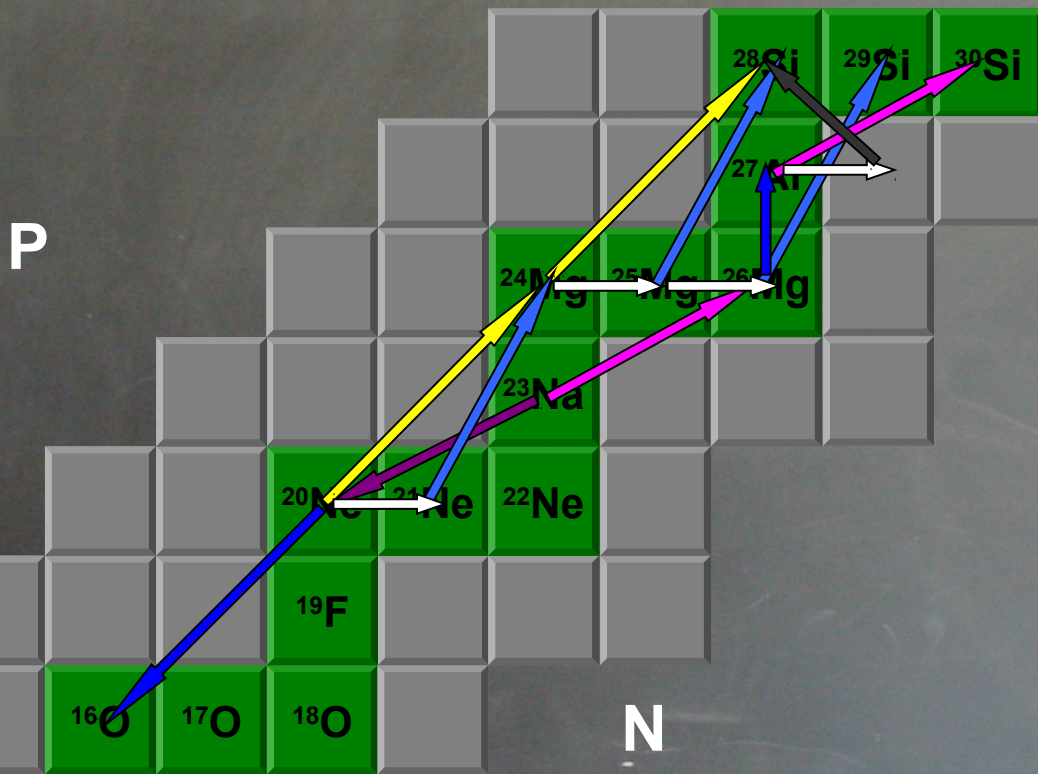


C burning

FIG. 6.—Most efficient nuclear processes during the central carbon burning. (a) The first part of the carbon burning; (b) the second part of carbon burning.

Ne burning

Typical temperature: 1.3-1.6 BK



Just the main processes in ...

Ne burning

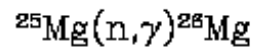
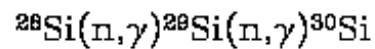
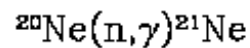
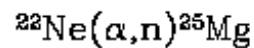
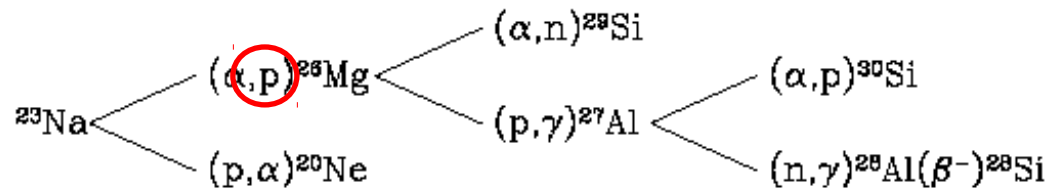
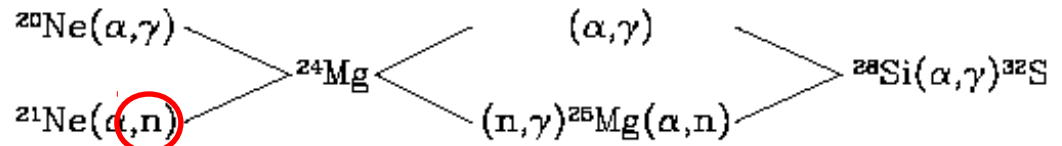
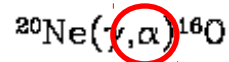
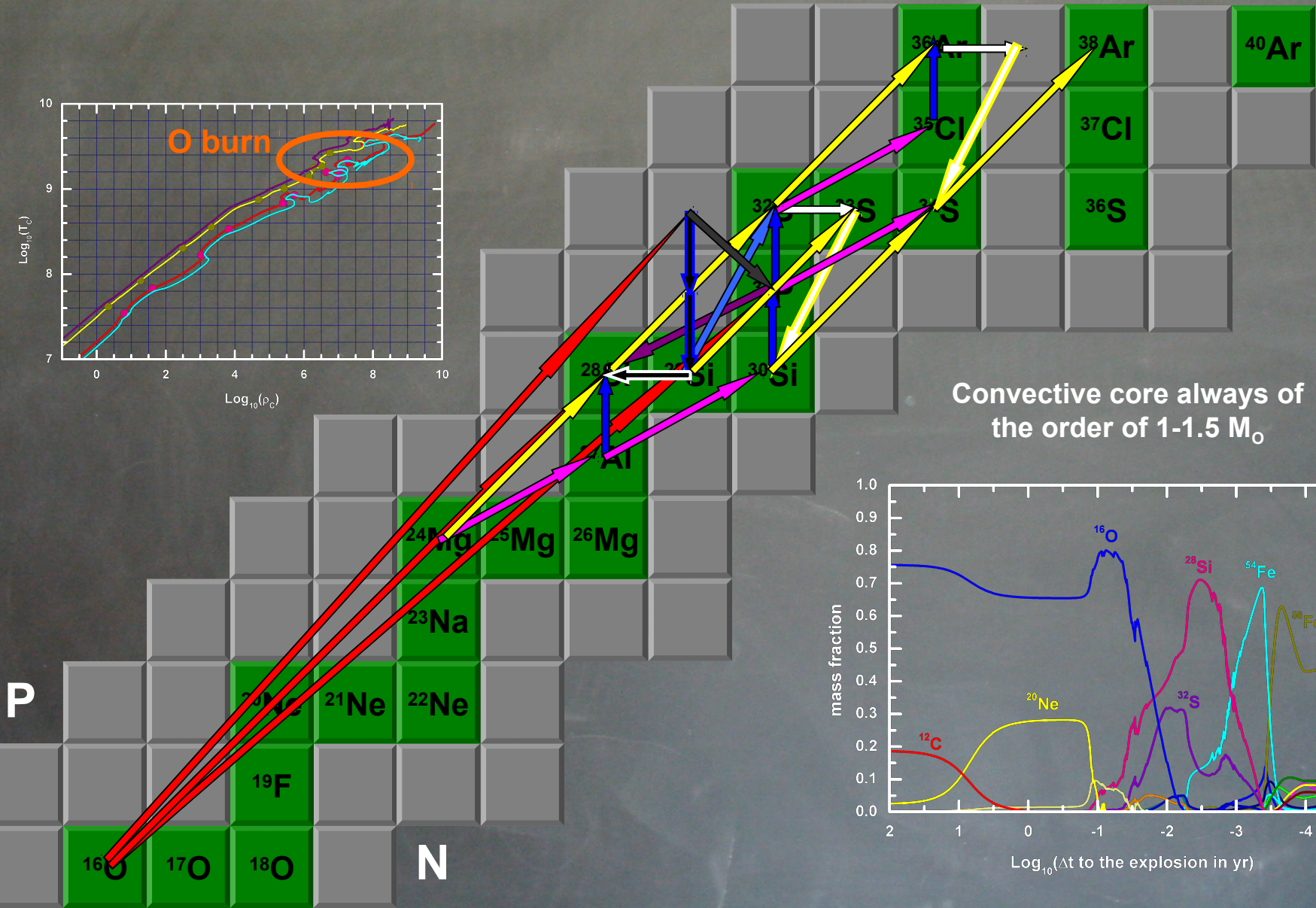
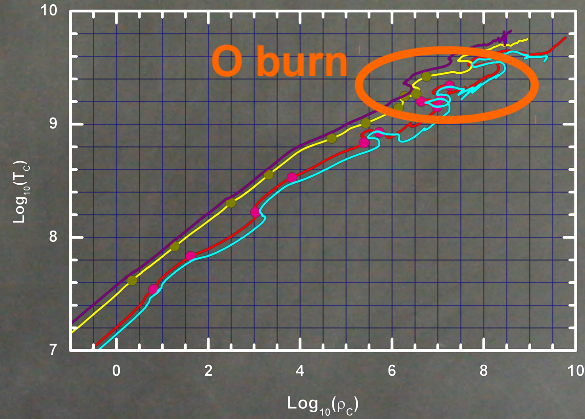


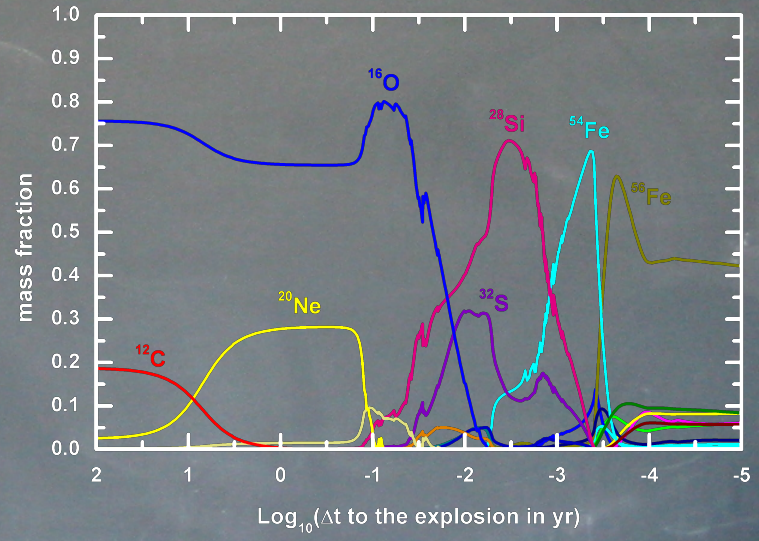
FIG. 8.—Most efficient nuclear processes during the central neon burning

O burning

Typical temperature: 2.-2.5 BK

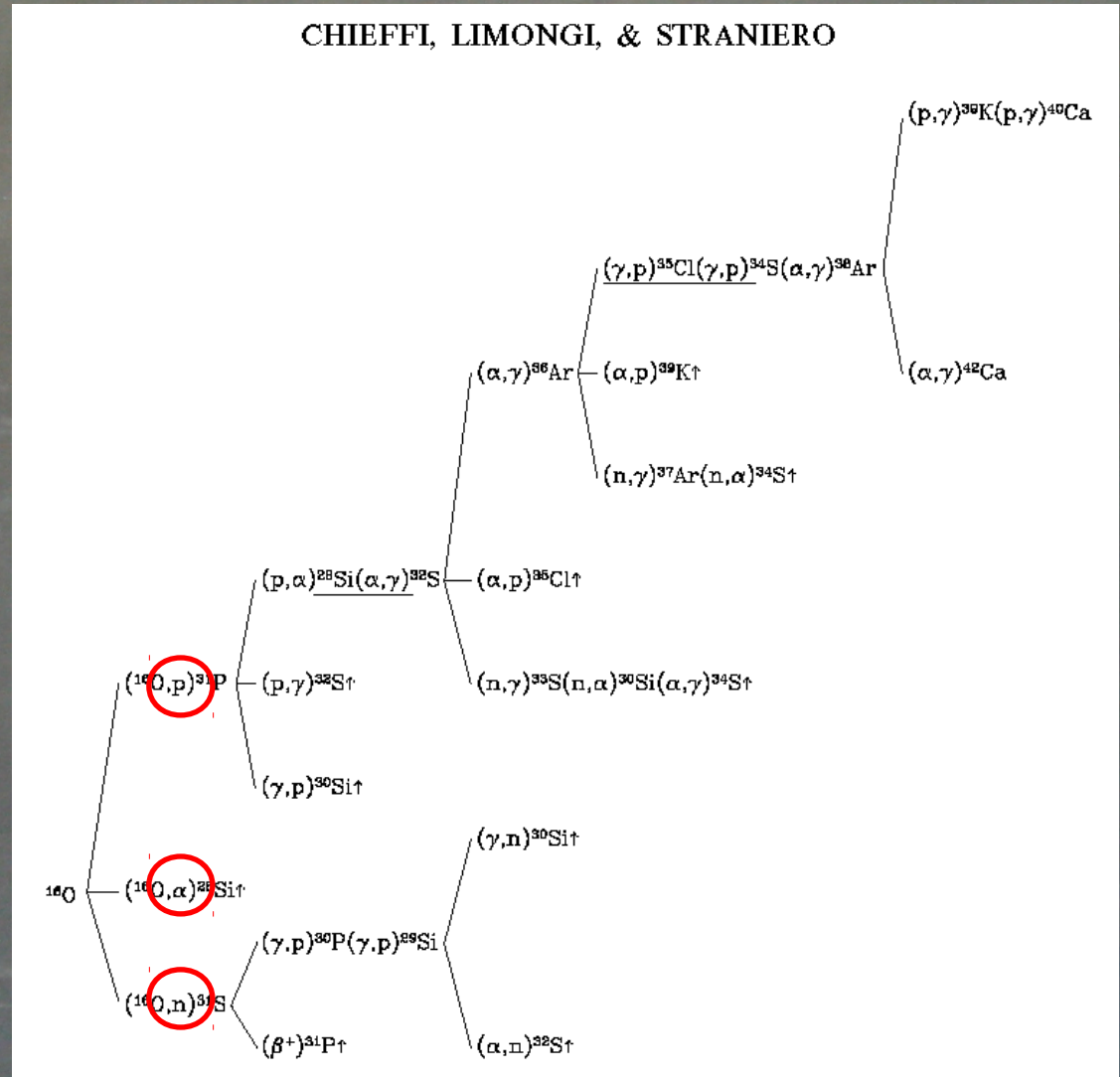


Convective core always of the order of 1-1.5 M_{\odot}



Just the main processes in ...

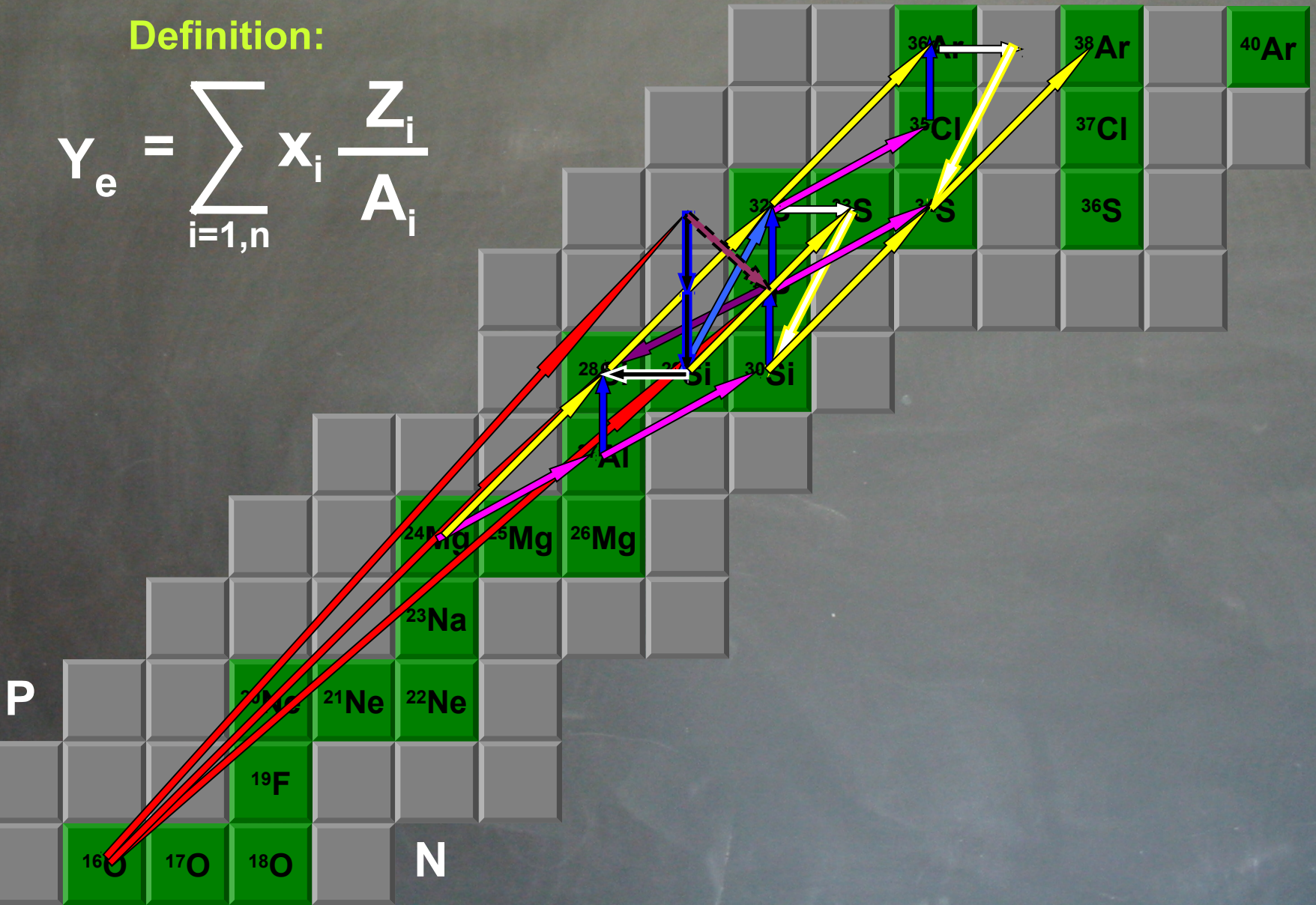
O burning



neutronization

Definition:

$$Y_e = \sum_{i=1,n} x_i \frac{Z_i}{A_i}$$

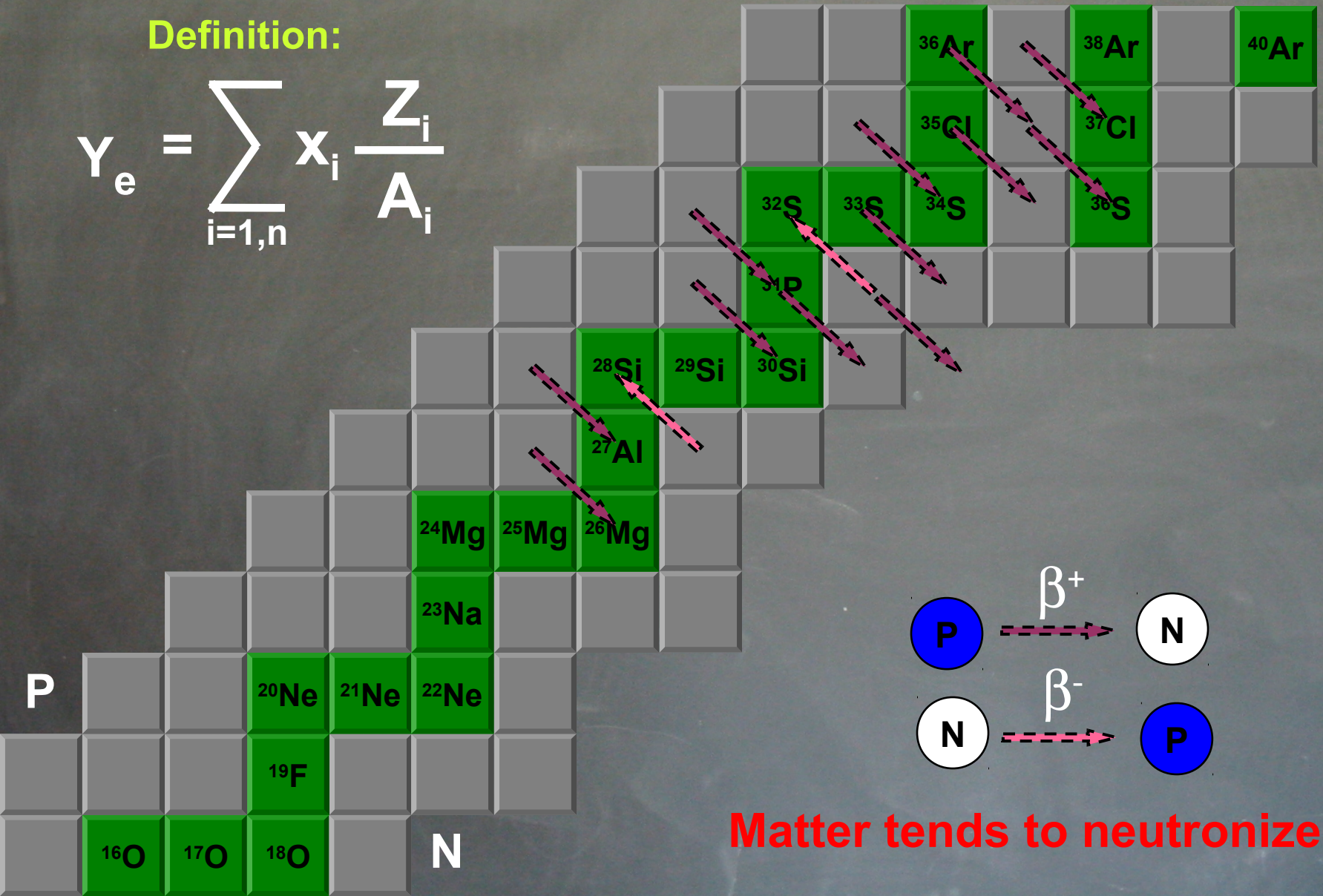


neutronization

Main weak processes in O burning

Definition:

$$Y_e = \sum_{i=1,n} x_i \frac{Z_i}{A_i}$$



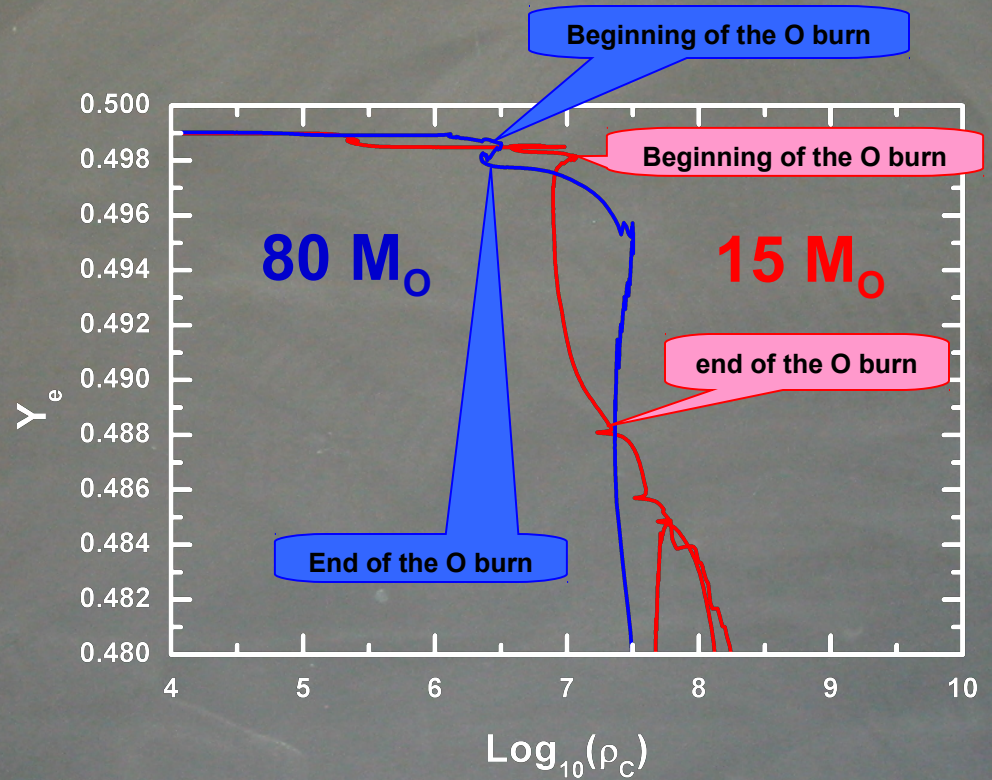
Matter tends to neutronize !

neutronization

Definition:

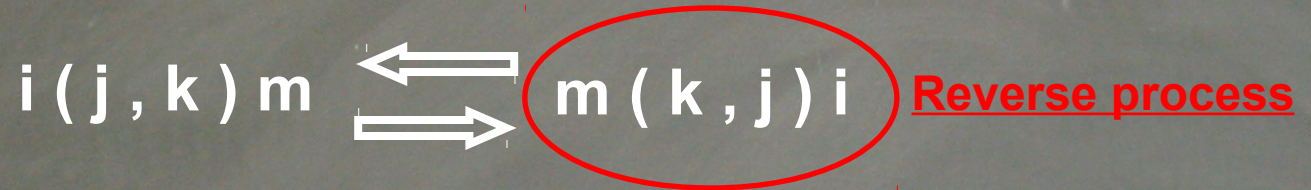
$$Y_e = \sum_{i=1,n} x_i \frac{Z_i}{A_i}$$

Degree of neutronization depends on the initial mass (actually on CO core mass)



RULE: The smaller the M_{CO} the higher the degree of neutronization at the end of the central O burning.

Beyond the O burning



R_{ij} \rightarrow Rate of the $i(j,k)m$ process

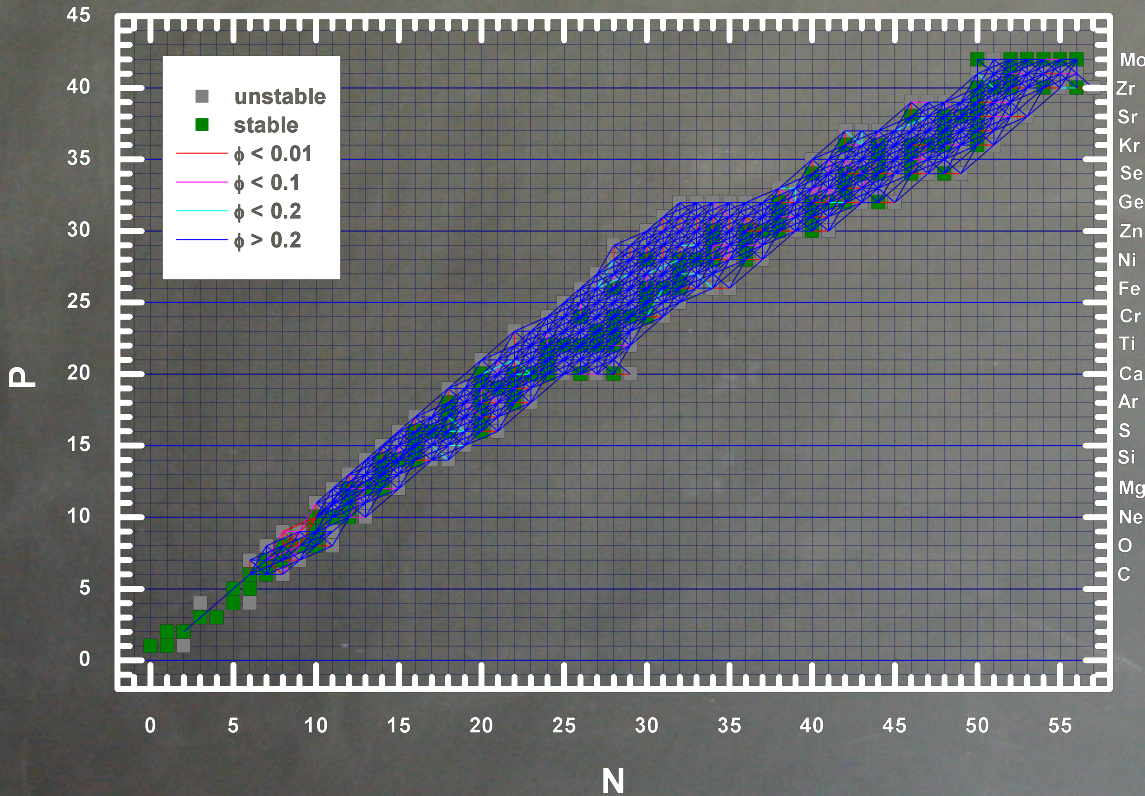
R_{mk} \rightarrow Rate of the $m(k,j)i$ process

$$\Phi_{ij} = \frac{|R_{ij} - R_{mk}|}{\text{Max}(R_{ij}, R_{mk})}$$

$\Phi = 0$ means perfect equilibrium

$\Phi = 1$ means one process dominates over the other

The approach to the Nuclear Statistical Equilibrium



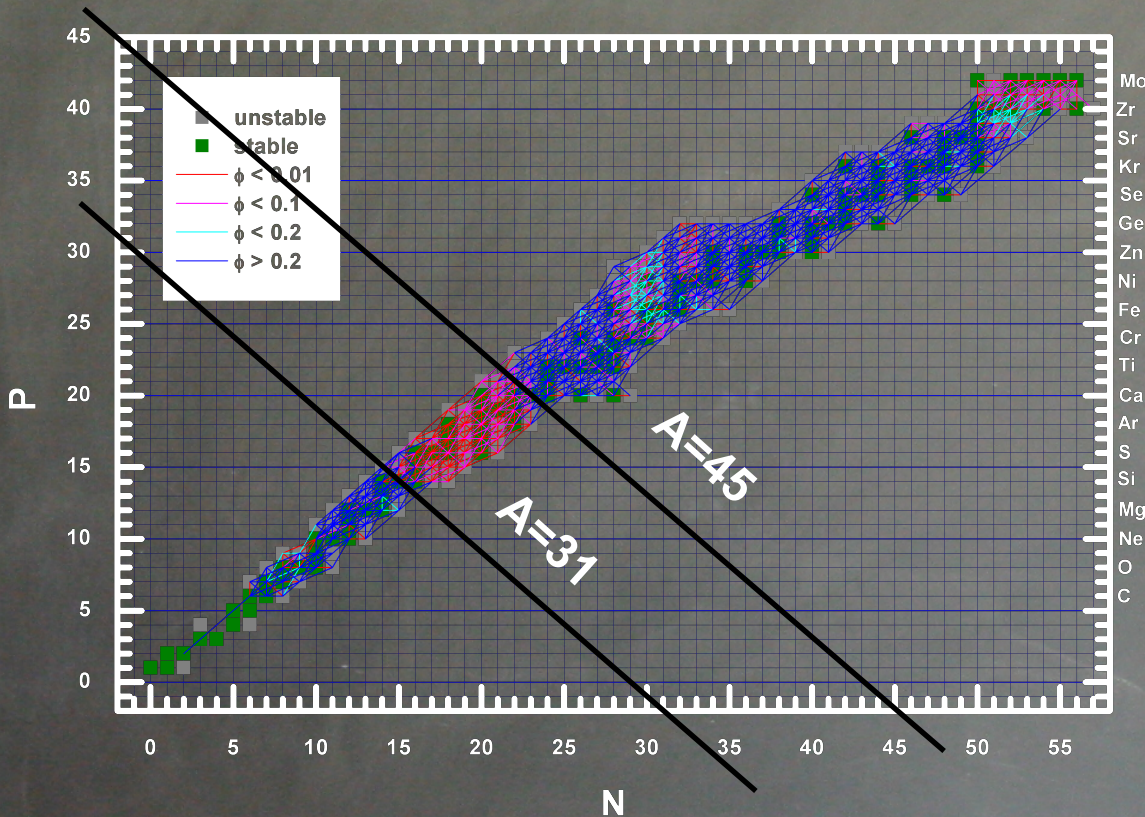
T=1.9 BK

Beginning of the O burning

$Y_e=0.4987$

**Almost all the processes
are far from the
equilibrium with their
reverse**

The approach to the Nuclear Statistical Equilibrium



$T=2.7 \text{ BK}$

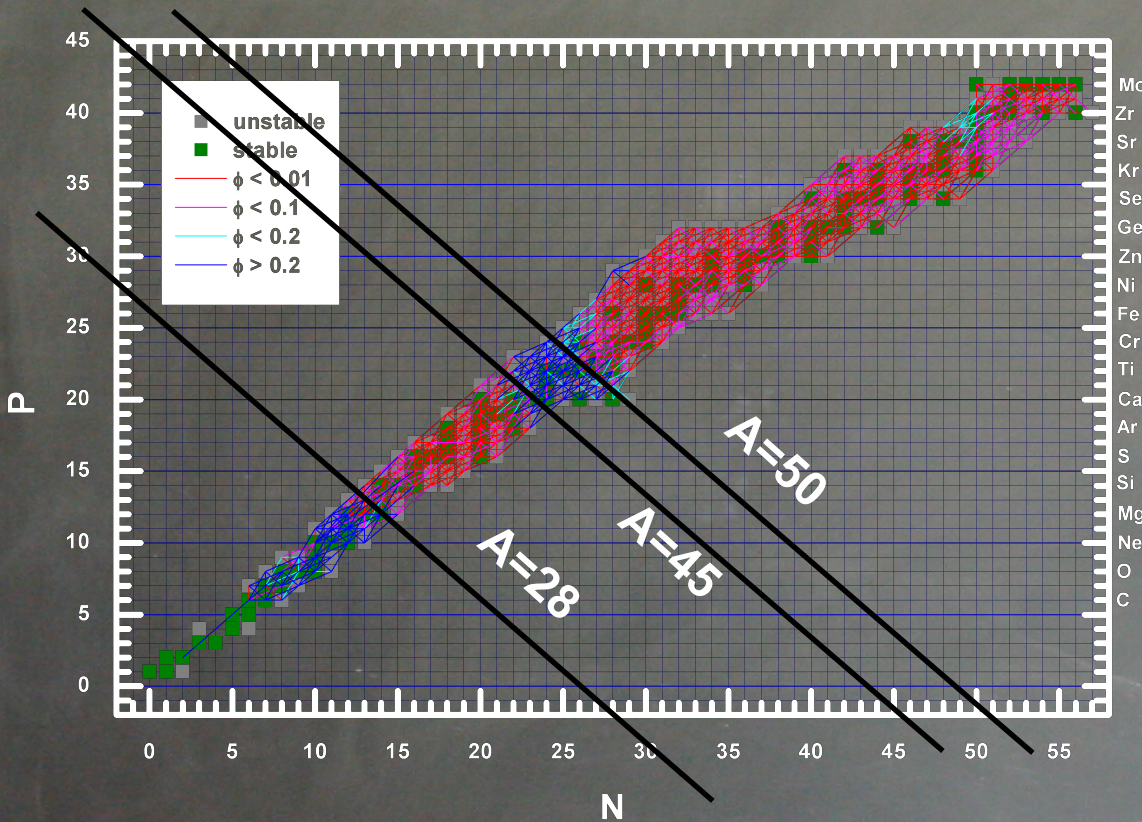
End of the O burning

$Y_e=0.4978$

A first cluster forms
between $A=31$ and $A=45$

Definition: a CLUSTER is a group of nuclei connected by processes that are at the equilibrium with their reverse.

The approach to the Nuclear Statistical Equilibrium



T=2.9 BK

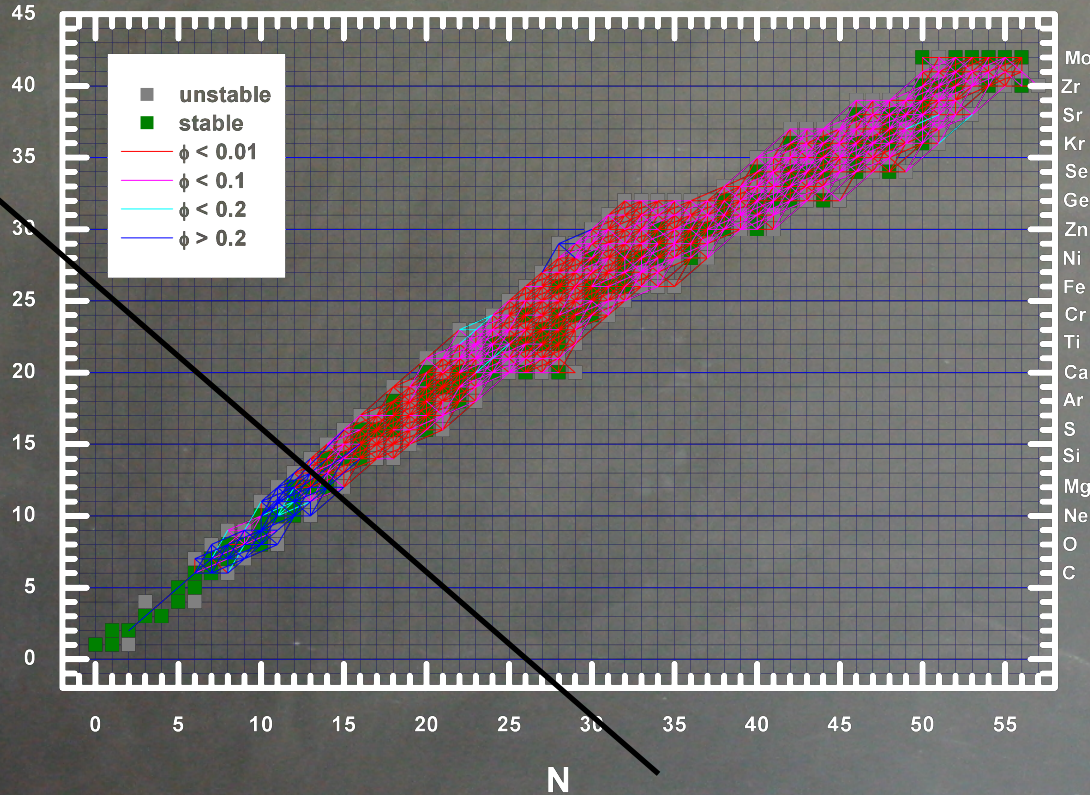
A little bit before the Si burning
 $Y_e = 0.4976$

The first cluster extends now
between A=28 and A=45
but
a second one forms at A>50

Within each cluster the abundances of the various nuclei depend on their equilibrium with respect to the sea of α and p. Such an equilibrium abundance is determined by an equation that looks like a Saha equation:

$$Y(n,z) = f(\rho, T, \text{a nucleus not in equilibrium})$$

The approach to the Nuclear Statistical Equilibrium



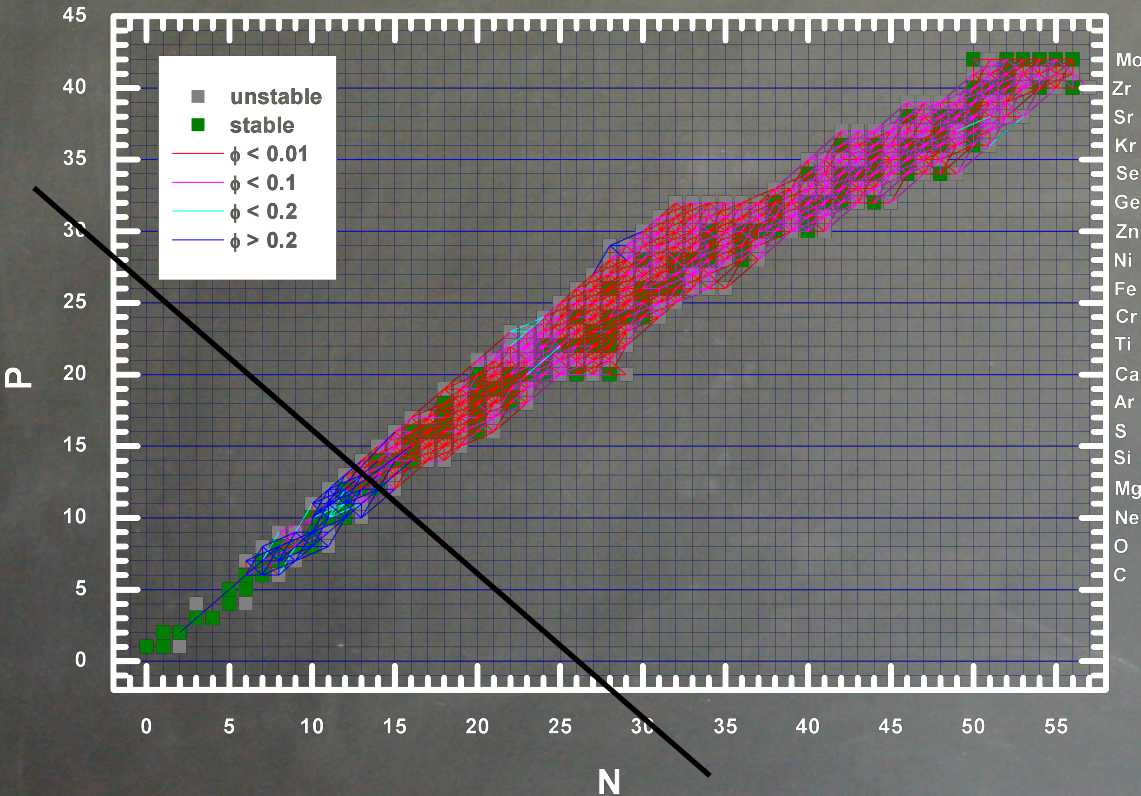
T=3.4 BK

Beginning of the Si burning
 $Y_e = 0.4955$

The two clusters begin to merge and form an unique cluster that starts at A=28

^{28}Si is not at the equilibrium. It is destroyed by the γ, α photodisintegration

The approach to the Nuclear Statistical Equilibrium



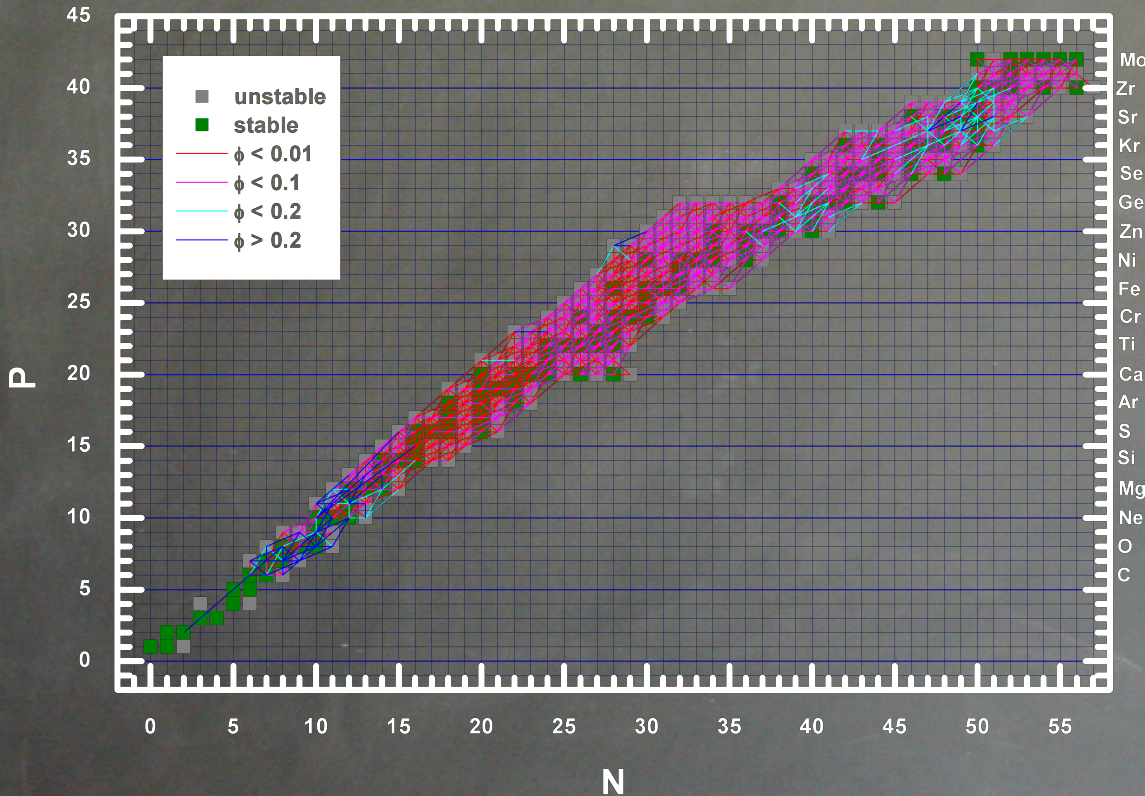
T=3.4 BK

Beginning of the Si burning
 $Y_e = 0.4955$

The two clusters begin to merge and form an unique cluster that starts at A=28

^{28}Si is not at the equilibrium. It is destroyed by the γ, α photodisintegration

The approach to the Nuclear Statistical Equilibrium



$T=4.1$ BK

End of the Si burning

$Y_e=0.4780$

Nuclei with $A < 28$ do not reach the equilibrium

Most of the matter located in the nucleus(i) that has(ve) the highest binding energy for the Y_e present at the moment. Remember that the weak processes are not at the equilibrium and must be taken into account explicitly!

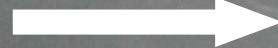
At 5 BK full Nuclear Statistical equilibrium is attained

The abundances of the various nuclei are governed by a set of equations of this kind:

$$Y(n, z) = f(\rho, T) \cdot e^{\frac{-Q(n, z)}{KT}} \cdot Y_p^z \cdot Y_n^n$$

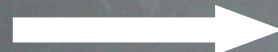
The system is closed by the conditions:

Mass conservation



$$\frac{\sum Y_i \cdot A_i}{\sum Y_i} = 1$$

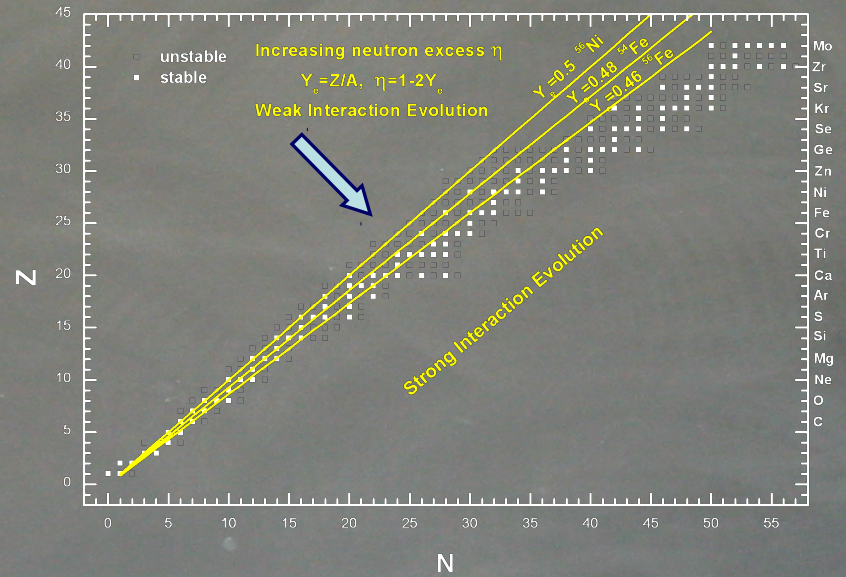
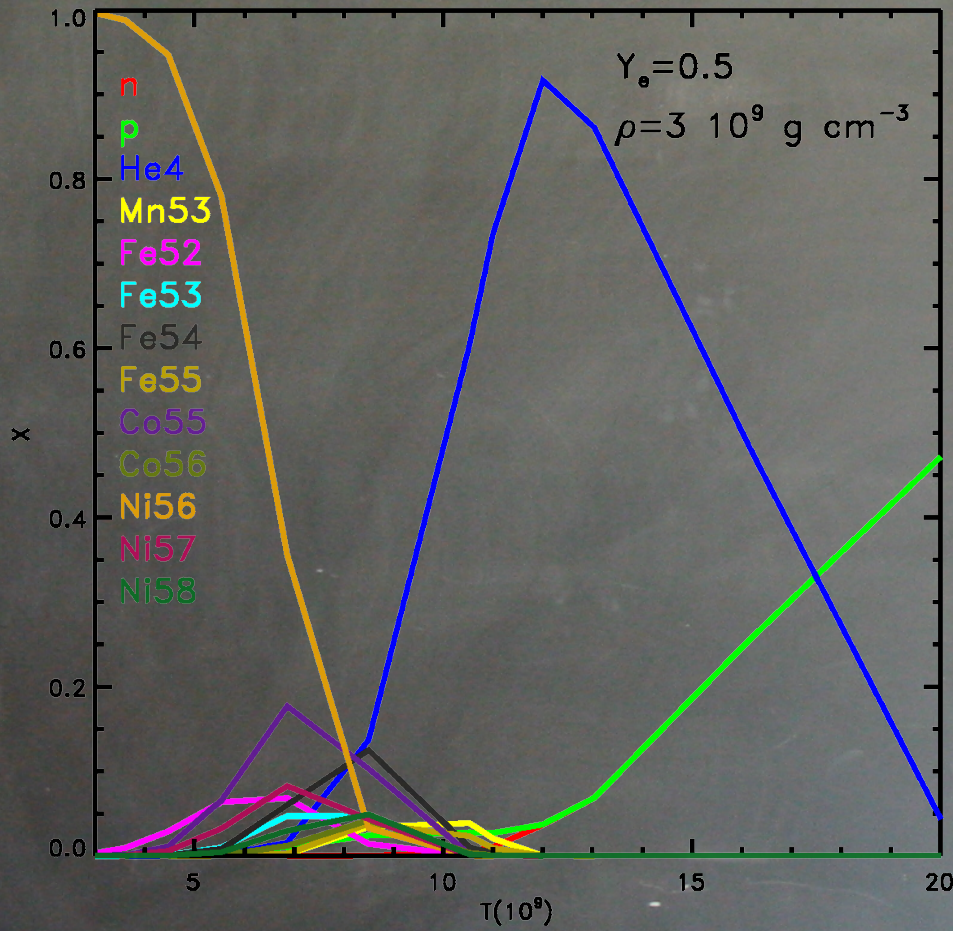
Electron mole number
conservation



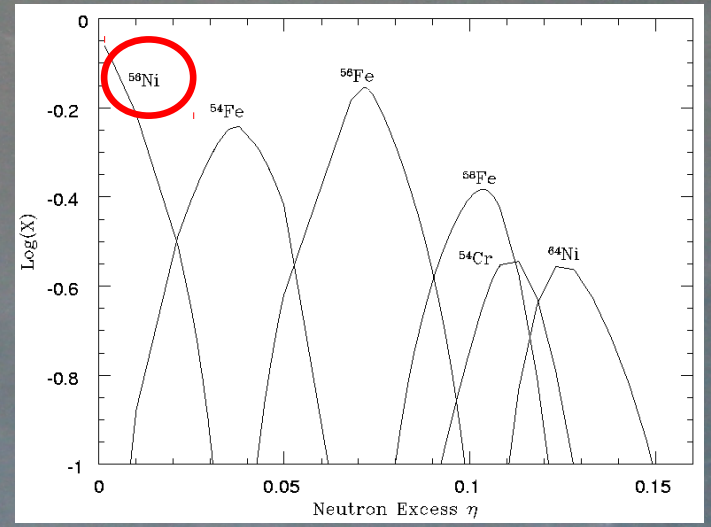
Ye = constant (at each time)

The abundances of all nuclei depend only on ρ , T and Y_e

Most abundant elements in NSE conditions as a function of the temperature

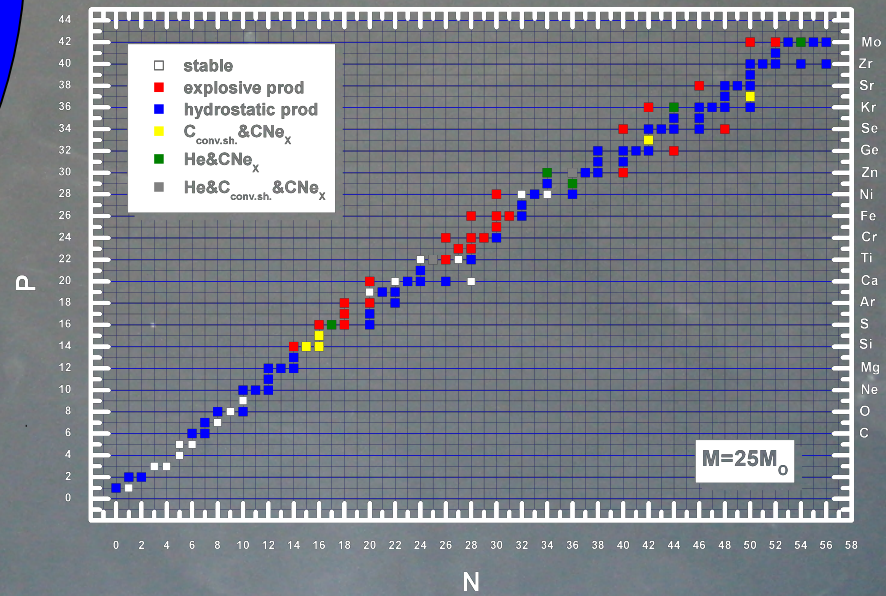
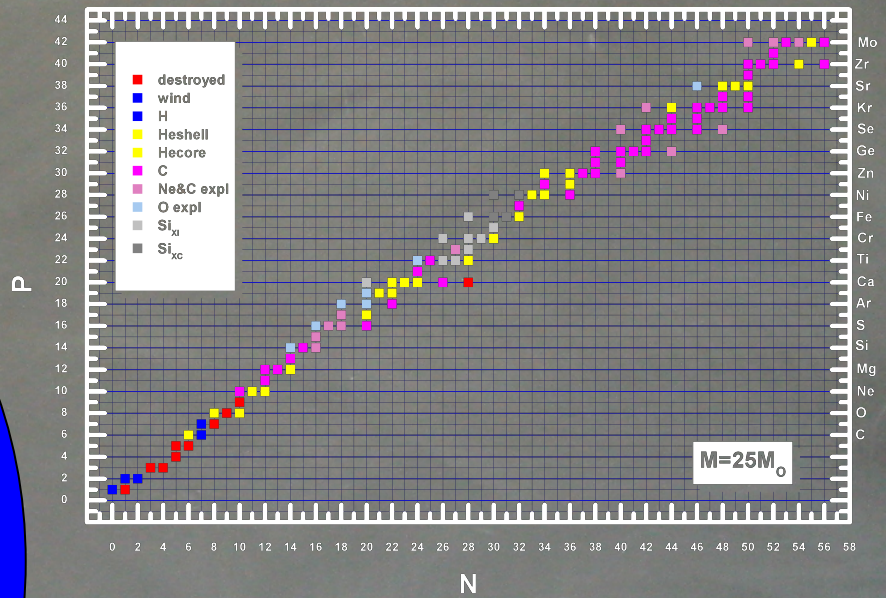
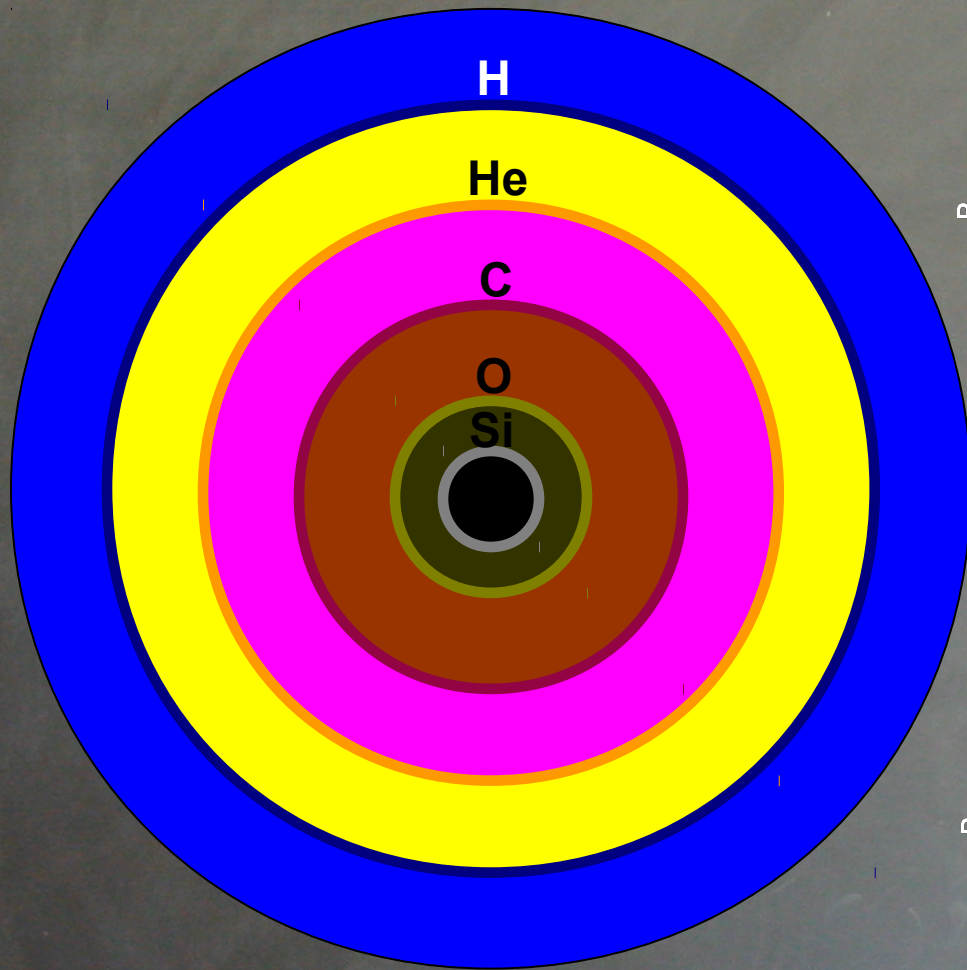


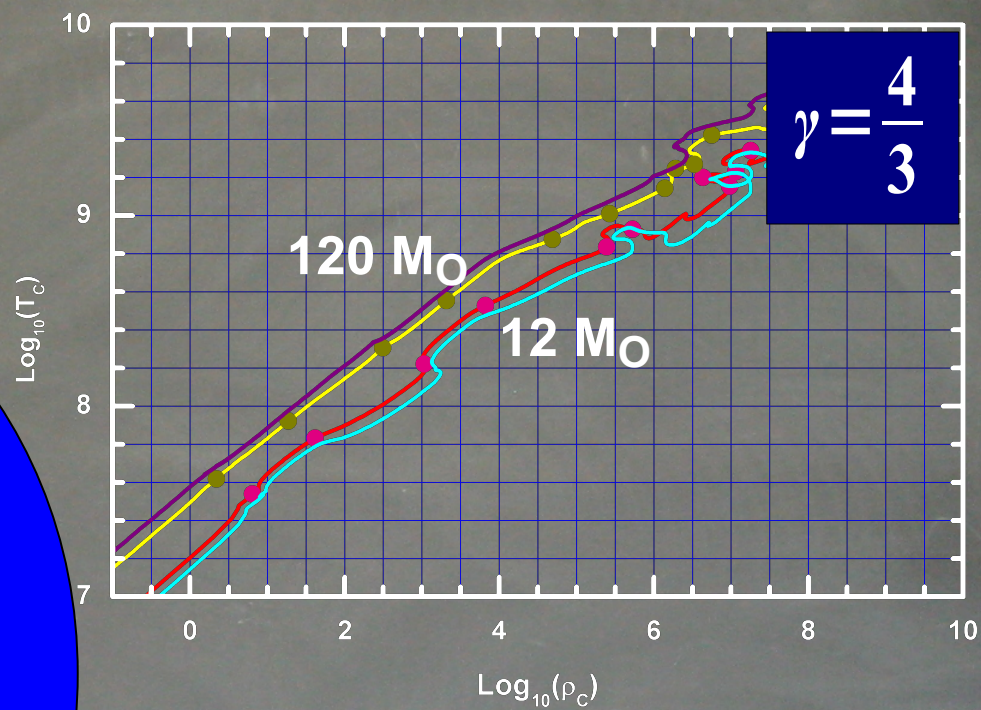
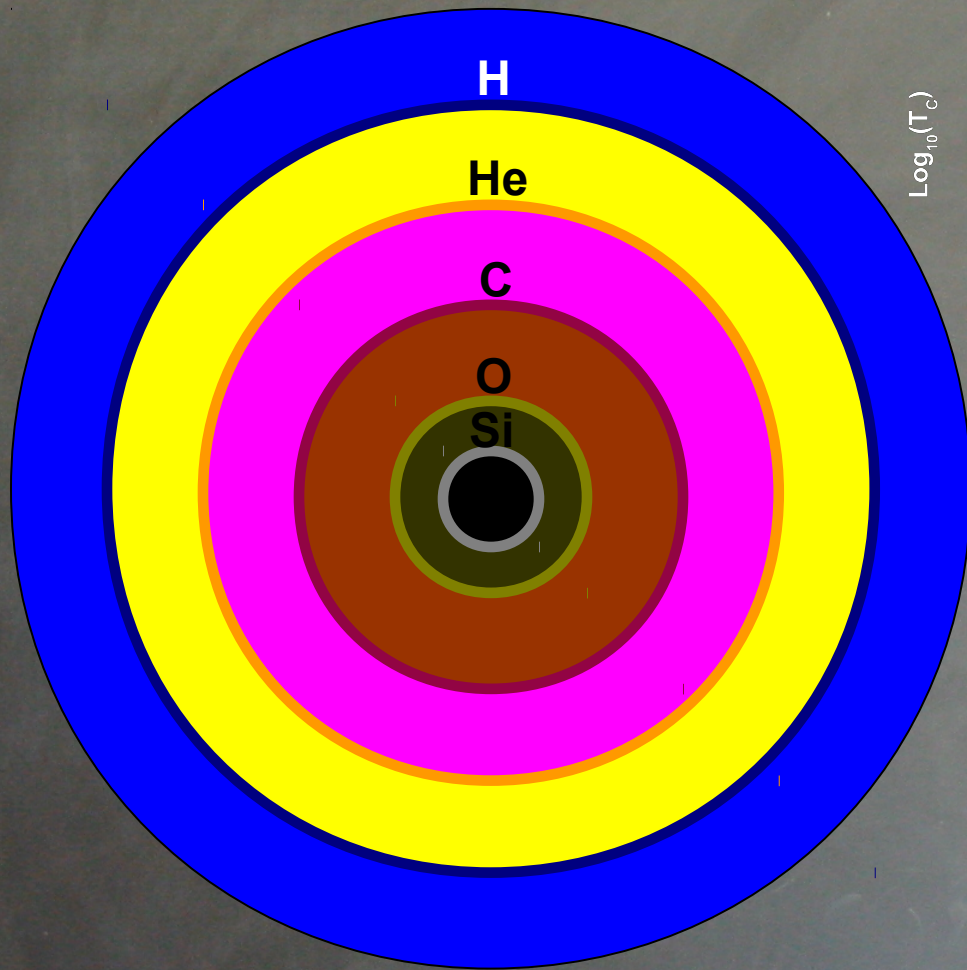
$T = 5 \cdot 10^9 \text{ K}$ $\rho = 10^8 \text{ g/cm}^3$



Trailer time!

C, Ne, O & Si burning movie





Virial theorem

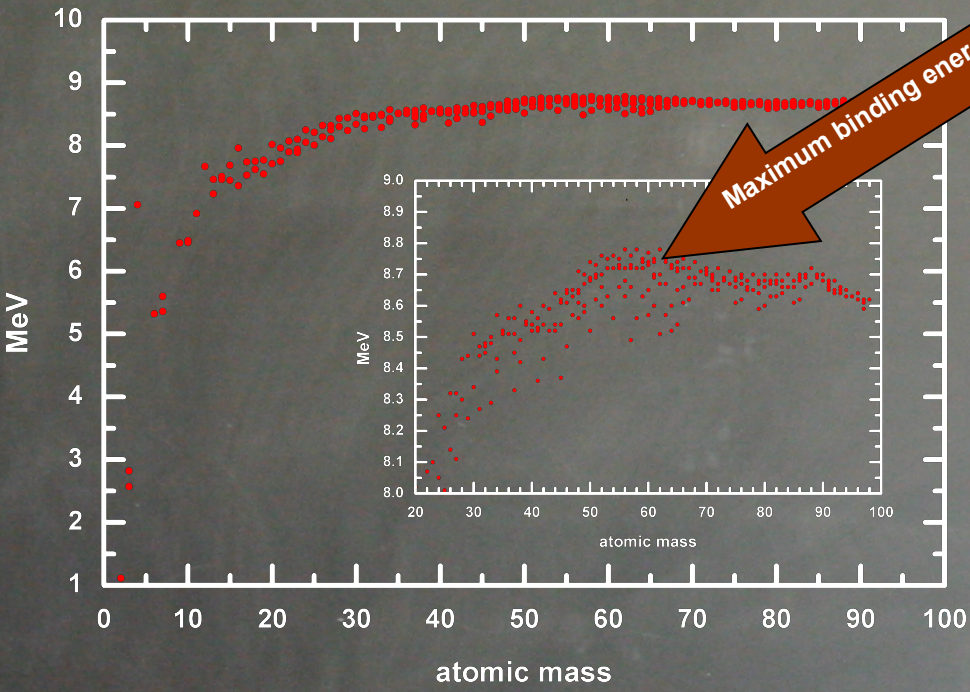
$$3(\gamma - 1)U + \Omega = 0$$

$$U + \Omega \equiv E_{TOT} = 0$$

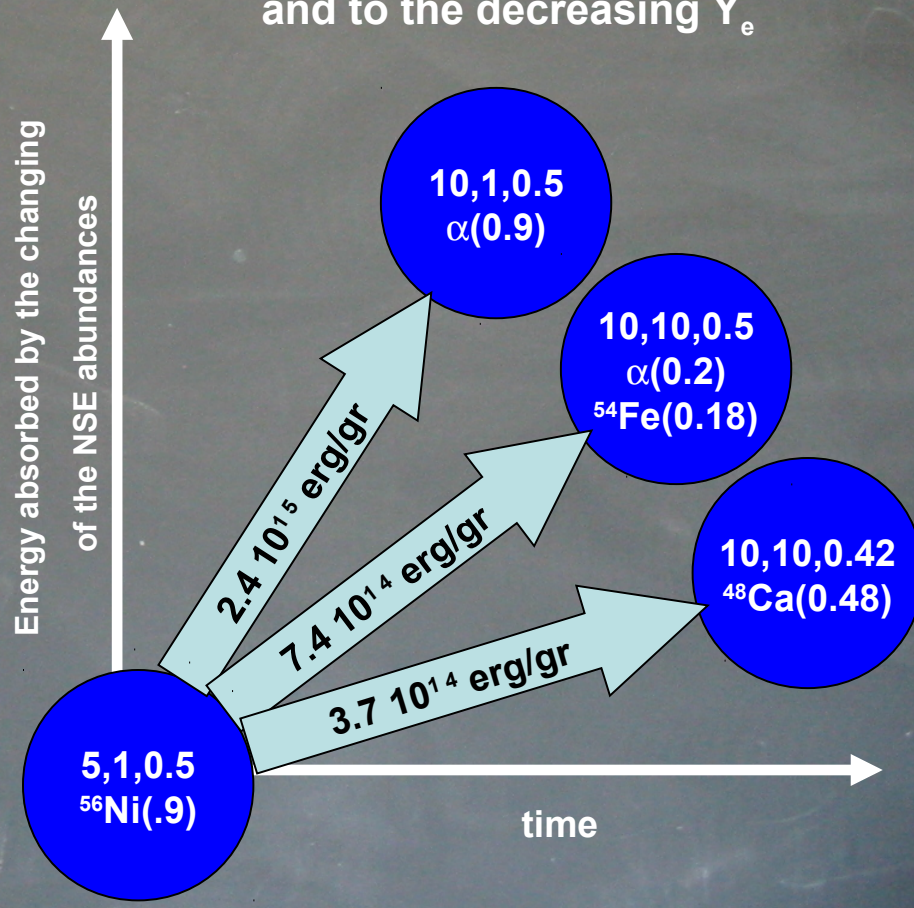
$$\Delta E_{TOT} \equiv 0$$

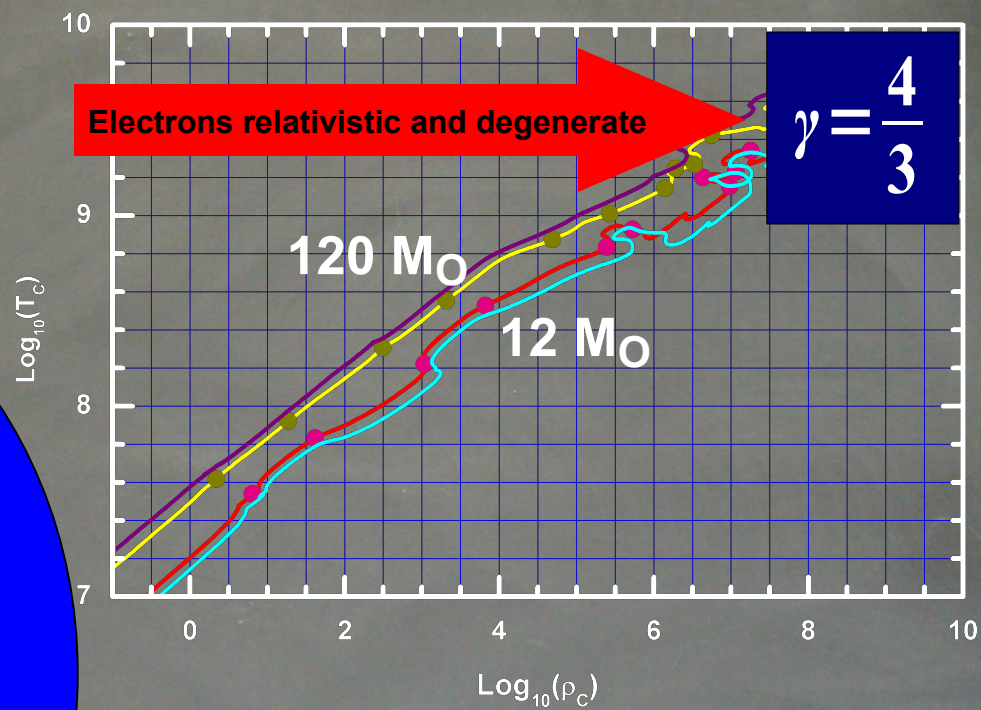
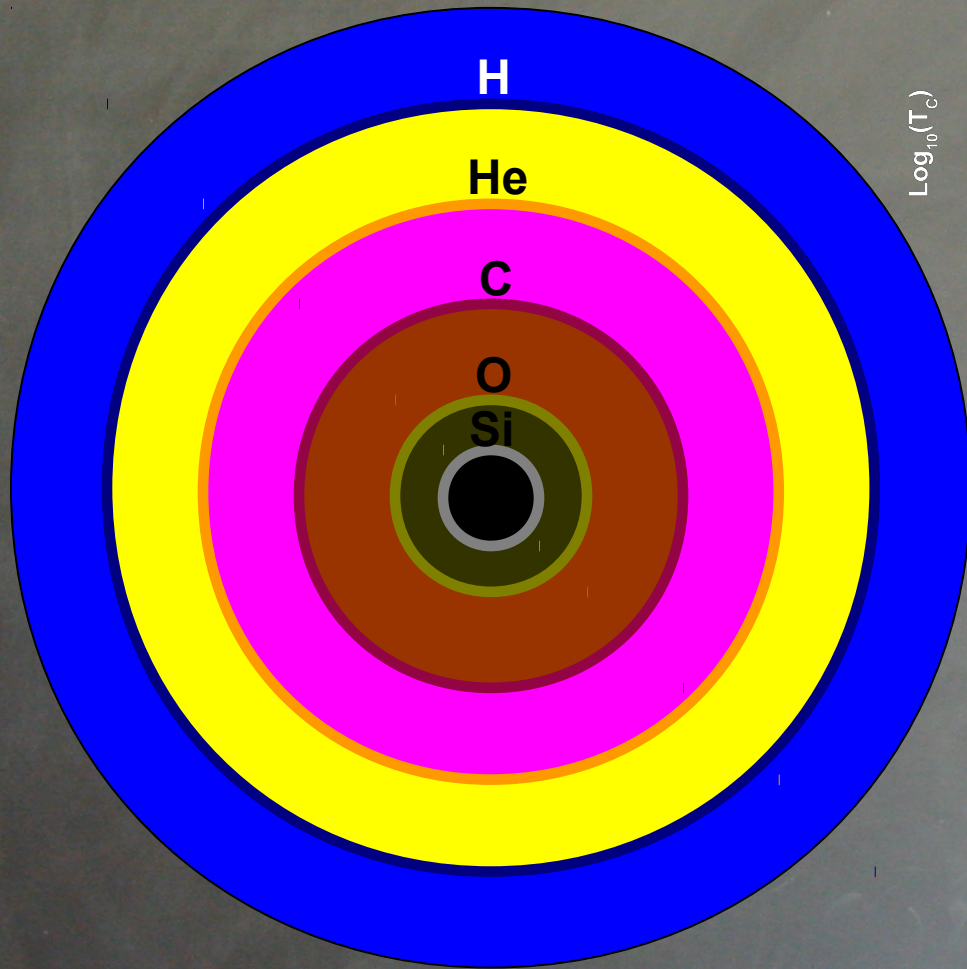
NO delay is required for a contraction,
the structure is only marginally stable

What happens to the inner core after the central Si burning?



NSE abundances
readjust to the increasing T and ρ
and to the decreasing Y_e





Virial theorem

$$3(\gamma - 1)U + \Omega = 0$$

$$U + \Omega \equiv E_{TOT} = 0$$

$$\Delta E_{TOT} \equiv 0$$

NO delay is required for a contraction,
the structure is only marginally stable

Sequence of events that lead to the collapse

The passage from an NSE configuration to another one of higher T, ρ and lower Y_e absorbs energy and hence speeds up the contraction.

Electrons become relativistic degenerate, so that $\gamma=4/3$

The weak processes subtract electrons and hence pressure.

The reduction of the pressure worsens the problem because it translates in a further contraction, electron more relativistically degenerate and stronger weak processes.

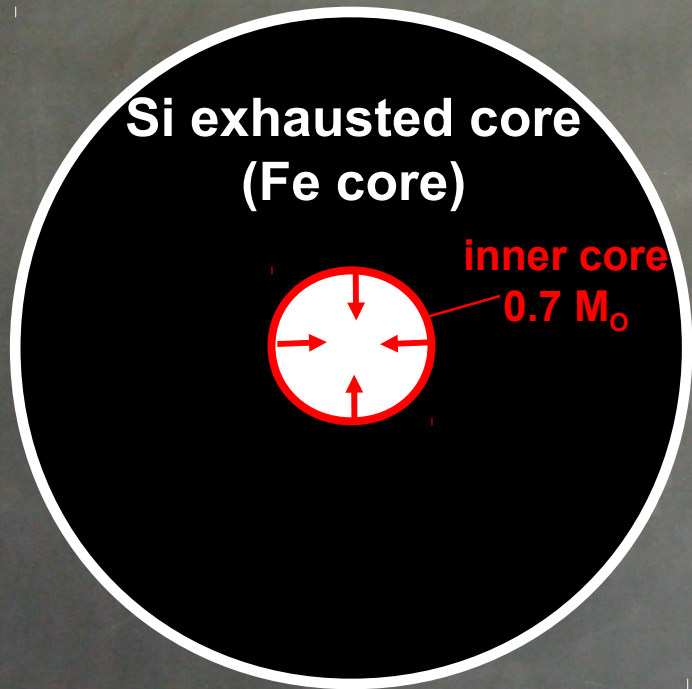
The Chandrasekhar mass reduces because $M_{CH} = 5.76 (Y_e)^2$

No configuration equilibrium exists any more and the collapse starts

T



Basic core collapse scenario



The inner 0.7-1 M_o starts collapsing

The collapse stops only when matter reaches the nuclear densities:

$$\rho \simeq 10^{14} \text{ g cm}^{-3}$$

because at this stage matter becomes incompressible

If we assume that the density is constant throughout the collapsing core, we can easily estimate the final radius of a giant "NUCLEUS" of 1 M_o:

$$M = \frac{4}{3} \pi R^3 \rho \quad \Rightarrow \quad R = \left(\frac{3}{4 \pi} \frac{M}{\rho} \right)^{1/3} \simeq 17 \text{ Km}$$

If, for simplicity, we assume constant density:

$$\Omega = - \int_0^M \frac{G M}{R} dm = - \frac{3}{5} \frac{G M^2}{R}$$

$$\Delta \Omega = \Omega_{final} - \Omega_{initial} = - \frac{3}{5} G M^2 \left(\frac{1}{R_{final}} - \frac{1}{R_{initial}} \right) \simeq -1.58 \cdot 10^{59} \left(\frac{1}{R_{final}} \right)$$

$$\simeq 1.6 \cdot 10^{53} \text{ erg}$$

(assuming a radius of 10 Km)

Is this energy enough to drive a successful explosion?

Inventory:

$$\text{Initial pot: } 1.6 \cdot 10^{53} \text{ erg } M_{\odot}^{-1}$$

As T and ρ increase, NSE favors P and N so we must consider the energy required to dissociate nuclei in P and N:

$$\Delta E = (28M_P + 28M_N - M(^{56}\text{Ni})) 1.49 \cdot 10^{-3} \frac{6.022 \cdot 10^{23}}{56} 1.989 \cdot 10^{33} = 1.7 \cdot 10^{52} \text{ erg } M_{\odot}^{-1}$$

Most of the P tend to convert in N as nucleons begin to feel their fermion soul:

$$\Delta E = (M_N - M_P - M_e) 1.49 \cdot 10^{-3} \frac{6.022 \cdot 10^{23}}{1} 1.989 \cdot 10^{33} = 1.5 \cdot 10^{51} \text{ erg } M_{\odot}^{-1}$$

But in this process also neutrinos are emitted:

$$\Delta E = (20) 1.6 \cdot 10^{-6} \frac{6.022 \cdot 10^{23}}{1} 1.989 \cdot 10^{33} = 3.8 \cdot 10^{52} \text{ erg } M_{\odot}^{-1} \quad \text{assuming } E_{\nu} = 20 \text{ MeV}$$

The energy available to drive the explosion is therefore given by:

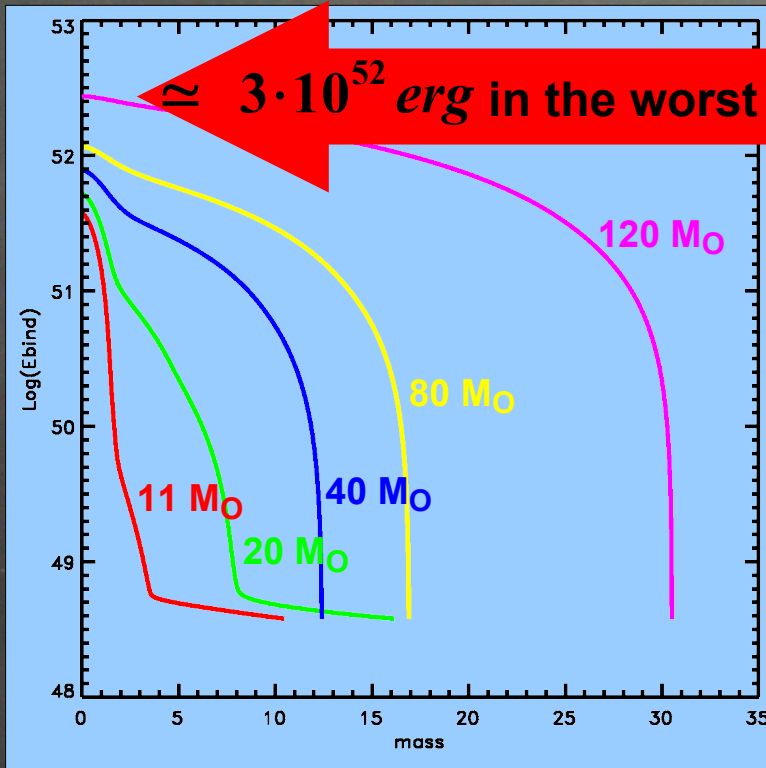
$$1.6 \cdot 10^{53} - 1.7 \cdot 10^{52} - 1.5 \cdot 10^{51} - 3.8 \cdot 10^{52} \simeq 10^{53} \text{ erg } M_{\odot}^{-1}$$

Is this energy enough to drive a successful explosion?

Inventory:

... so we are left with $\simeq 10^{53} \text{ erg } M_{\odot}^{-1}$

Binding energy of the mantle as a function of the mass coordinate



Observations show that some kinetic energy is provided to the ejecta and it ranges, roughly, between:

$$10^{50} - 10^{52} \text{ erg}$$

So in principle there is plenty of energy to drive a successful explosion!

Basic core collapse scenario

Unfortunately most of the energy gained during the collapse is emitted as ν and not as γ !

The reason is that, the relative proportions between P and N in the giant “nucleus” are kept at their equilibrium value by the two very efficient processes :



The mean free path λ between two successive interactions between the particles i and j is given by:

$$\lambda = \frac{1}{\kappa \rho}$$

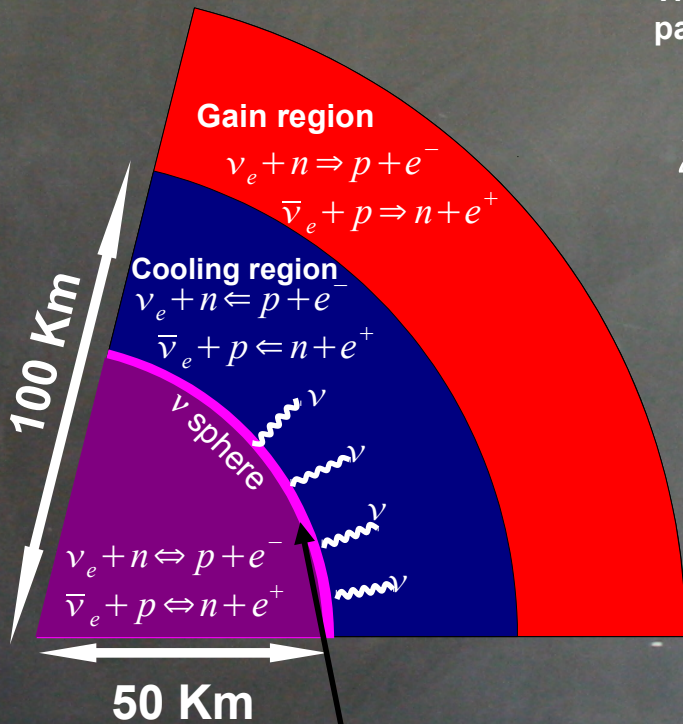
Where κ , the “opacity”, may be expressed in terms of the probability σ_{ij} that an interaction between the particles i and j occurs:

$$\kappa = \frac{N_A}{A} \sigma_{\nu A}$$

The basic interaction between ν and a nucleus A is given by the neutral current coherent scattering, whose cross section is given by:

$$\sigma_{\nu A} \simeq 10^{-44} N^2 \left(\frac{E_\nu}{\text{MeV}} \right)^2 \text{ cm}^2$$

$$\lambda = \frac{A}{N_A \sigma_{\nu A} \rho} = \frac{1}{6.022 \cdot 10^{23} 10^{-44} 10^2 \rho} \simeq \frac{1.7 10^{18}}{\rho} \text{ cm}$$



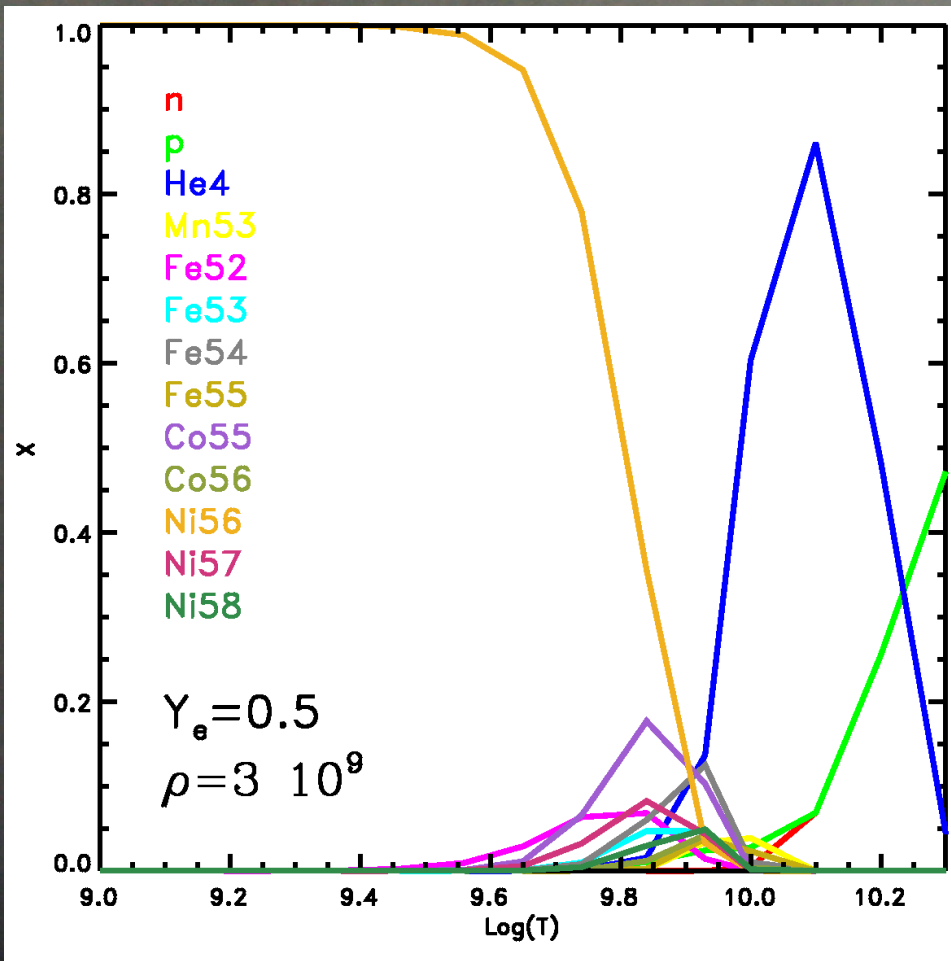
ν diffusion timescale: 10 s

Basic core collapse scenario

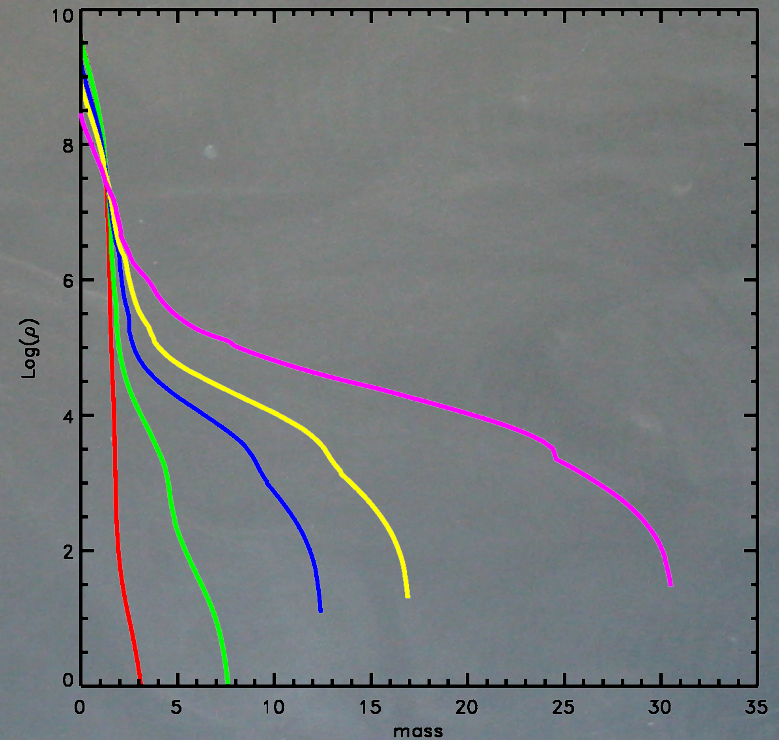
What it is even worst is that the shock wave lose a large amount of energy on its way out:

The reason is that it fully photodisintegrates matter as it advances in mass.

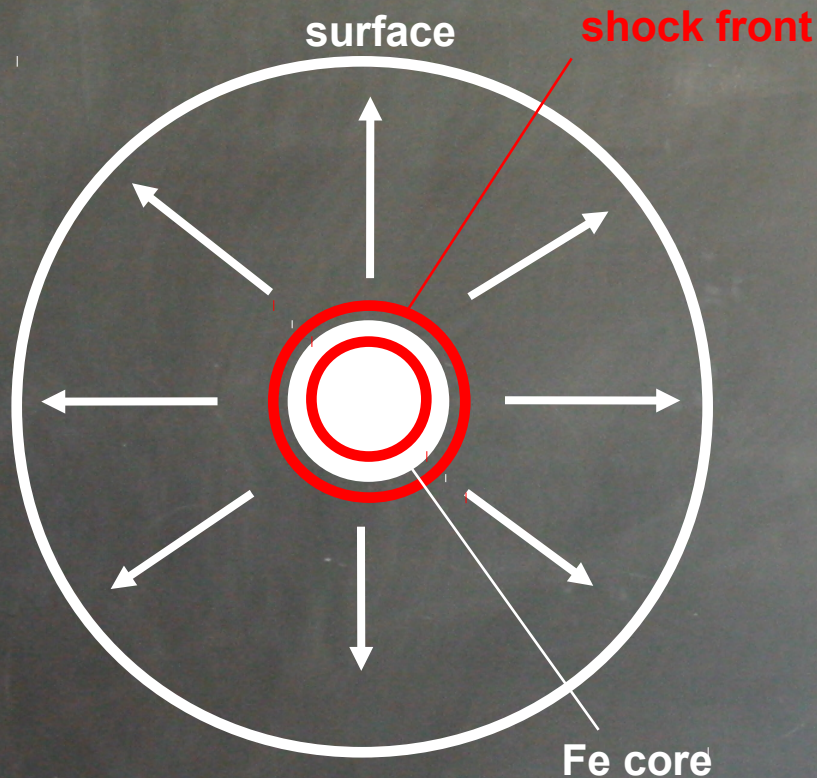
For example: $^{56}\text{Fe} \Rightarrow 30 \text{ n} + 26 \text{ p}$ requires the absorbtion of $7.87 \cdot 10^{-4} \text{ erg}$ (492 MeV)



$8.47 \cdot 10^{18} \text{ erg/gr} \Rightarrow 10^{51} \text{ erg} / 0.1 M_{\odot}$



In spite of the many efforts, no successful explosion has been obtained yet



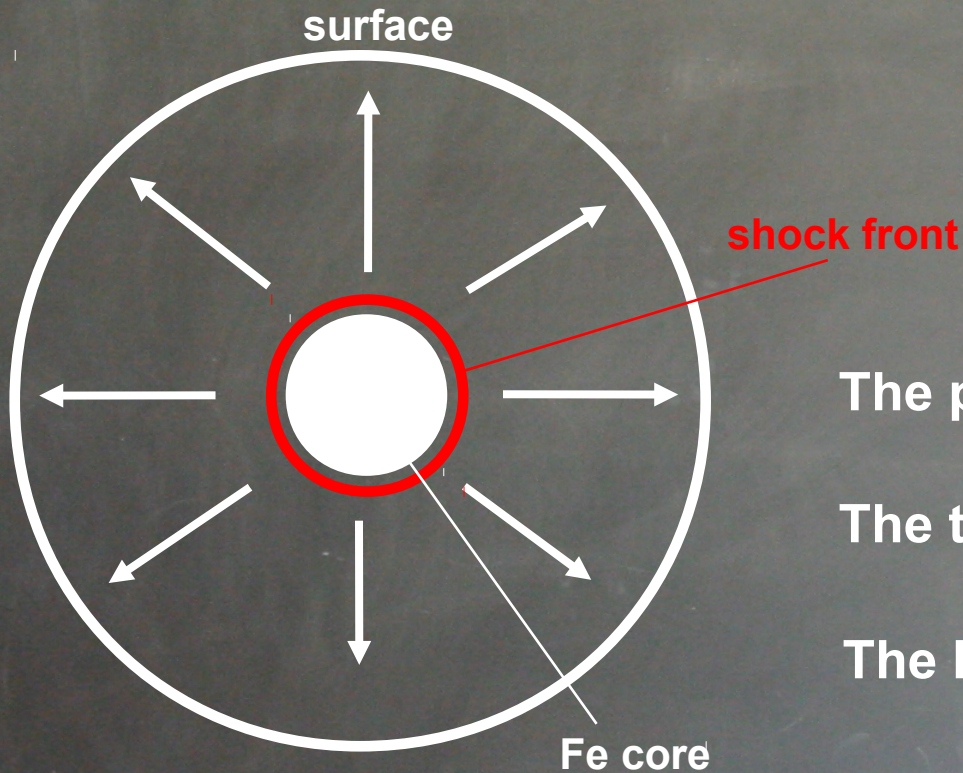
Escamotage:

Assume that the shock wave escapes the dense core (roughly the Fe core)

Since the explosion is not obtained “naturally” a few assumptions are unavoidable:

- 1) Energy deposited in the shock front
- 2) Formation of a shock driven convective zone

Three different techniques have been used up to now:



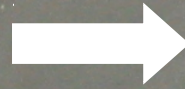
The piston (Woosley and coworkers)

The thermal bomb (Nomoto and coworkers)

The kinetic bomb (Limongi and Chieffi)

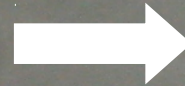
BASIC PROPERTIES OF THE SHOCK WAVE AFTER IT HAS ESCAPED THE DENSE FE CORE:

RADIATION DOMINATED:



$$E = \frac{4}{3} \pi r^3 a T^4$$

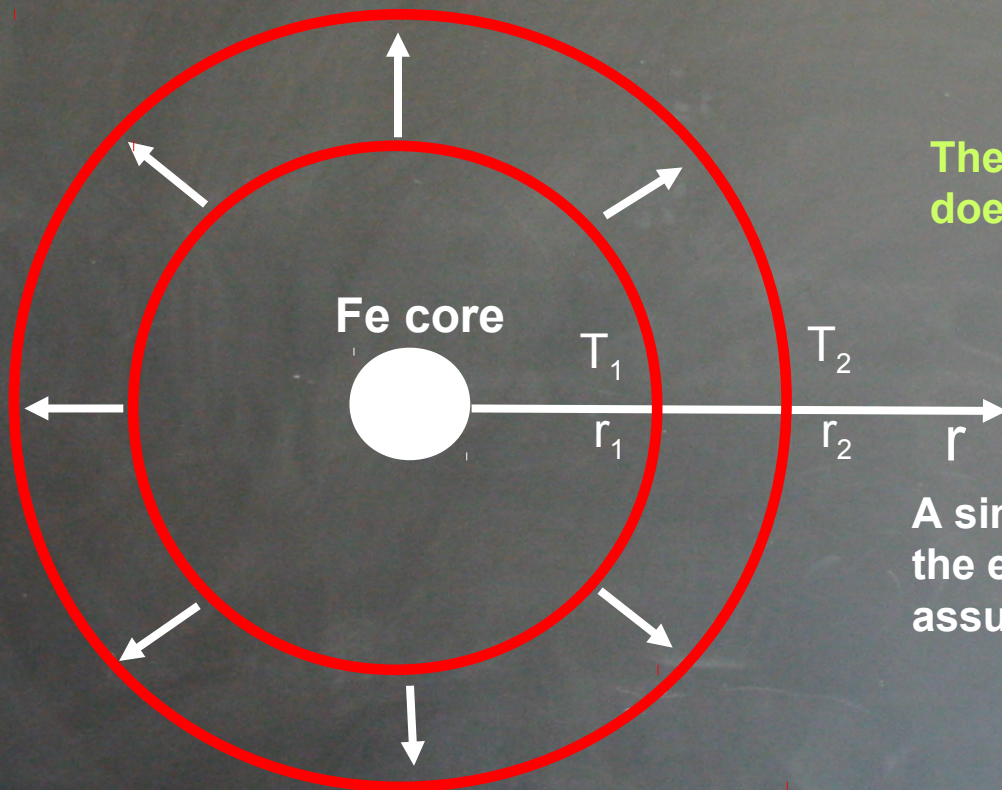
ADIABATIC EXPANSION:



$$T = \text{const} \cdot r^{\frac{3}{4}}$$

CONSEQUENCE:

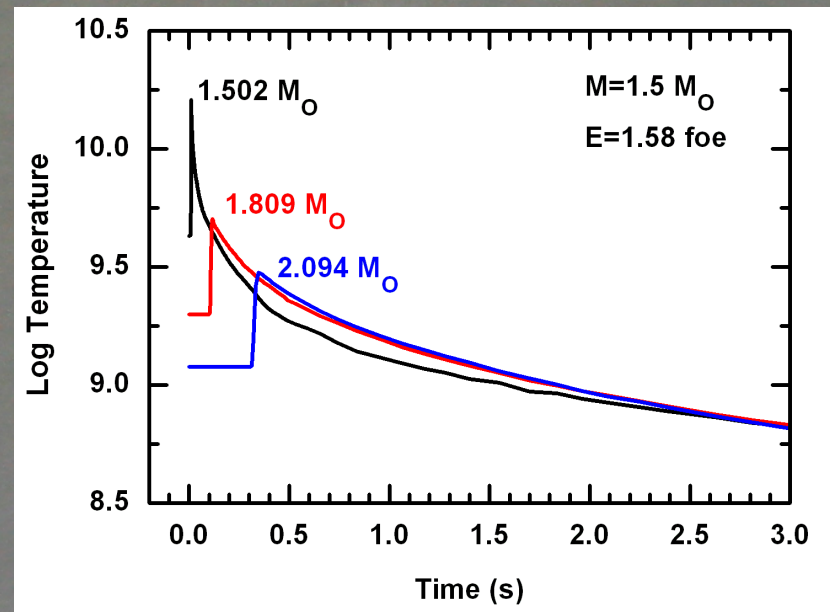
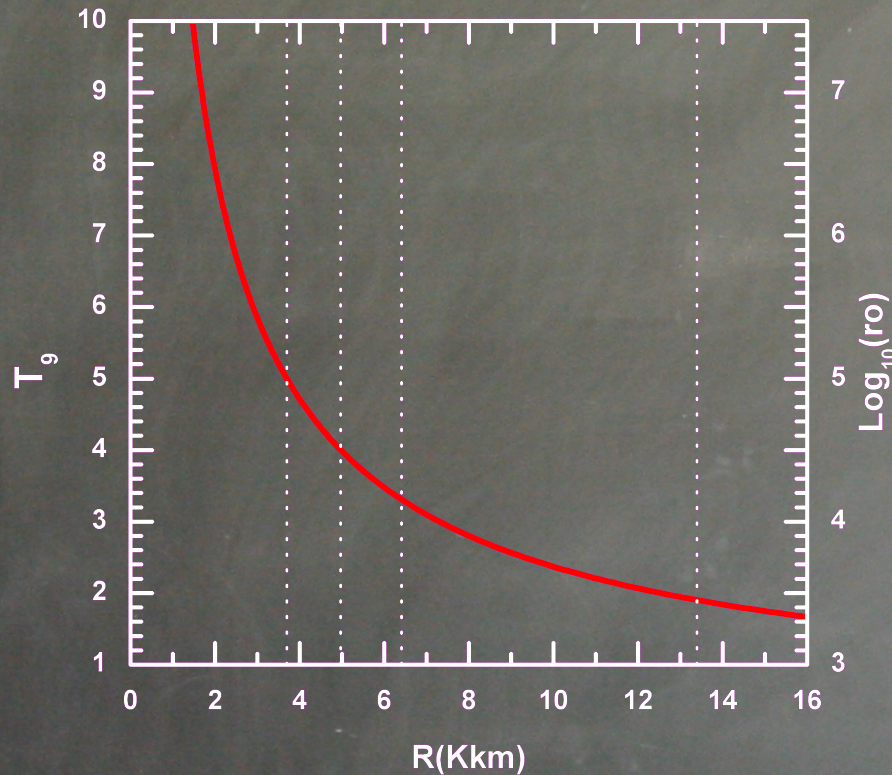
The peak temperature of the blast wave does not depend on the stellar structure.



A simple but quite effective computation of the explosive yields may be obtained by assuming:

$$T(t) = T_{\text{peak}} e^{-t/\tau}$$

Basic properties of the explosive burnings

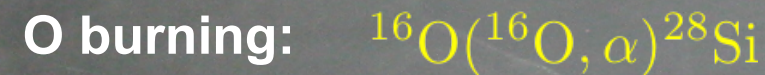
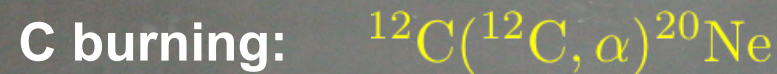
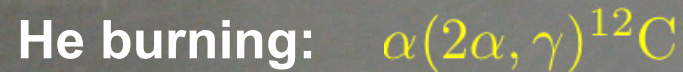


$$\dot{Y}_i = \sum_j c_i(j) \lambda_j Y_j + \sum_{j,k} c_i(j,k) \rho N_A \langle \sigma v \rangle_{j,k} Y_j Y_k + \sum_{j,k,l} c_i(j,k,l) \rho^2 N_A^2 \langle \sigma v \rangle_{j,k,l} Y_j Y_k Y_l$$

The typical burning timescale for the destruction of any given nuclear specie is given by: $\tau_i \simeq \left| \frac{Y_i}{\dot{Y}_i} \right|$

CHARACTERISTIC EXPLOSIVE BURNING TEMPERATURES

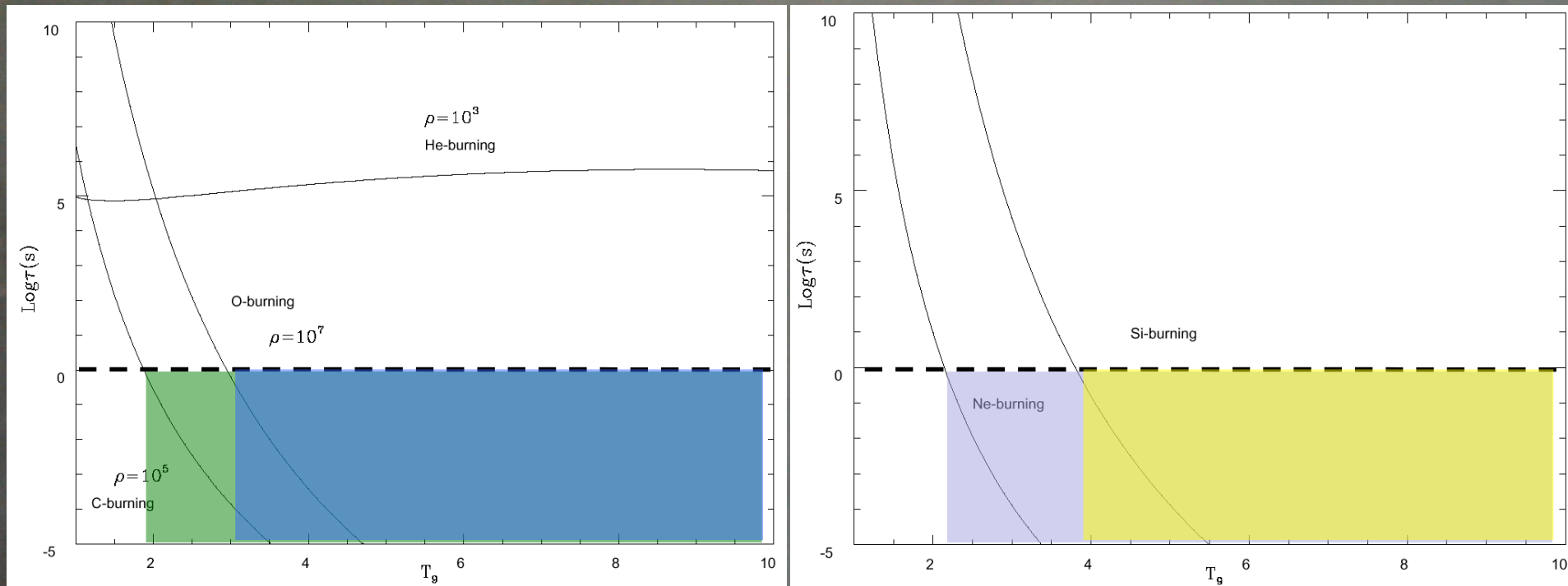
The timescales for the destruction of H, He, C, Ne, O and Si are determined by these nuclear reactions:



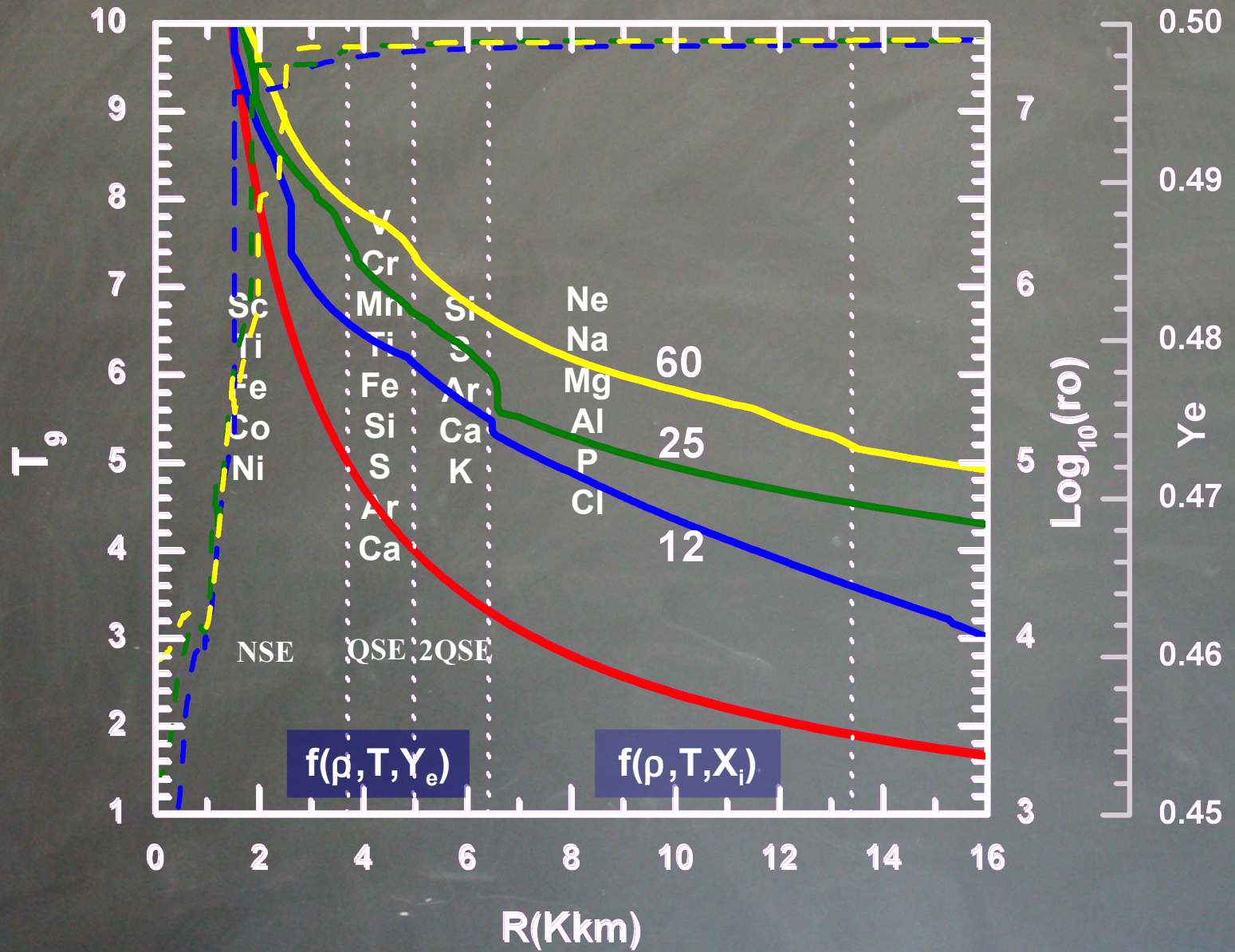
and in general are functions of temperature and density: $\tau_i = f(T, \rho)$

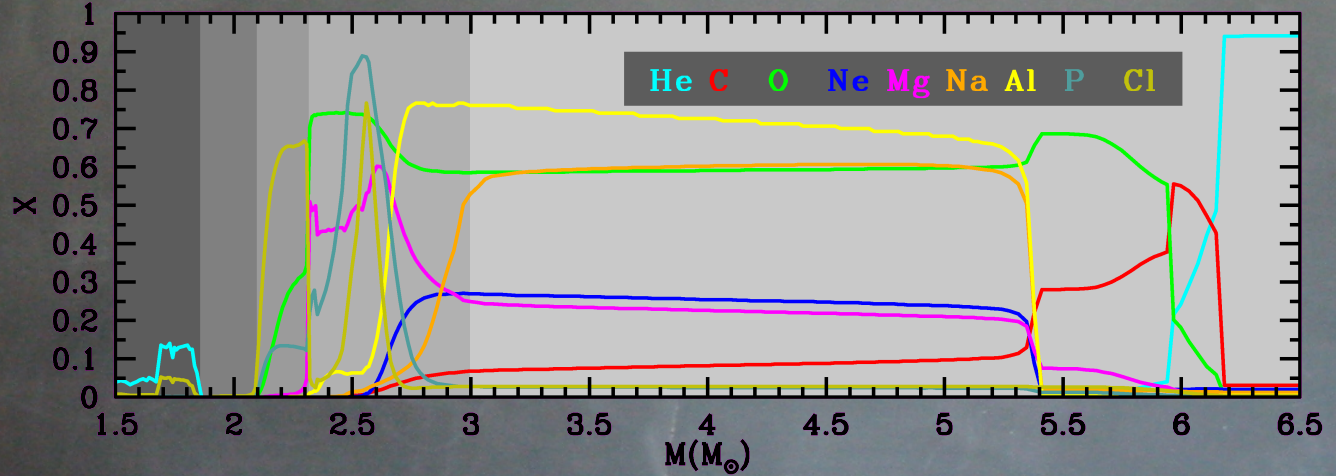
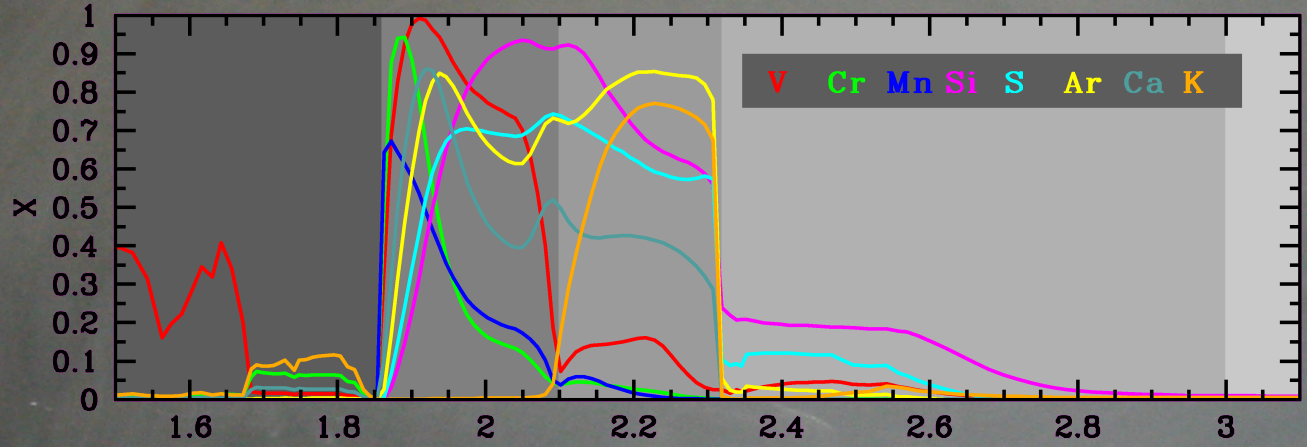
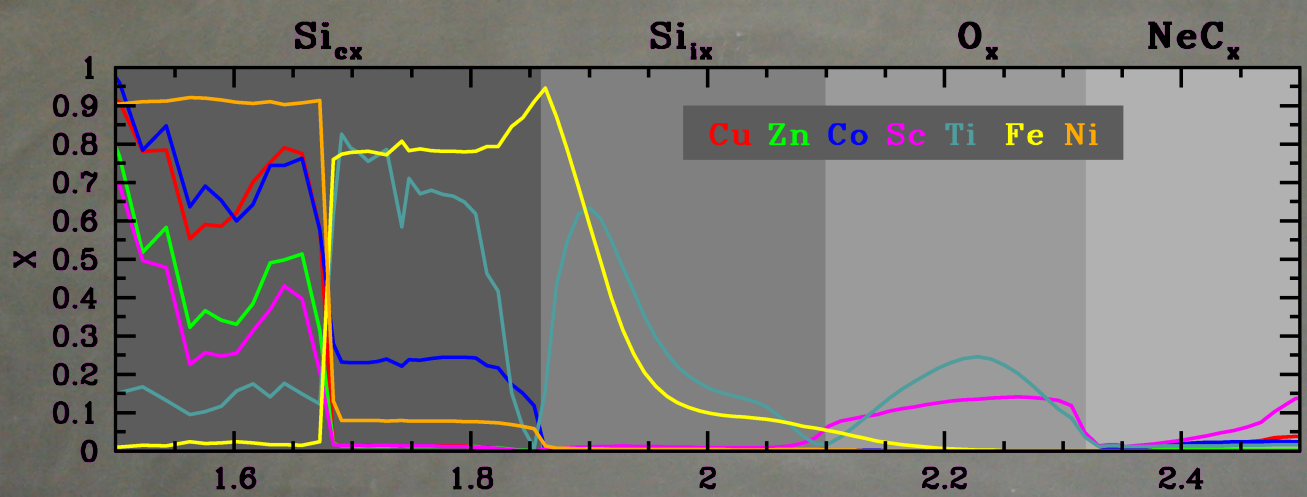
CHARACTERISTIC EXPLOSIVE BURNING TEMPERATURES

If we take typical explosive burning timescales of the order of 1s



$T_{9,\text{peak}} \gtrsim 1.9$	Explosive C burning
$T_{9,\text{peak}} \gtrsim 2.1$	Explosive Ne burning
$T_{9,\text{peak}} \gtrsim 3.3$	Explosive O burning
$T_{9,\text{peak}} \gtrsim 4.0$	Explosive Si burning

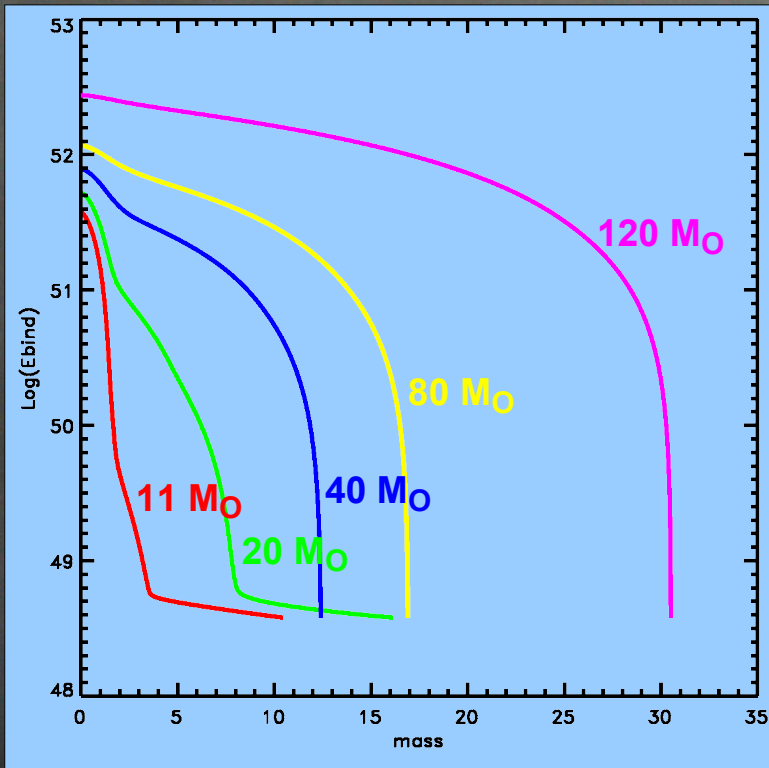




Let's come back to the exploding star. Since a self consistent determination of the energy escaping the Fe core is not yet available, we are forced to fix “by hand” a value.

The energy of the shock wave is fixed by imposing that some “observable” is reproduced:
usually
the kinetic energy of the ejecta
and/or
the amount of ^{56}Ni ejected

General considerations



If the energy assumed to escape the Fe core is too low, all the star will fall back in to the remnant (no matter will be ejected).

If the energy assumed to escape the Fe core is high, all the mantle will be ejected.

In the intermediate cases part of the mantle will fall back on the remnant and part will be ejected in the interstellar medium.

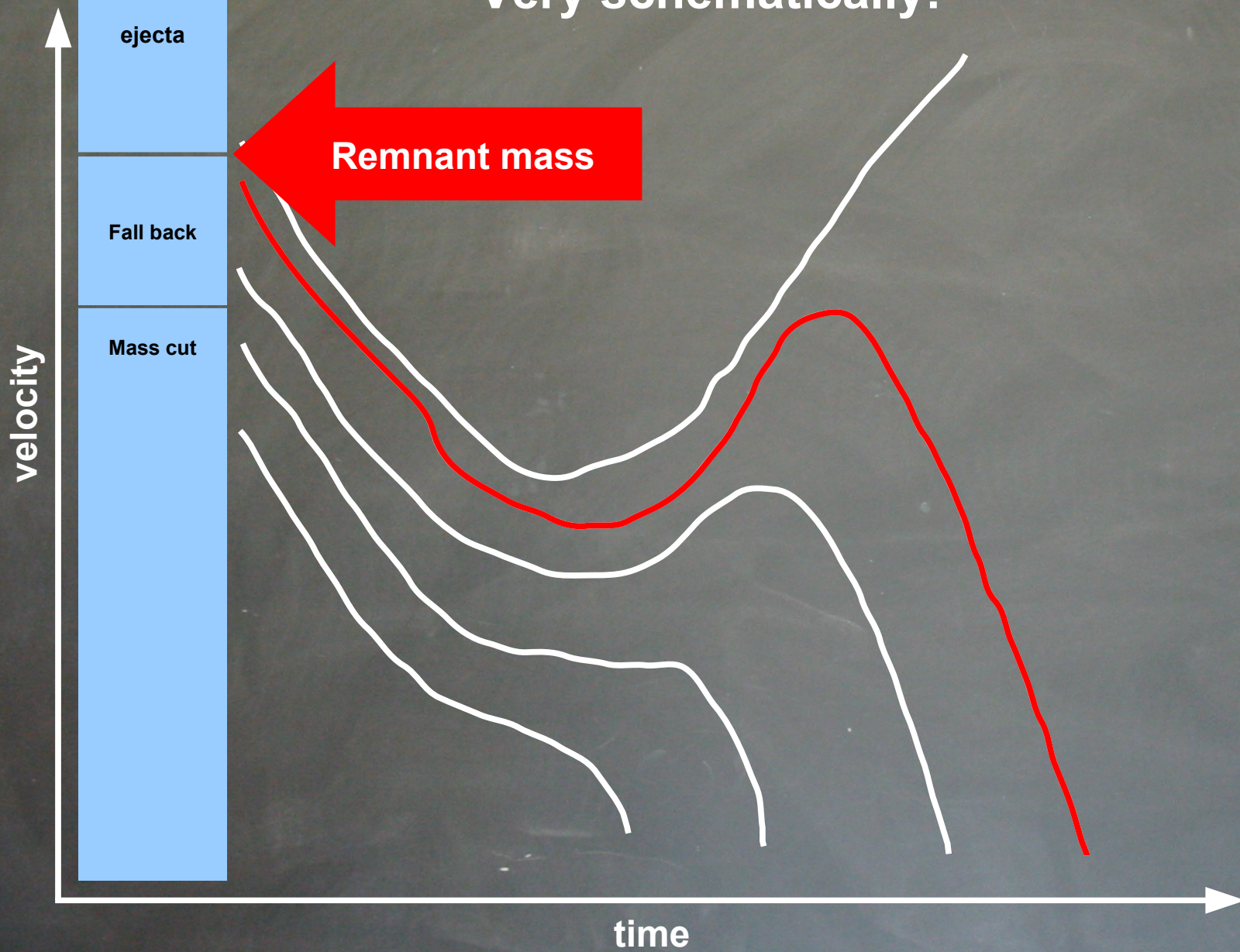
Basic definitions:

Mass cut: maximum mass that will always move inward

Fall back: mass that initially is kicked off but that then falls back on the collapsed core.

Remnant mass: mass cut + fall back (final mass size of the collapsed core).

Very schematically:



Remnant mass

ejecta

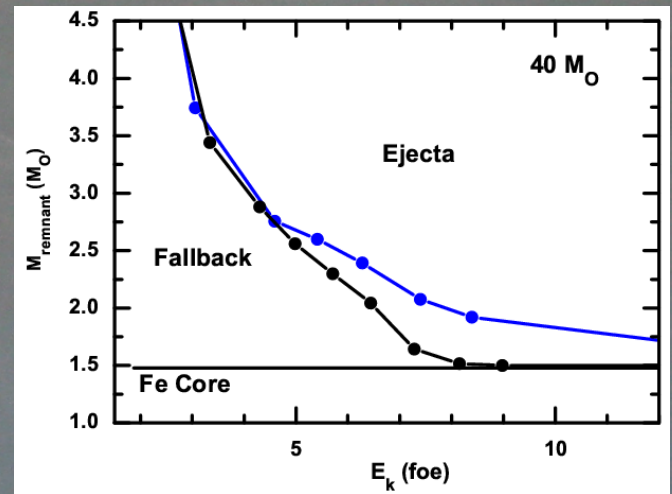
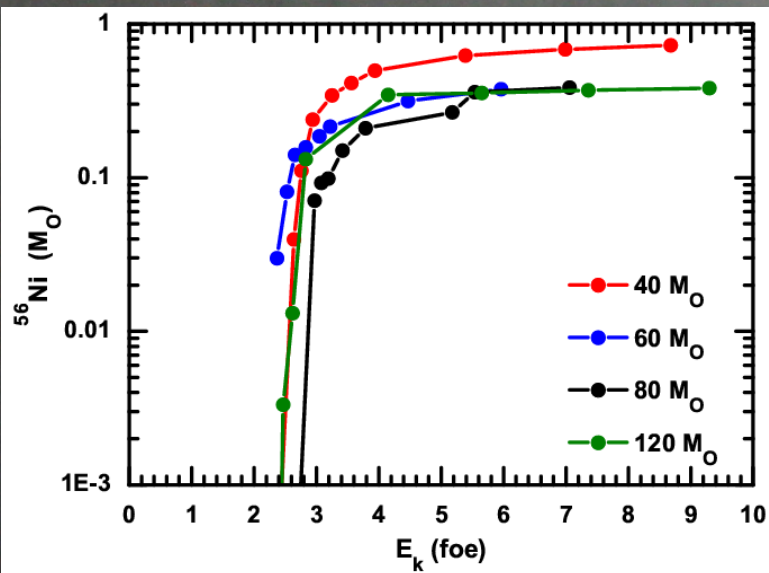
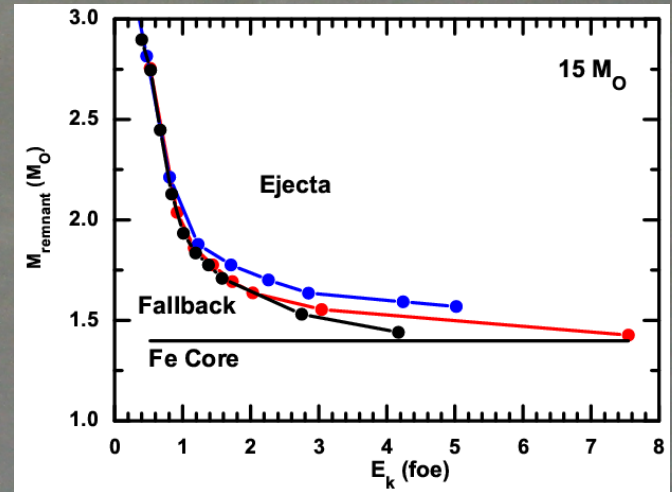
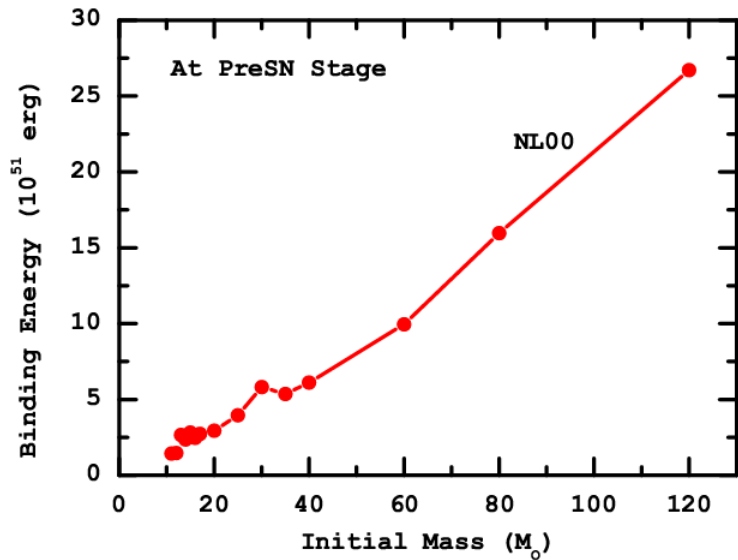
Fall back

Mass cut

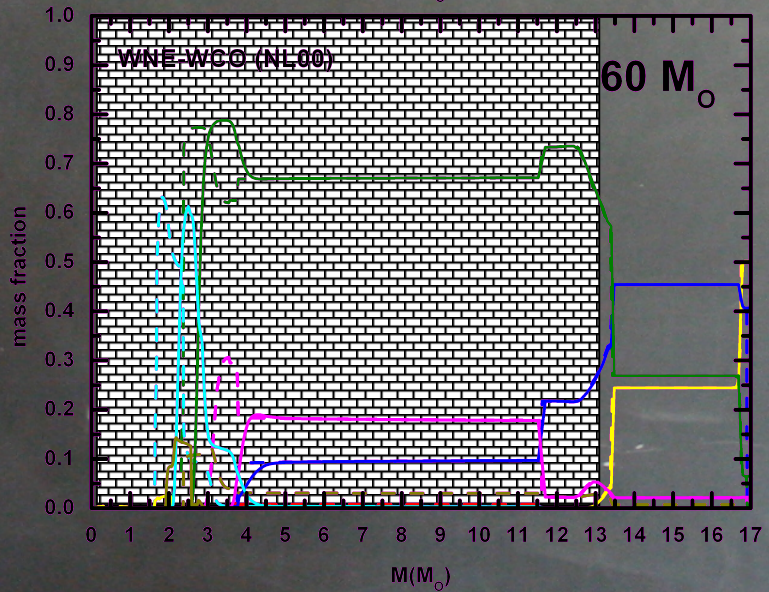
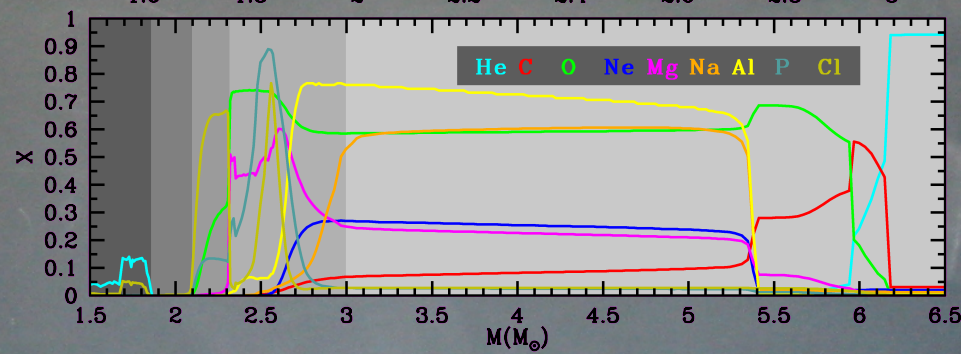
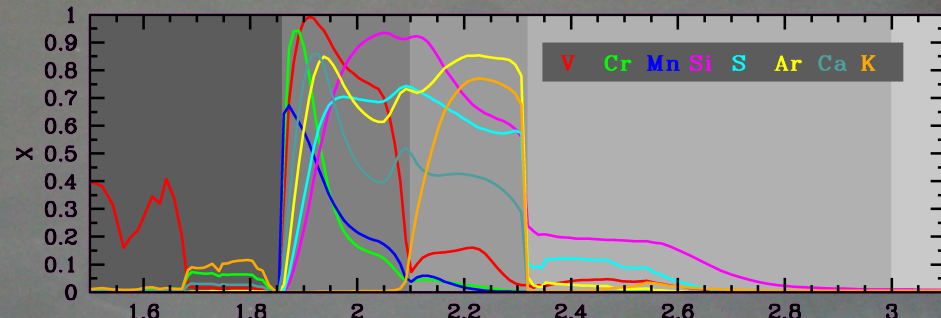
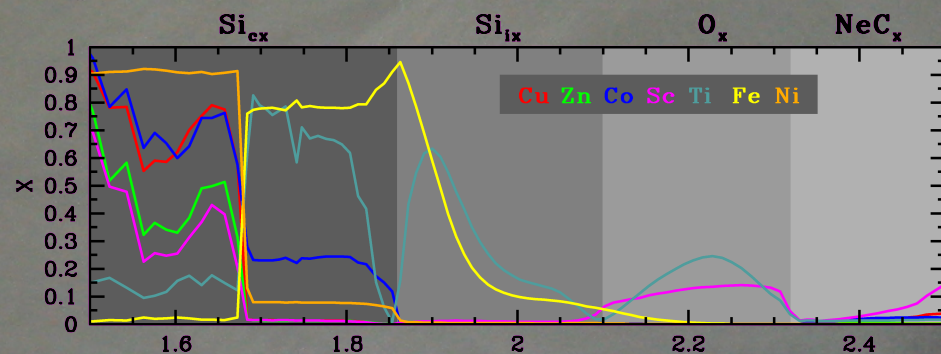
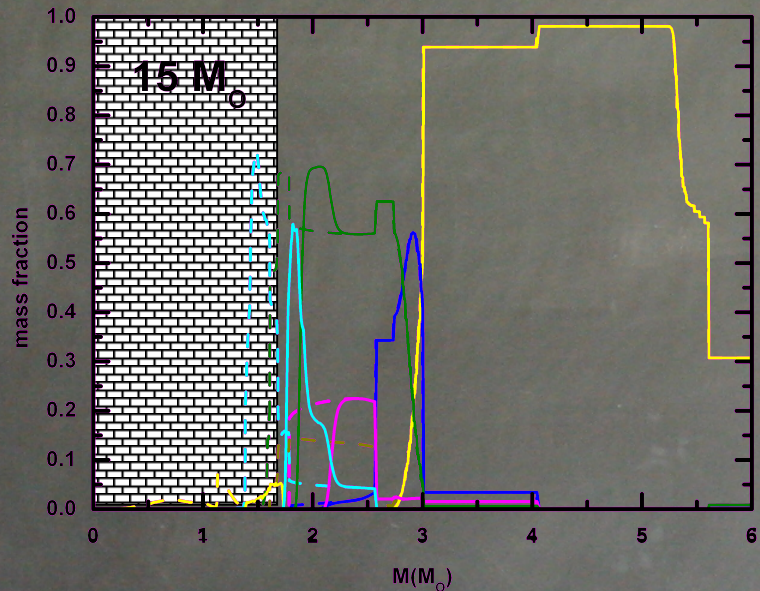
velocity

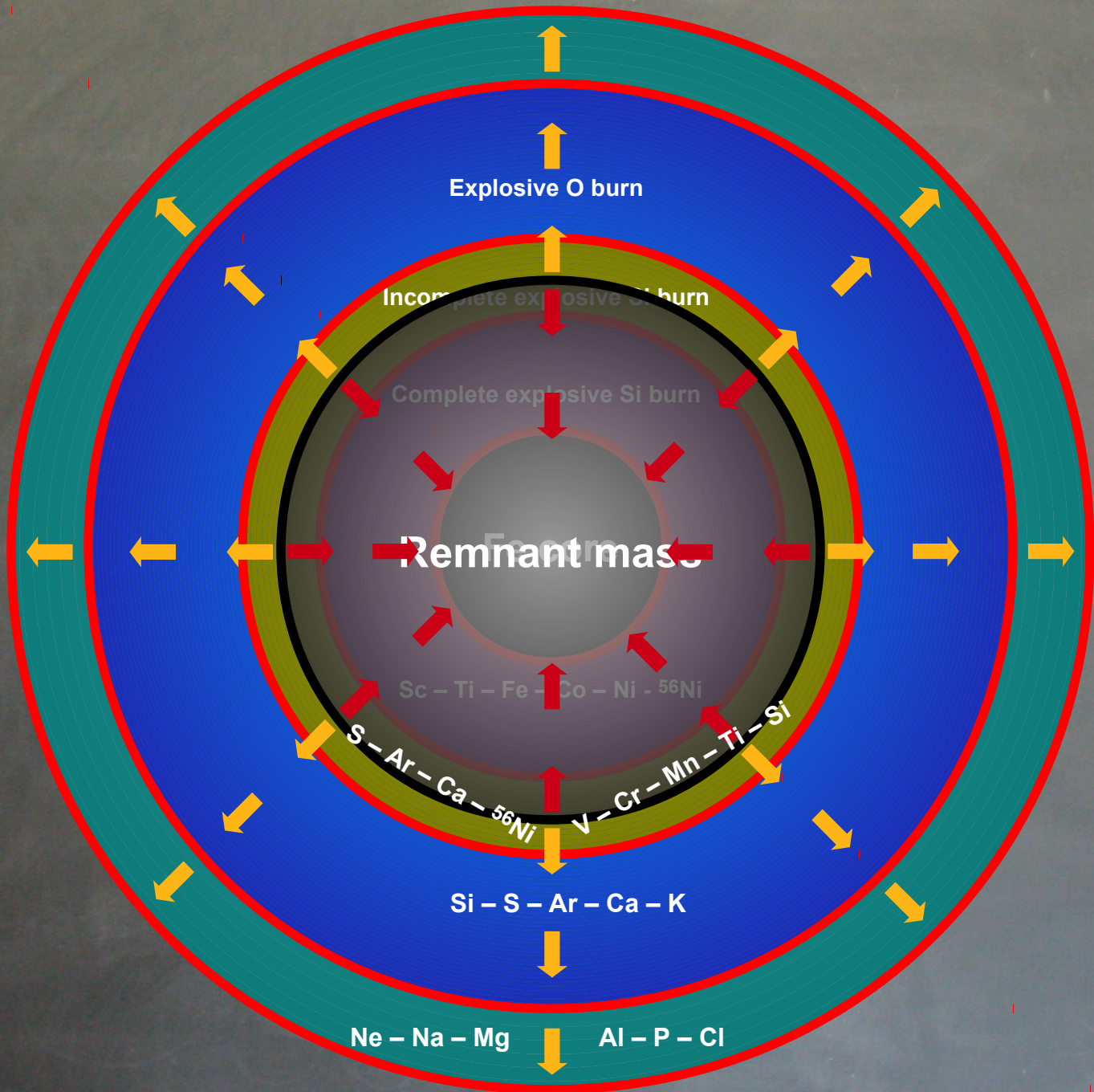
time

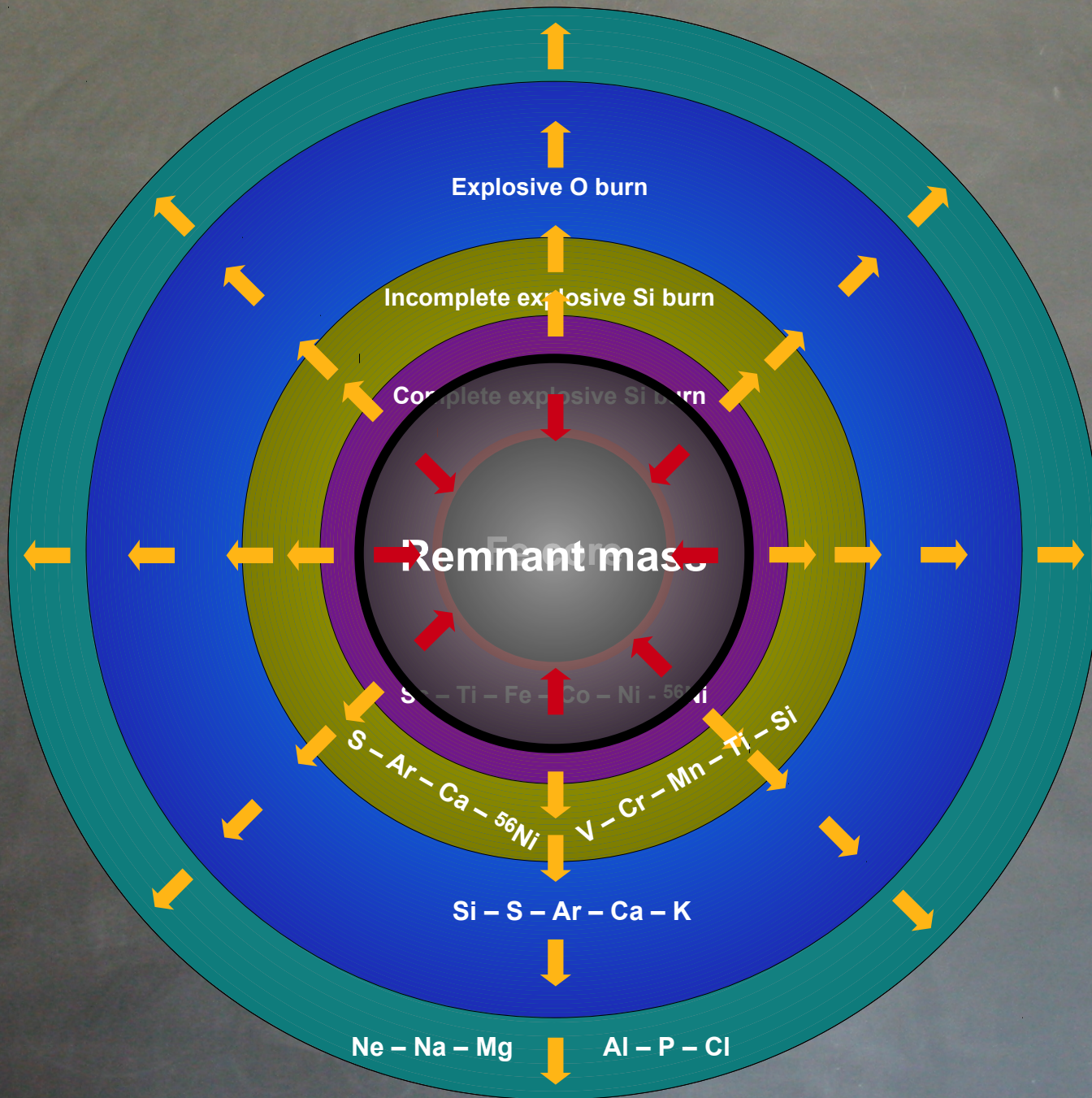
FALL BACK AND FINAL REMNANT



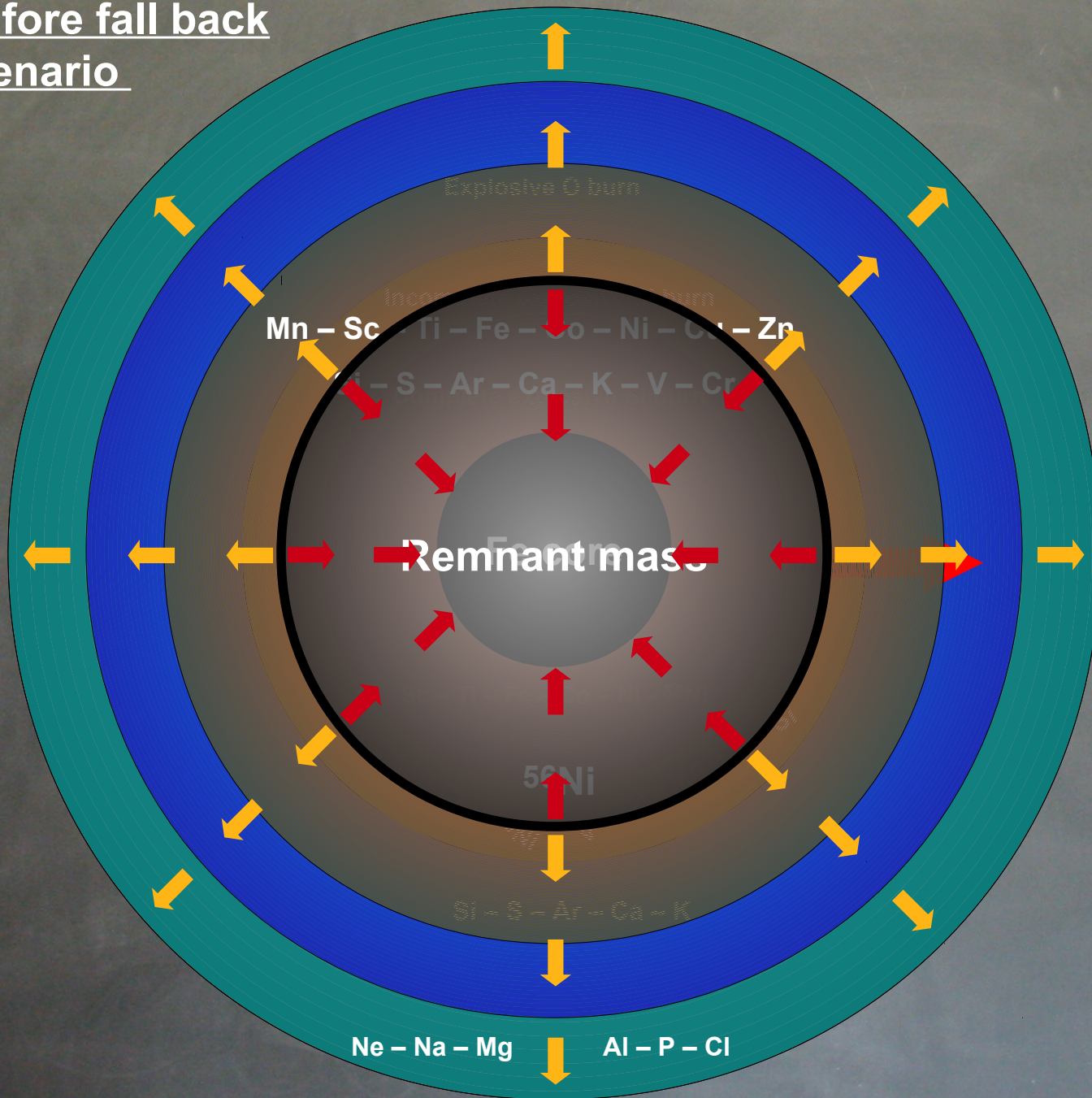
Final kinetic energy = 1 foe (10^{51} erg)



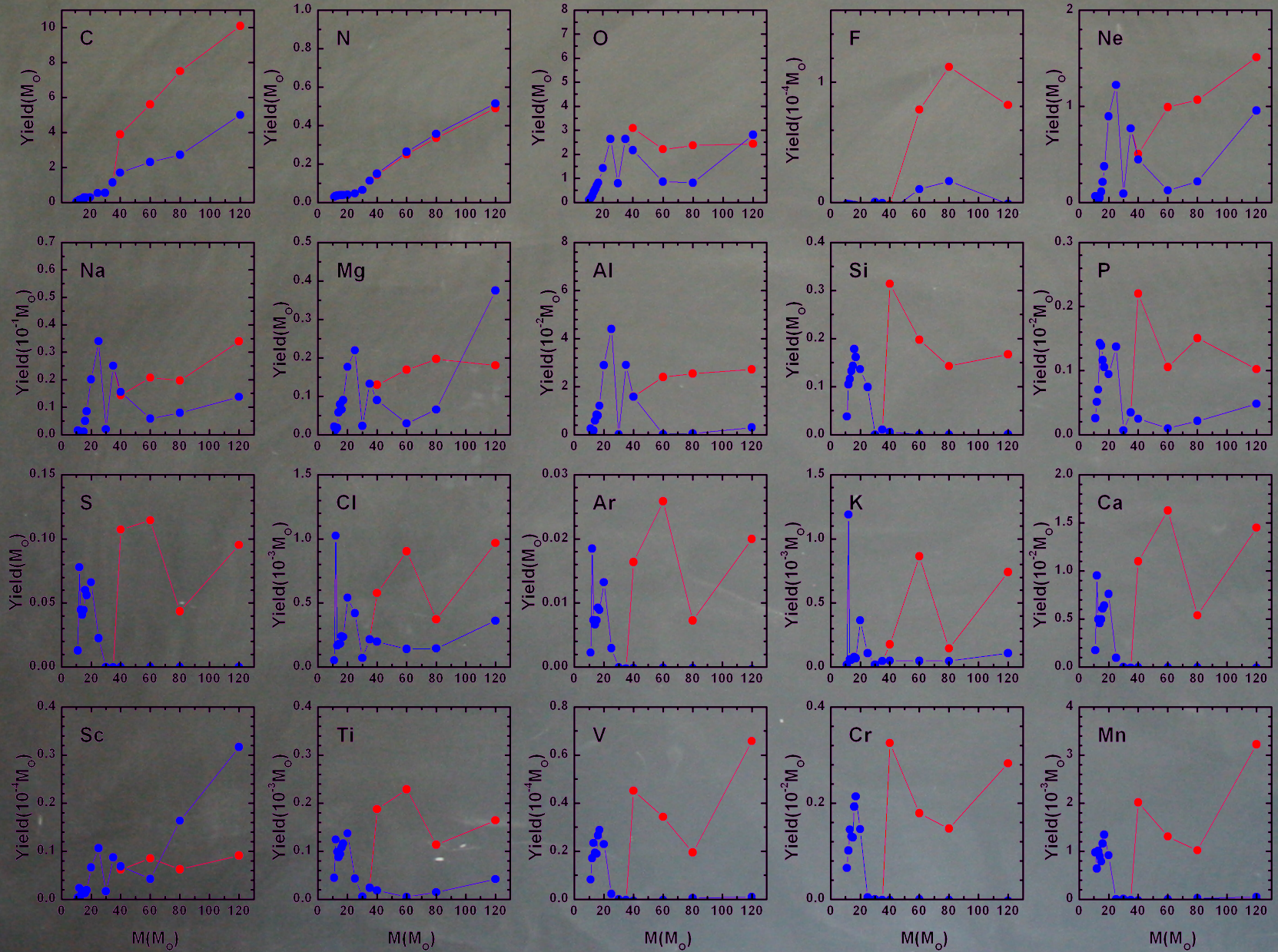




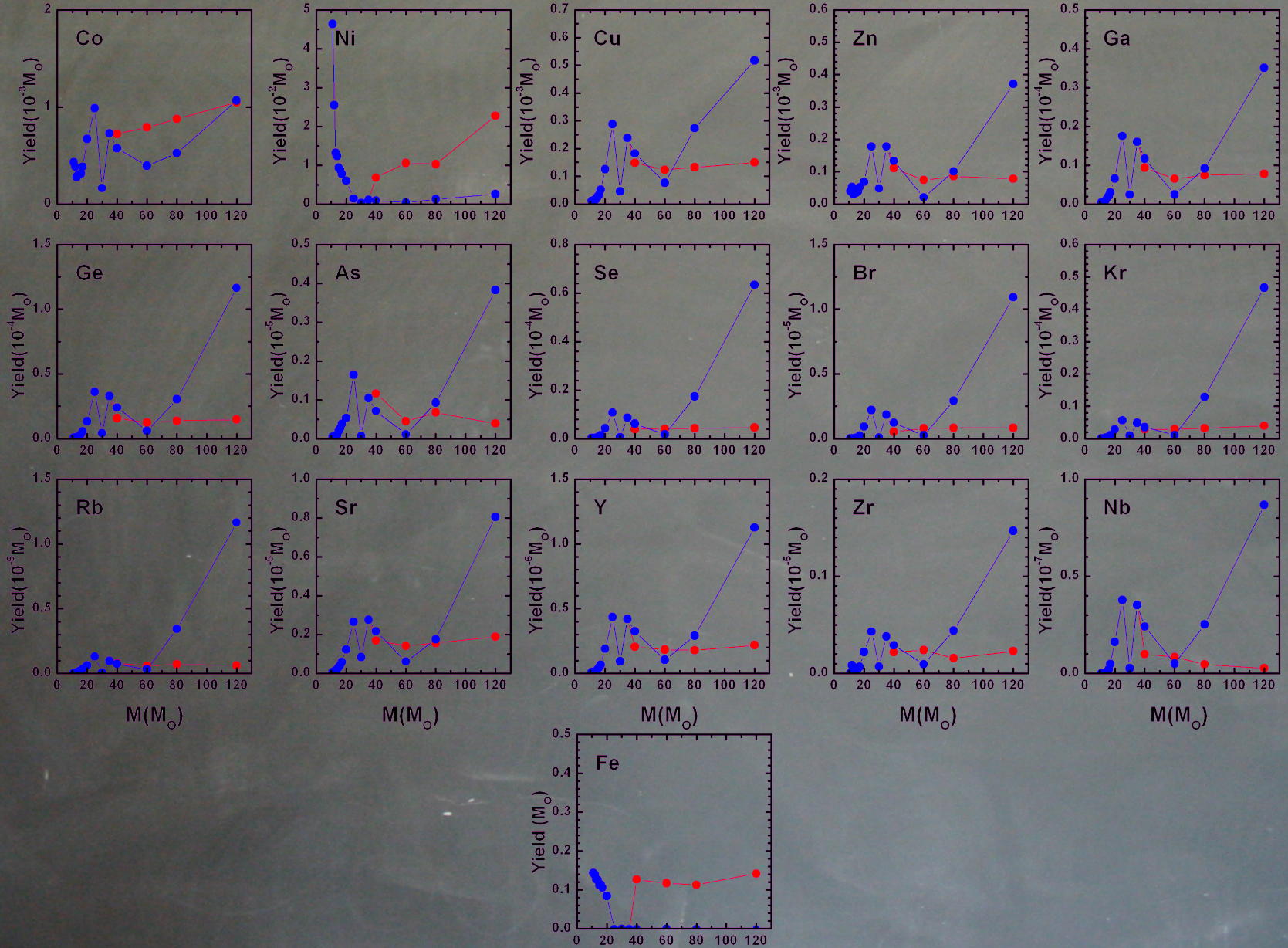
Mixing before fall back
scenario



Mass Loss in the WNE / WCO phases: **Langer89** - **Nugis & Lamers 00**



Mass Loss in the WNE / WCO phases: **Langer89** - **Nugis & Lamers 00**



Yields produced by a generation of massive stars


$$\frac{dN}{dm} = km^{-(1+x)}$$

Salpeter initial mass function

X=1.35 classical

X=1.7-1.8 Kroupa (for massive stars)

$M_1 < \text{massive stars} < M_2$

$$Y_i = k \int_{m_1}^{m_2} Y_i(m) \frac{dN}{dm} dm$$


total yield

Normalization:

$$N_{m_1}^{m_2} = k \int_{m_1}^{m_2} \frac{dN}{dm} dm = k \frac{(m_2^{-x} - m_1^{-x})}{-x}$$

$$N_{m_1}^{m_2} = 1 \Rightarrow k = \frac{-x}{(m_2^{-x} - m_1^{-x})}$$

$$M_{m_1}^{m_2} = k \int_{m_1}^{m_2} m \frac{dN}{dm} dm = k \frac{(m_2^{-x+1} - m_1^{-x+1})}{-x+1}$$

$$M_{m_1}^{m_2} = 1 \Rightarrow k = \frac{-x+1}{(m_2^{-x+1} - m_1^{-x+1})}$$

Production factor  $PF = \frac{Yield}{X_{initial} M_{ejected}}$

PF > 1 (produced)

PF < 1 (destroyed)

PF = 1 (untouched)

A flat PF factor implies that the initial relative scaling among the various nuclei is preserved

This means that an initial scaled solar distribution is preserved if the PF is flat

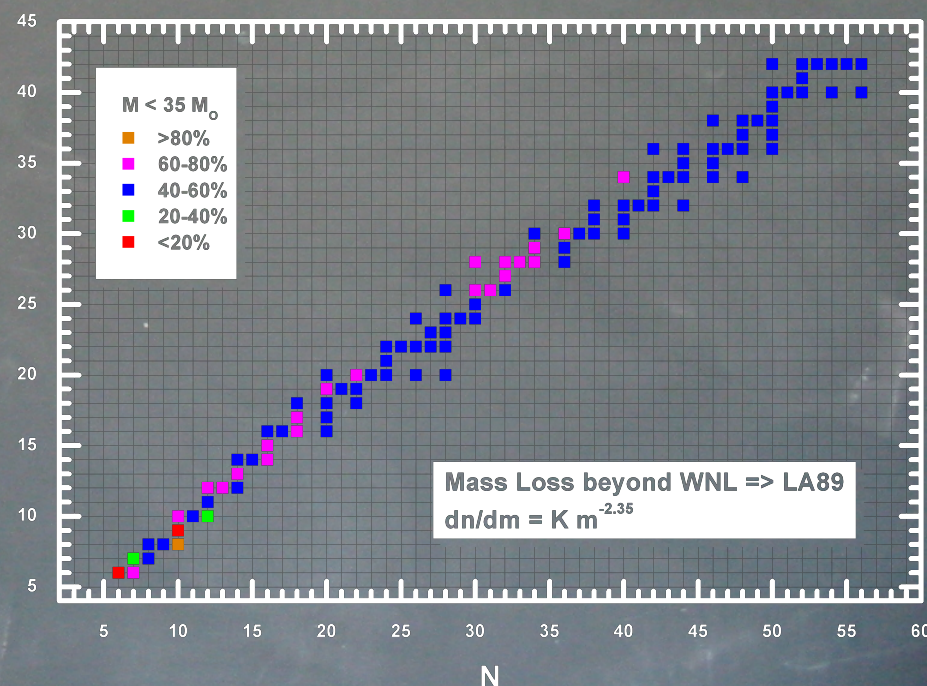
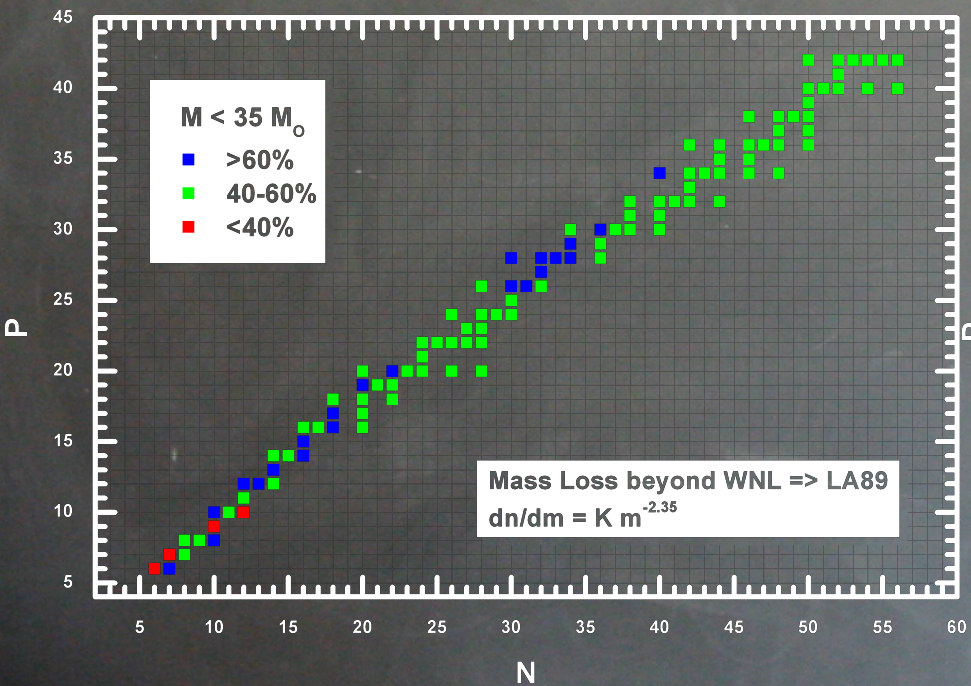
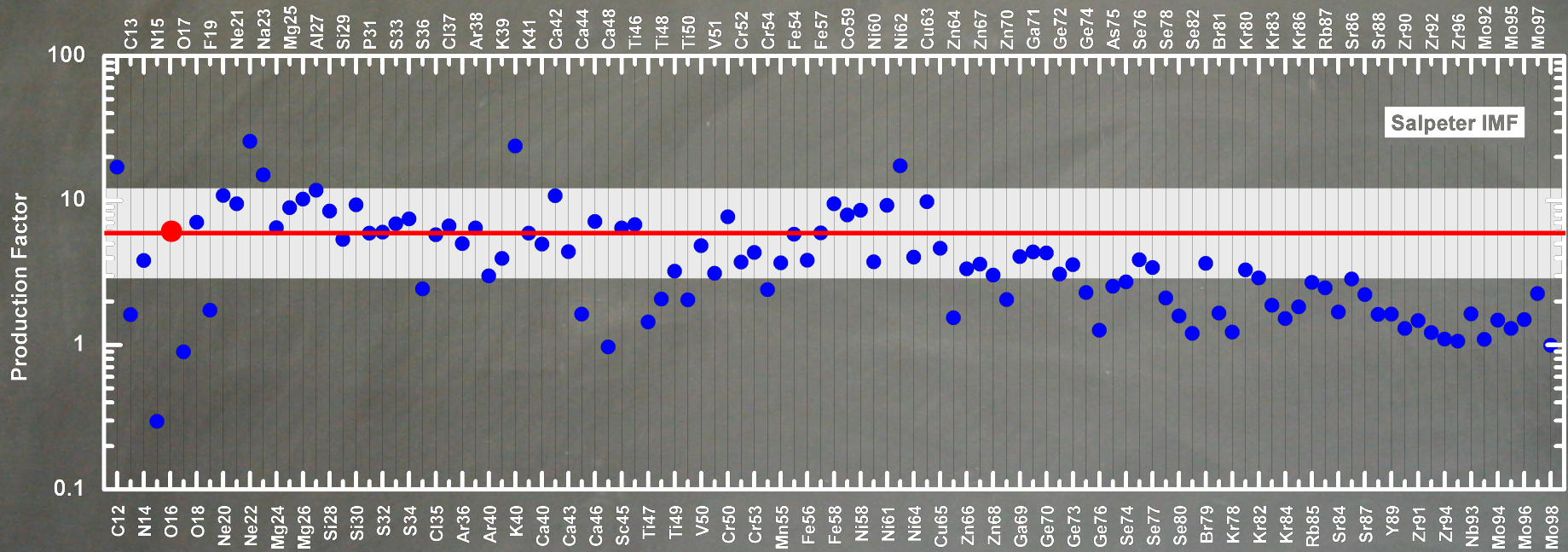
Since the solar chemical composition mainly reflects the ejecta of star having “quasi” solar c.c.,
a “roughly” flat PF should be typical of a generation of stars having a solar metallicity

A natural, robust “leader” nucleus is ^{16}O because it is certainly produced only by massive stars
and it is also the most abundant nucleus in nature (beyond H and ^4He)

If a nucleus has a PF at same level of that of the O, this means that it comes from massive stars only

If a nucleus has a PF larger than that of the O, this could be a problem since it would imply that it
is overproduced (note however that “secondary” nuclei must be slightly overproduced)

If a nucleus has a PF lower than that of the O, in principle this would simply mean that massive stars
are not the main producers of that nucleus



P

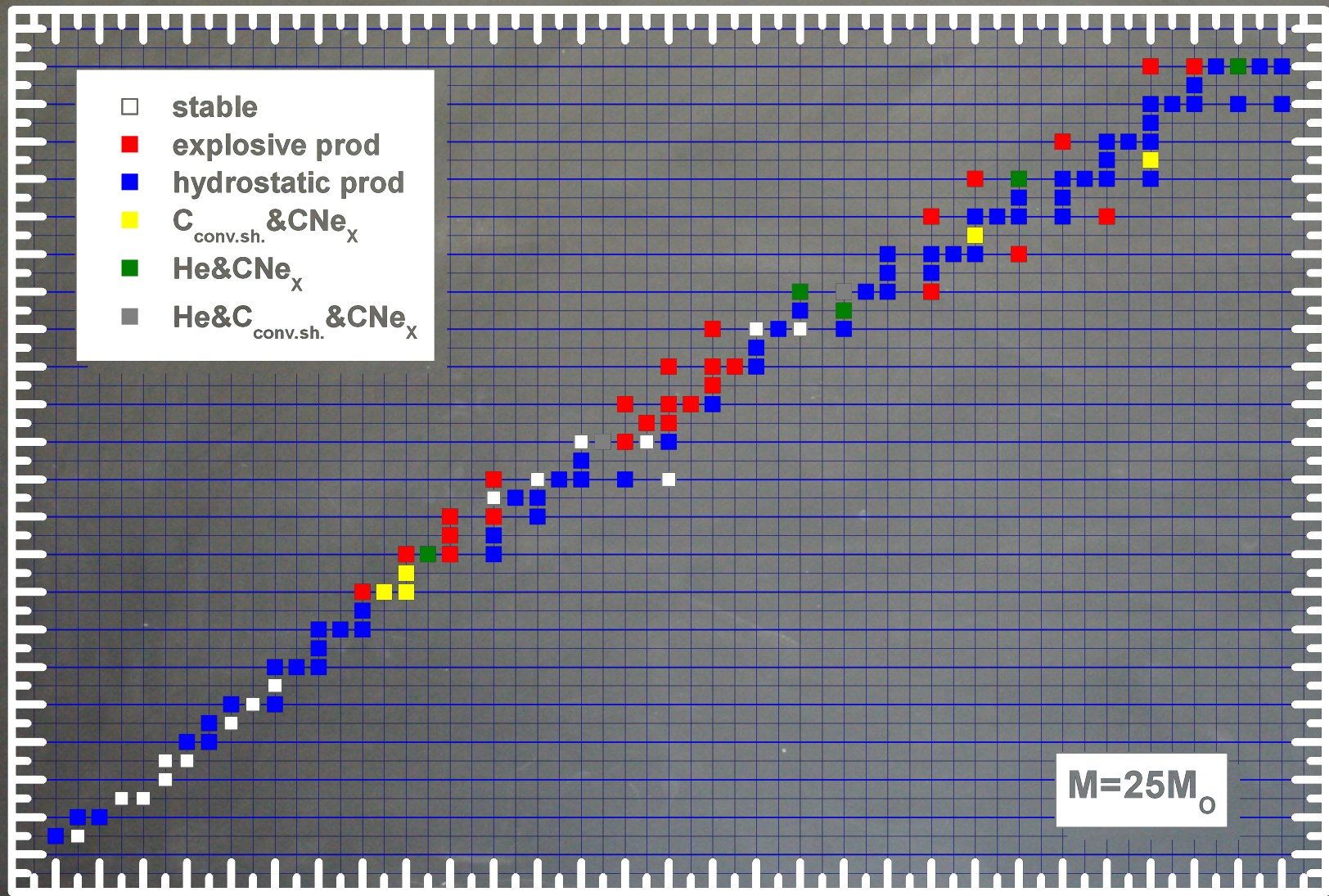
Mo
Zr
Sr
Kr
Se
Ge
Zn
Ni
Fe
Cr
Ti
Ca
Ar
S
Si
Mg
Ne
O
C

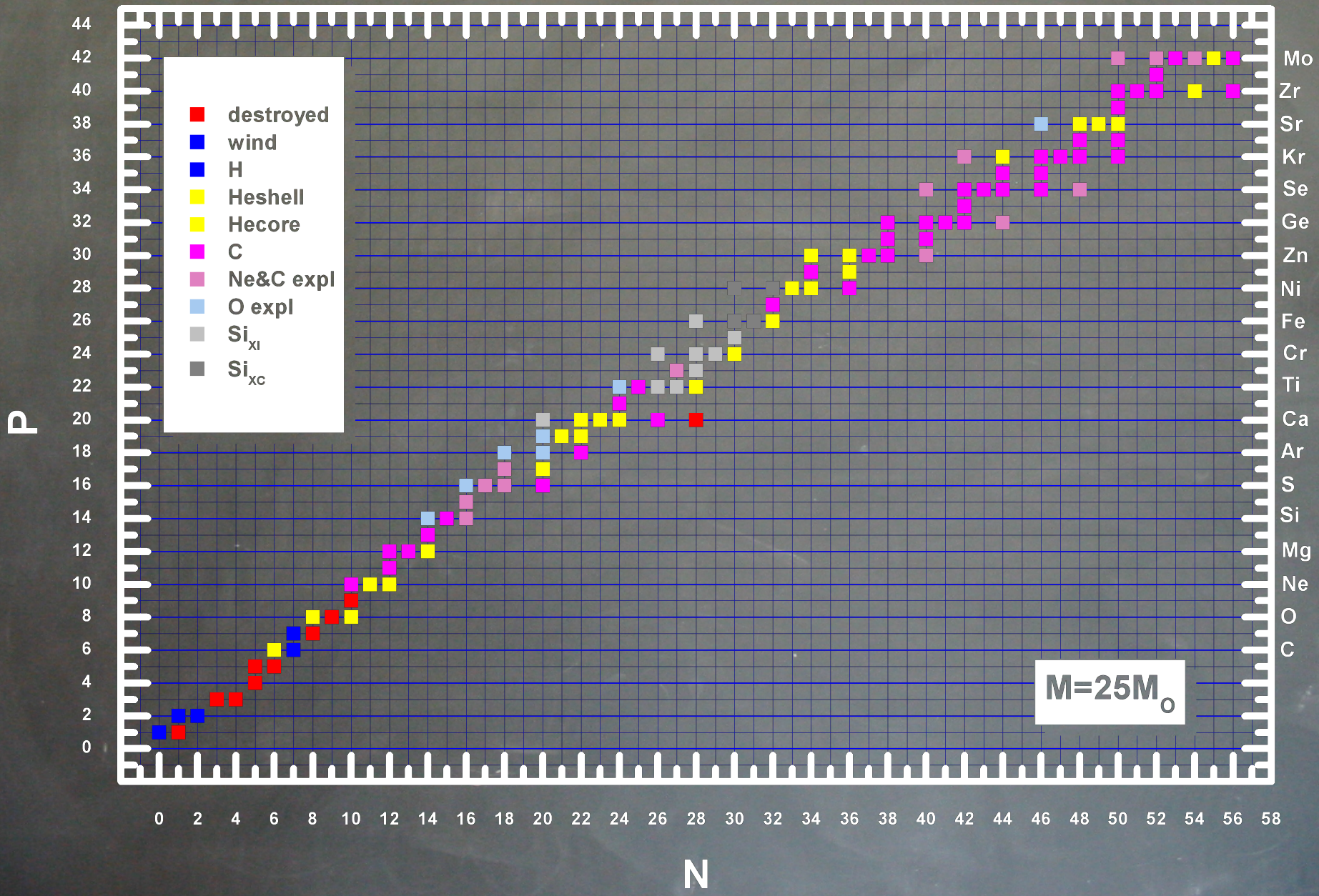
- stable
- explosive prod
- hydrostatic prod
- $C_{\text{conv.sh.}} \& CNe_x$
- He & CNe_x
- He & $C_{\text{conv.sh.}} \& CNe_x$

$M=25M_{\odot}$

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58

N





$He(^4 He) - H$

$C(^{12} C) - He$

$N(^{14} N) - H$

$O(^{16} O) - He$

$F(^{19} F)$ Destroyed by H

$Ne(^{20} Ne) - C$

$Na(^{23} Na) - C$

$Mg(^{24} Mg) - C$

$Al(^{27} Al) - C$

$Si(^{28} Si) - O_X, Si_{Xi}$

$P(^{31} P) - C_X, Ne_X$

$S(^{32} S) - O_X, Si_{Xi}$

$Cl(^{35} Cl) - C_X, Ne_X$

$Ar(^{36} Ar) - O_X, Si_{Xi}$

$K(^{39} K) - O_X$

$Ca(^{40} Ca) - O_X, Si_{Xi}$

$Sc(^{45} Sc) - C, Si_X$

$Ti(^{48} Ti) - Si_{Xi}$

$V(^{51} V) - Si_{Xi}$

$Cr(^{52} Cr) - Si_{Xi}$

$Mn(^{55} Mn) - Si_{Xi}$

$Fe(^{56} Fe) - Si_{Xi}, Si_X$

$Co(^{59} Co) - C, Si_X$

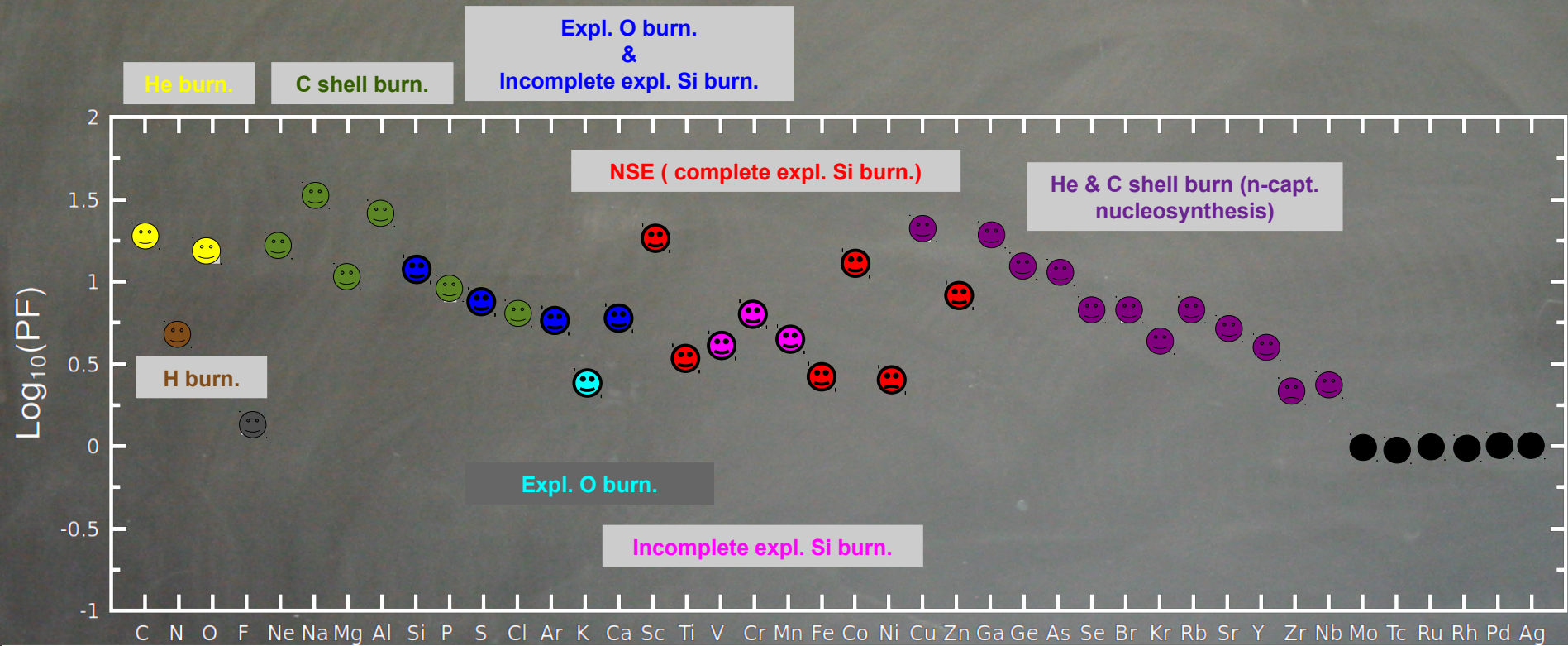
$Ni(^{58} Ni) - Si_X$

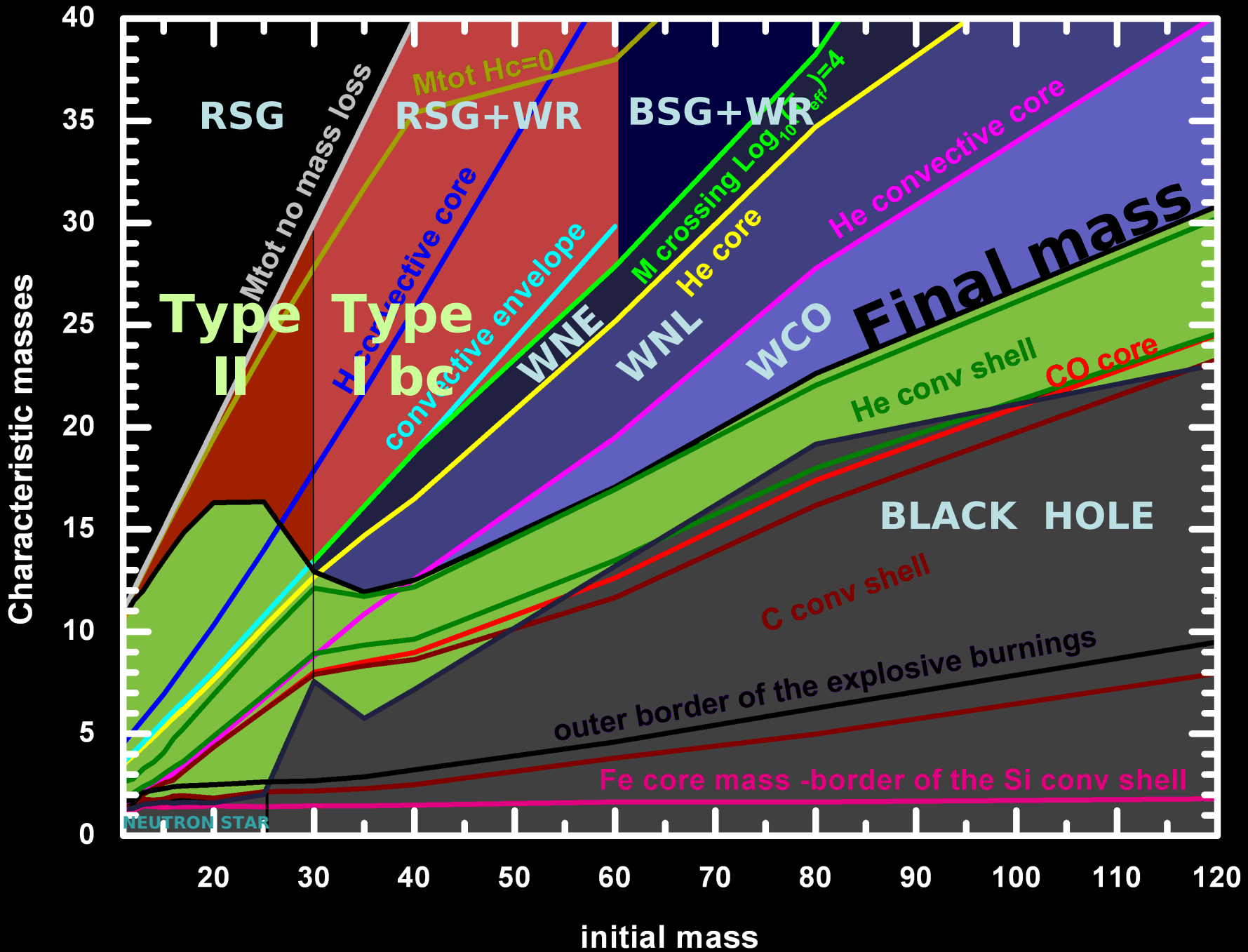
$Cu(^{63} Cu) - C, Si_X$

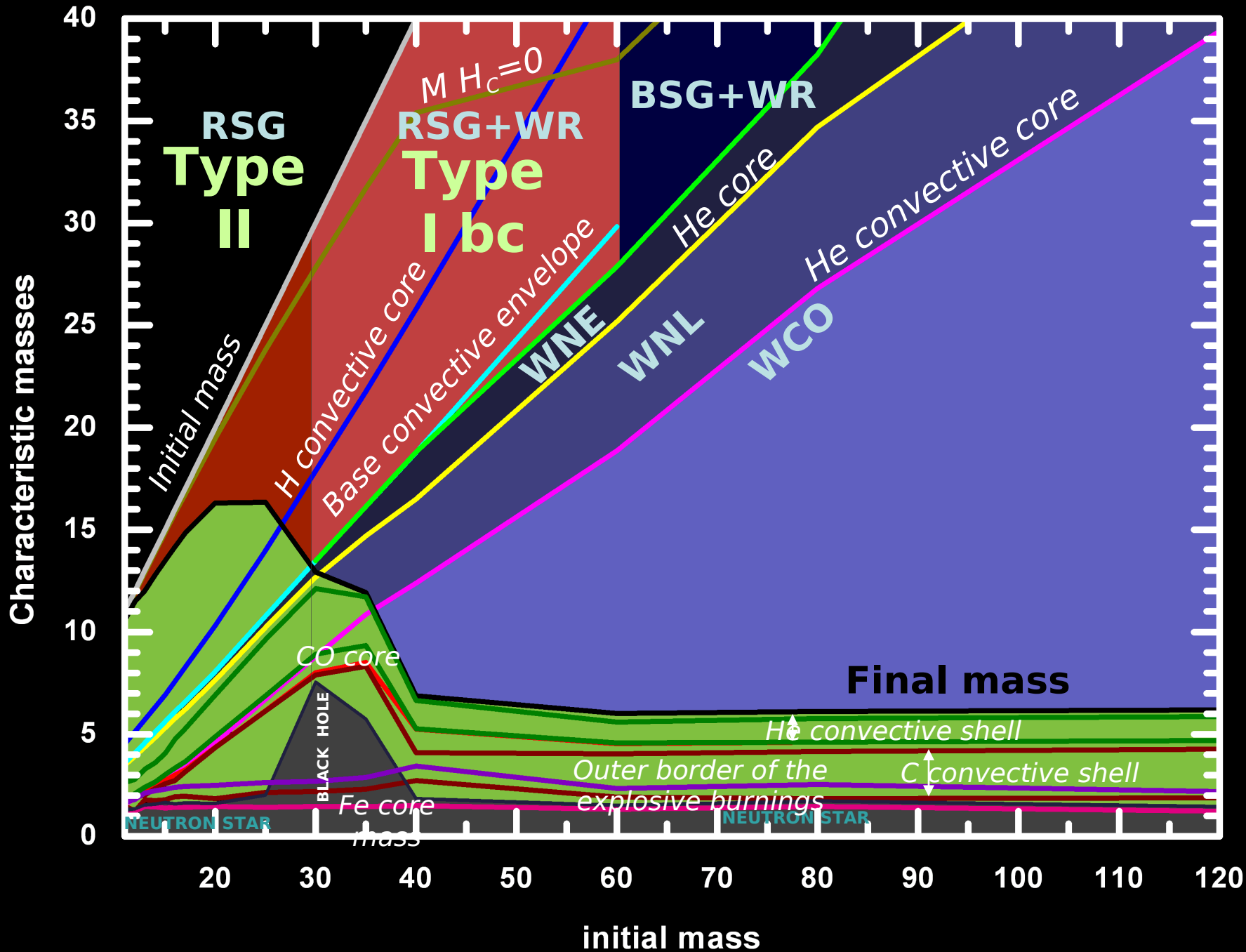
$Zn(^{64} Zn) - He, Si_X$

WARNING

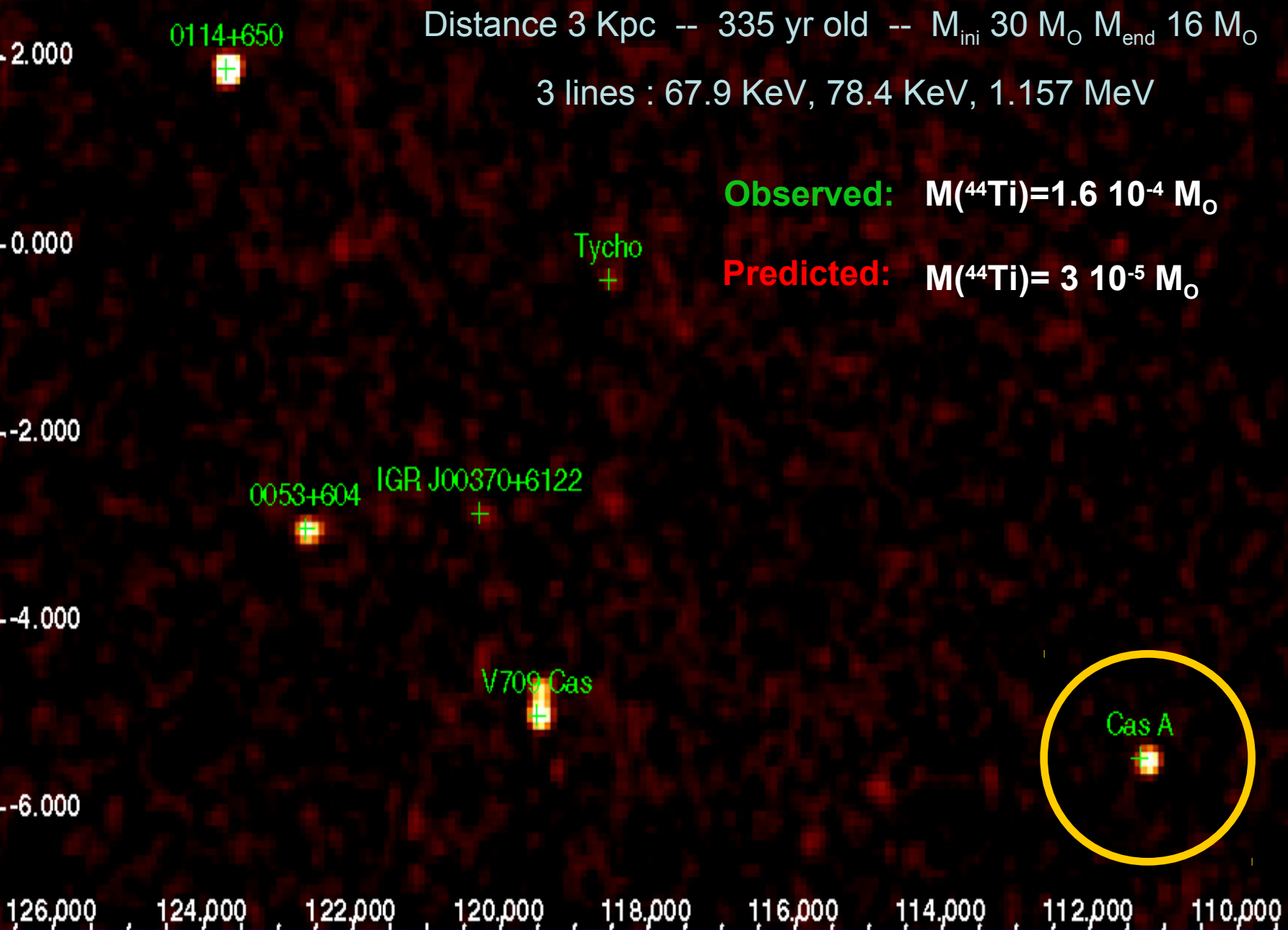
The production site of many elements depends on the mass of the star and the initial chemical composition.







Cas A as seen by IBIS – ISGRI aboard INTEGRAL at 25 - 40 KeV

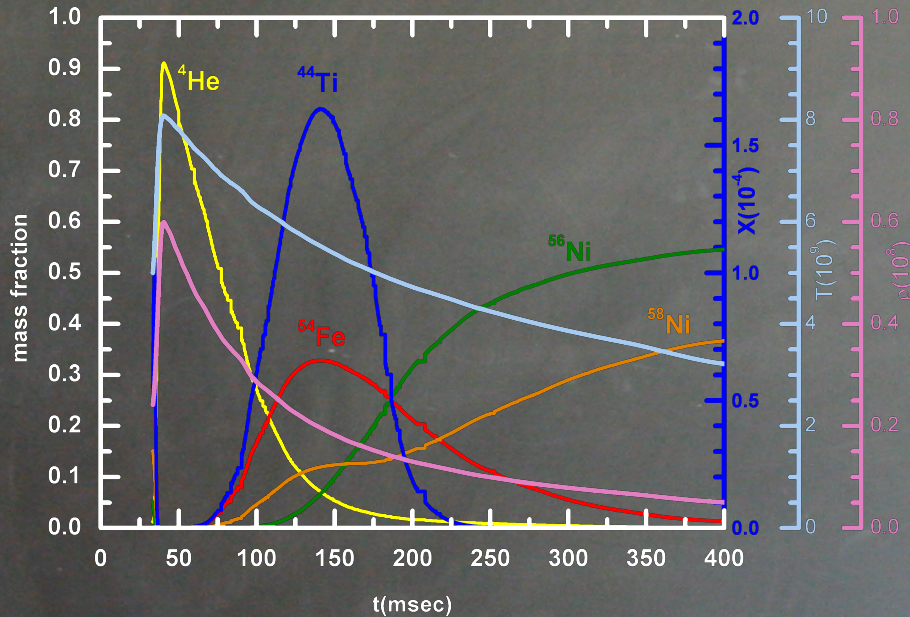


^{44}Ti

Not produced in a normal freeze out

$$\tau_{\text{cooling}} \gg \tau_{\text{build up}}$$

($3\alpha \text{ }^{12}\text{C}(\alpha,\gamma)^{16}\text{O}(\alpha,\gamma) \dots \text{NSE}$)



Produced in the α -rich freeze-out of zones exposed to the complete explosive Si burning

$$\tau_{\text{cooling}} \ll \tau_{\text{build up}}$$

