Matter Under Extreme Conditions: The Nuclear Equation of State and Neutron Stars

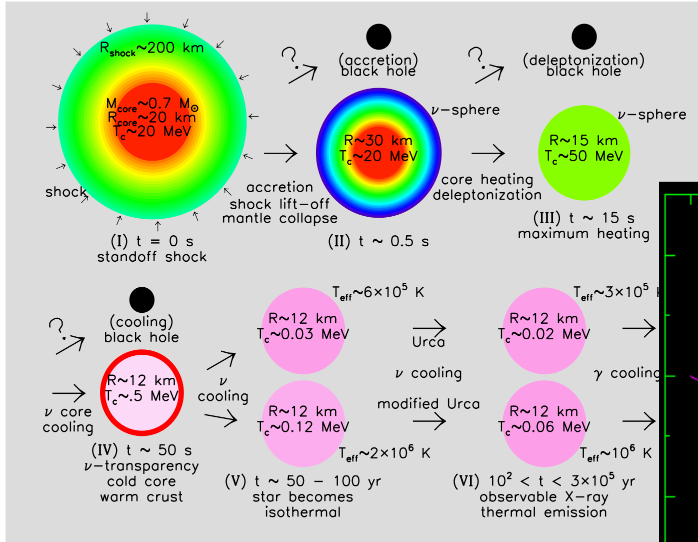
James M. Lattimer

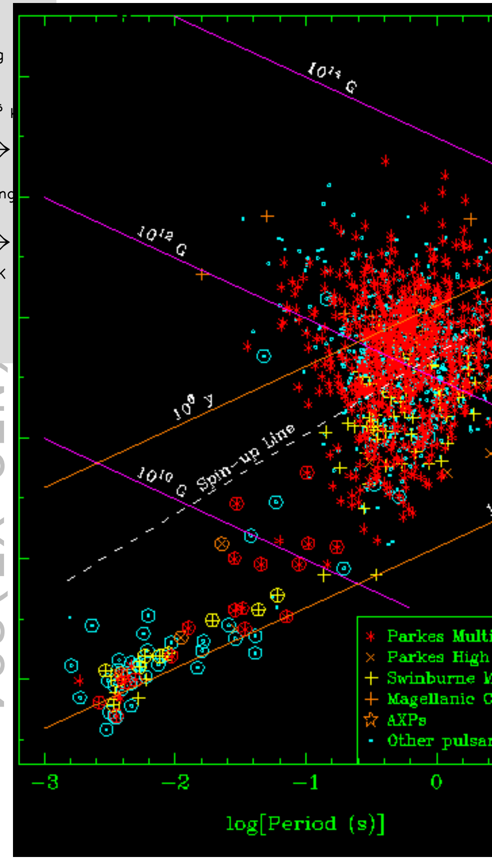
lattimer@astro.sunysb.edu

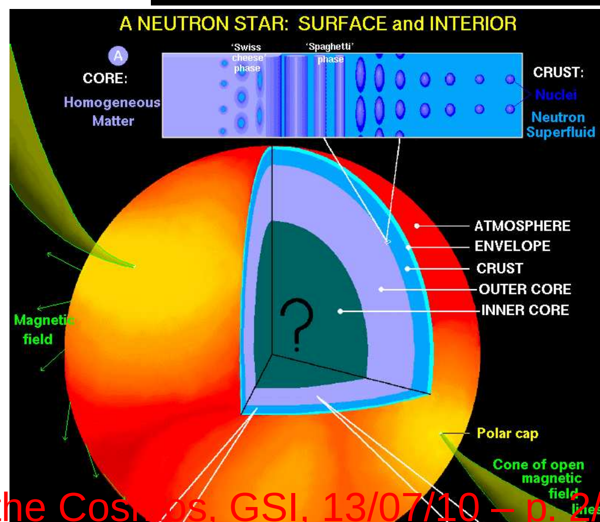
Department of Physics & Astronomy
Stony Brook University

Outline

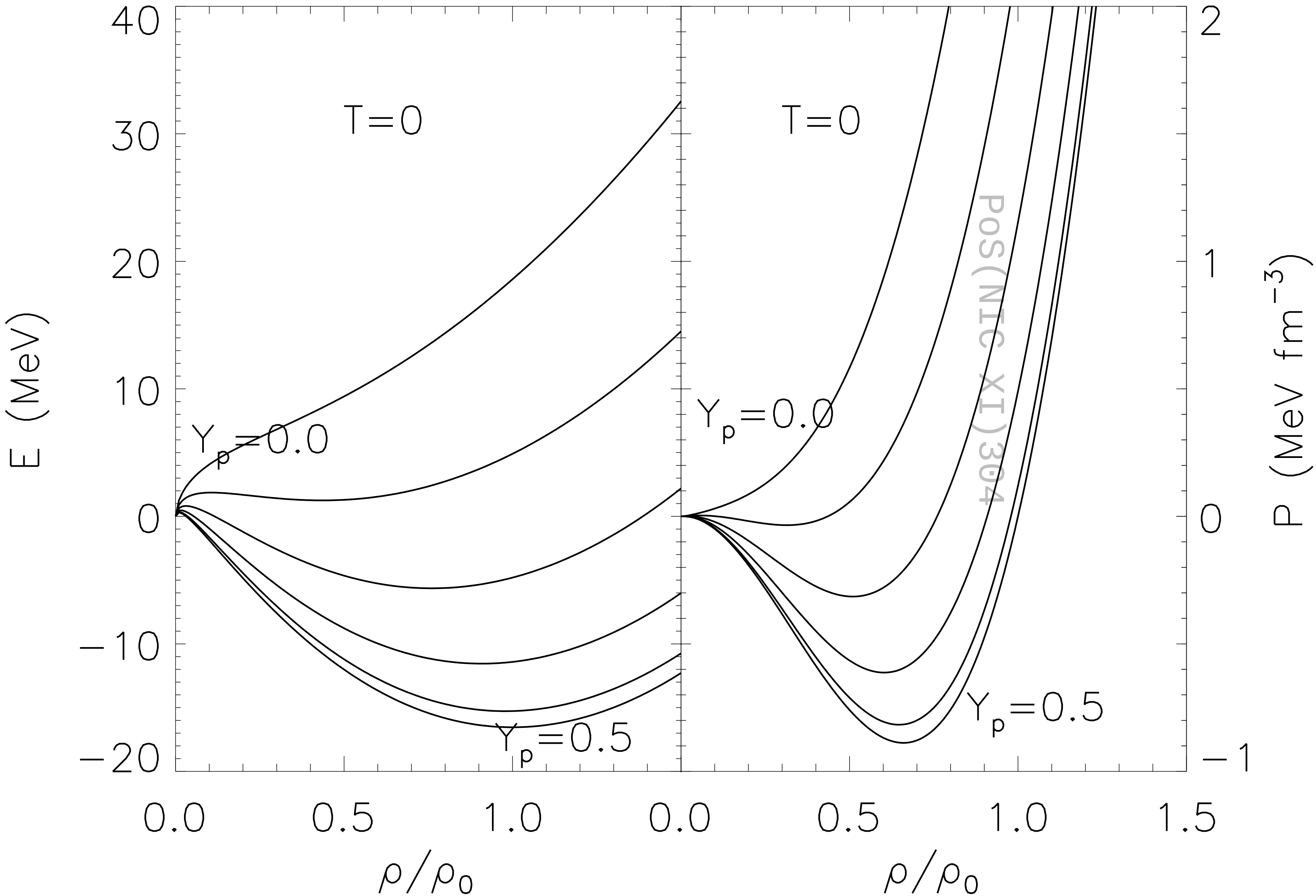
- Dense Matter EOS
- Lab Constraints
- Neutron Star Structure
- Mass, Radius Observations
- EOS Constraints







Bulk Matter Energy and Pressure



Schematic Energy Density

n: number density; x: proton fraction; T: temperature

$n_s \simeq 0.16 \pm 0.01$ fm^{-3}: nuclear saturation density

$B \simeq -16 \pm 1$ MeV: saturation binding energy

$K \simeq 220 \pm 15$ MeV: incompressibility parameter

$S_v \simeq 30 \pm 6$ MeV: bulk symmetry parameter

$a \simeq 0.065 \pm 0.010$ MeV^{-1}: bulk level density parameter

$$\begin{aligned}
\epsilon(n,x,T) &= n\left[B + \frac{K}{18}\left(1-\frac{n}{n_s}\right)^2 + S_v\frac{n}{n_s}(1-2x)^2 + a\left(\frac{n_s}{n}\right)^{2/3}T^2\right] \\
P &= n^2\frac{\partial(\epsilon/n)}{\partial n} = \frac{n^2}{n_s}\left[\frac{K}{9}\left(\frac{n}{n_s}-1\right) + S_v(1-2x)^2\right] + \frac{2an}{3}\left(\frac{n_s}{n}\right)^{2/3}T^2 \\
\mu_n &= \frac{\partial\epsilon}{\partial n} - \frac{x}{n}\frac{\partial\epsilon}{\partial x} \\
&= B + \frac{K}{18}\left(1-\frac{n}{n_s}\right)\left(1-3\frac{n}{n_s}\right) + 2S_v\frac{n}{n_s}(1-2x) - \frac{a}{3}\left(\frac{n_s}{n}\right)^{2/3}T^2 \\
\hat{\mu} &= -\frac{1}{n}\frac{\partial\epsilon}{\partial x} = \mu_n - \mu_p = 4S_v\frac{n}{n_s}(1-2x) \\
s &= \frac{1}{n}\frac{\partial\epsilon}{\partial T} = 2a\left(\frac{n_s}{n}\right)^{2/3}T
\end{aligned}$$

Phase Coexistence

Schematic energy density

$$
\begin{aligned}
\epsilon &= n\left[B + \frac{K}{18}\left(1 - \frac{n}{n_s}\right)^2 + S_v \frac{n}{n_s}(1-2x)^2 + a\left(\frac{n_s}{n}\right)^{2/3} T^2\right] \\
P &= \frac{n^2}{n_s}\left[\frac{K}{9}\left(\frac{n}{n_s} - 1\right) + S_v(1-2x)^2\right] + \frac{2an}{3}\left(\frac{n_s}{n}\right)^{2/3} T^2 \\
\mu_n &= B + \frac{K}{18}\left(1 - \frac{n}{n_s}\right)\left(1 - 3\frac{n}{n_s}\right) + 2S_v \frac{n}{n_s}(1-2x) - \frac{a}{3}\left(\frac{n_s}{n}\right)^{2/3} T^2 \\
\hat{\mu} &= \mu_n - \mu_p = 4S_v \frac{n}{n_s}(1-2x)\,, \qquad s = 2a\left(\frac{n_s}{n}\right)^{2/3} T
\end{aligned}
$$

Free Energy Minimization With Two Phases

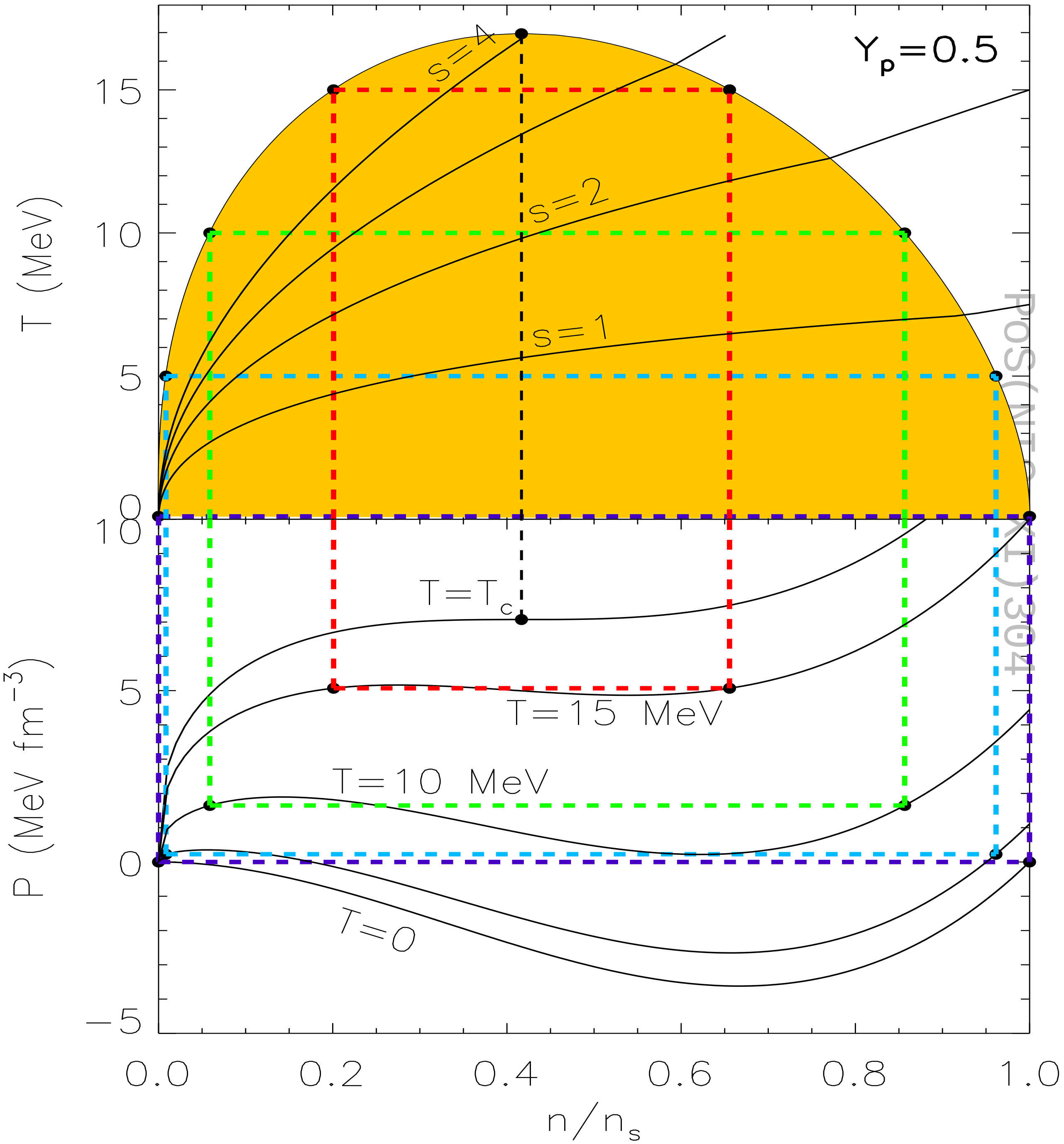
$$F = \epsilon - nTs = uF_I + (1-u)F_{II}\,, \qquad n = un_I + (1-u)n_{II}\,, \qquad nY_e = ux_I n_I + (1-u)x_{II}n_{II}$$

$$\frac{\partial F}{\partial n_I} = 0\,, \quad \frac{\partial F}{\partial x_I} = 0\,, \quad \frac{\partial F}{\partial u} = 0 \quad \Longrightarrow \mu_{nI} = \mu_{nII}\,, \qquad \mu_{pI} = \mu_{pII}\,, \qquad P_I = P_{II}$$

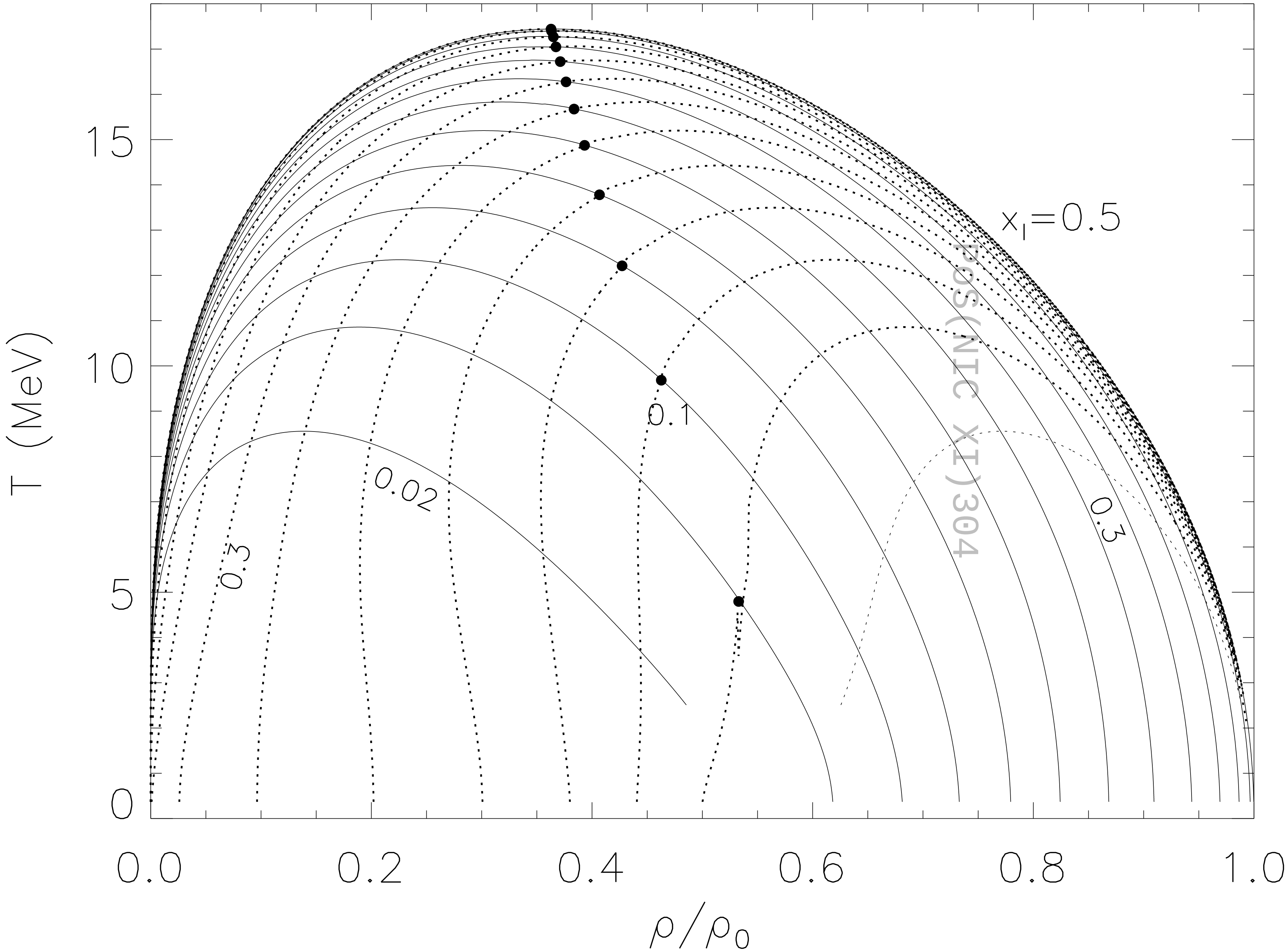
Critical Point ($Y_e = 0.5$)

$$\left(\frac{\partial P}{\partial n}\right)_T = \left(\frac{\partial^2 P}{\partial n^2}\right)_T = 0$$

$$n_c = \frac{5}{12} n_s\,, \qquad T_c = \left(\frac{5}{12}\right)^{1/3}\left(\frac{5K}{32a}\right)^{1/2}\,, \qquad s_c = \left(\frac{12}{5}\right)^{1/3}\left(\frac{5Ka}{8}\right)^{1/2}$$



$Y_p=0.5$
T (MeV)
s=4
s=2
s=1
P (MeV fm^{-3})
T=T$_c$
T=15 MeV
T=10 MeV
T=0
n/n$_s$
0.0
0.2
0.4
0.6
0.8
1.0
15
10
5
0
10
5
0
−5



15
10
5
0
T (MeV)
$x_l=0.5$
0.1
0.02
0.3
0.3
0.0
0.2
0.4
0.6
0.8
1.0
ρ/ρ_0

The Pressure of Neutron Star Matter

Expansion of cold nucleonic matter energy near n_s and isospin symmetry $x = 1/2$:

$$E(n,x) \simeq E(n,1/2) + E_{sym}(n)(1-2x)^2 + \frac{3\hbar c}{4} x(3\pi^2 nx)^{1/3}\,,$$

$$P(n,x) \simeq n^2\left[\frac{dE(n,1/2)}{dn} + \frac{dE_{sym}}{dn}(1-2x)^2\right] + \frac{\hbar c}{4} nx(3\pi^2 nx)^{1/3}\,,$$

$$\mu_e = \hbar c(3\pi^2 nx)^{1/3}\,, \qquad E(n,1/2) \simeq -B + \frac{K}{18}\left(1-\frac{n}{n_s}\right)^2.$$

Beta Equilibrium:

$$\left(\frac{\partial E}{\partial x}\right)_n = \mu_p - \mu_n + \mu_e = 0\,.$$

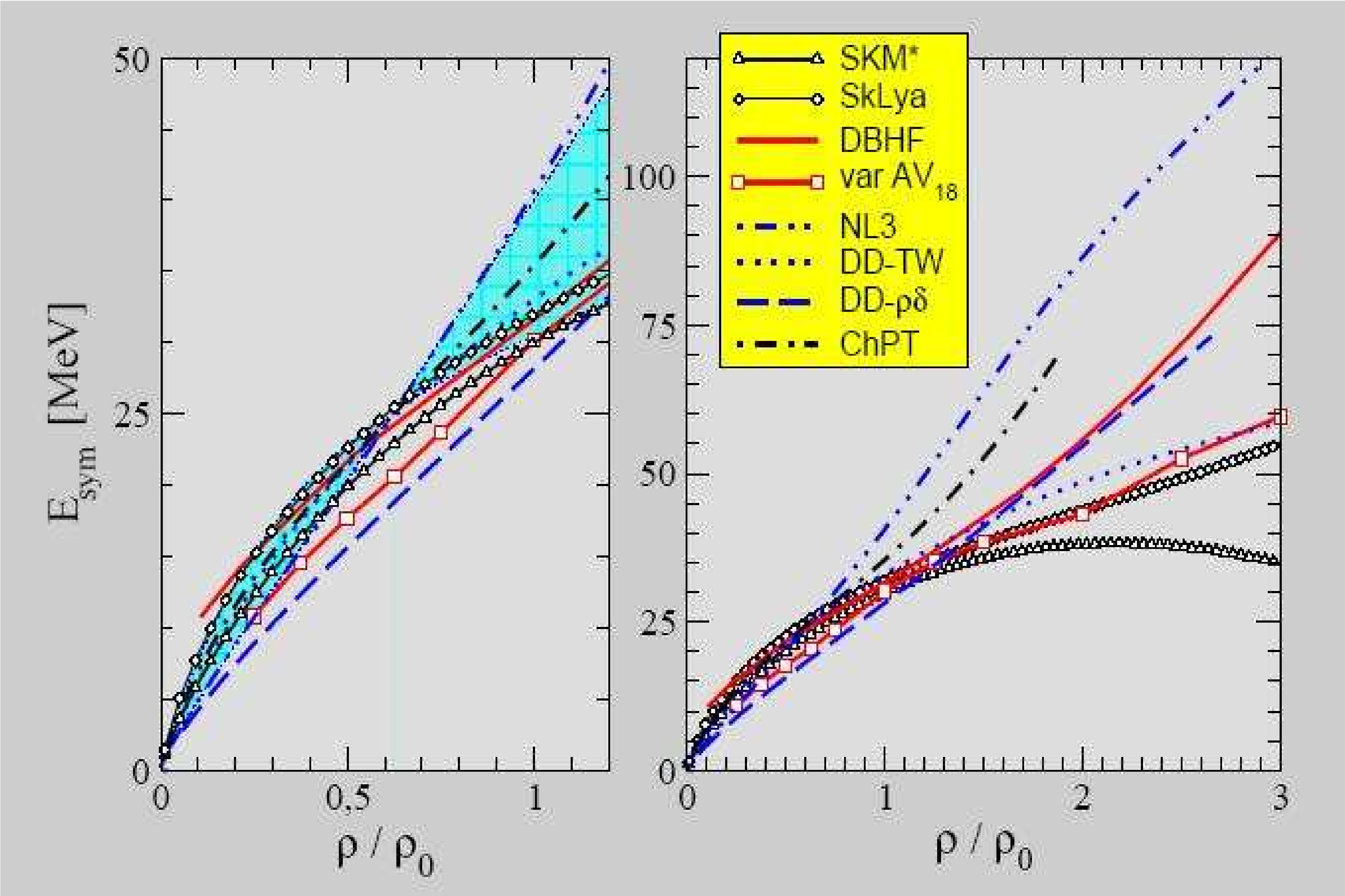
$$x_\beta \simeq (3\pi^2 n)^{-1}\left(\frac{4E_{sym}}{\hbar c}\right)^3\,,$$

$$P_\beta = \frac{Kn^2}{9n_0}\left(\frac{n}{n_s}-1\right) + n^2(1-2x_\beta)^2\frac{dE_{sym}}{dn} + E_{sym} n x_\beta (1-2x_\beta)$$

$E_{sym}(n_s) \equiv S_v \simeq$ 30 MeV, $\hbar c \simeq$ 200 MeV/fm, $\qquad n \to n_s \Longrightarrow$

$$x_\beta \to 0.04\,, \qquad P_\beta \to n_s^2 \left.\frac{dE_{sym}}{dn}\right|_{n_s}\,.$$

The Uncertain $E_{sym}(n)$



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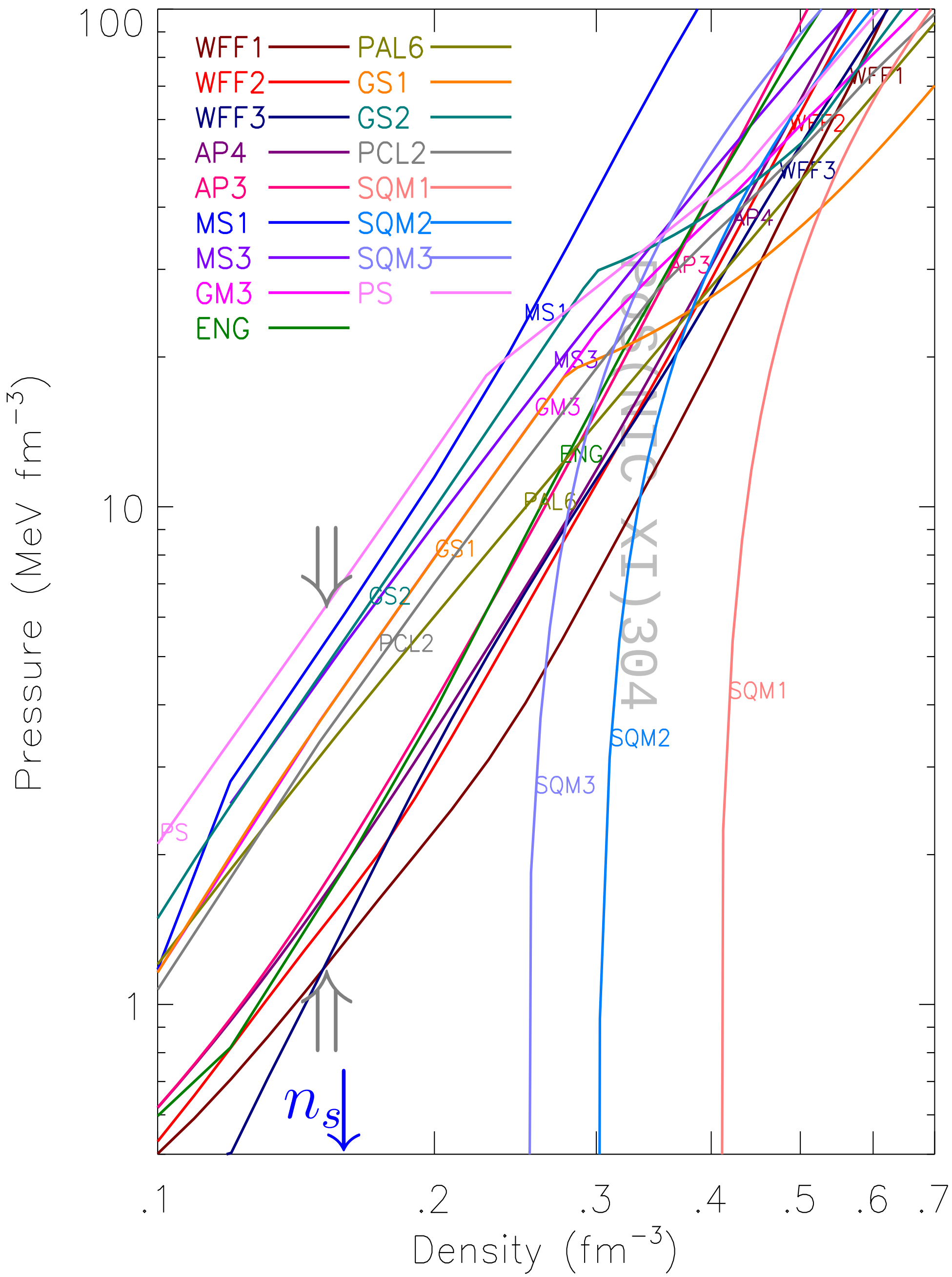
Neutron Star Matter Pressure

$$p \simeq K n^{\gamma}$$

$$\gamma = d\ln p / d\ln\ n \sim 2$$

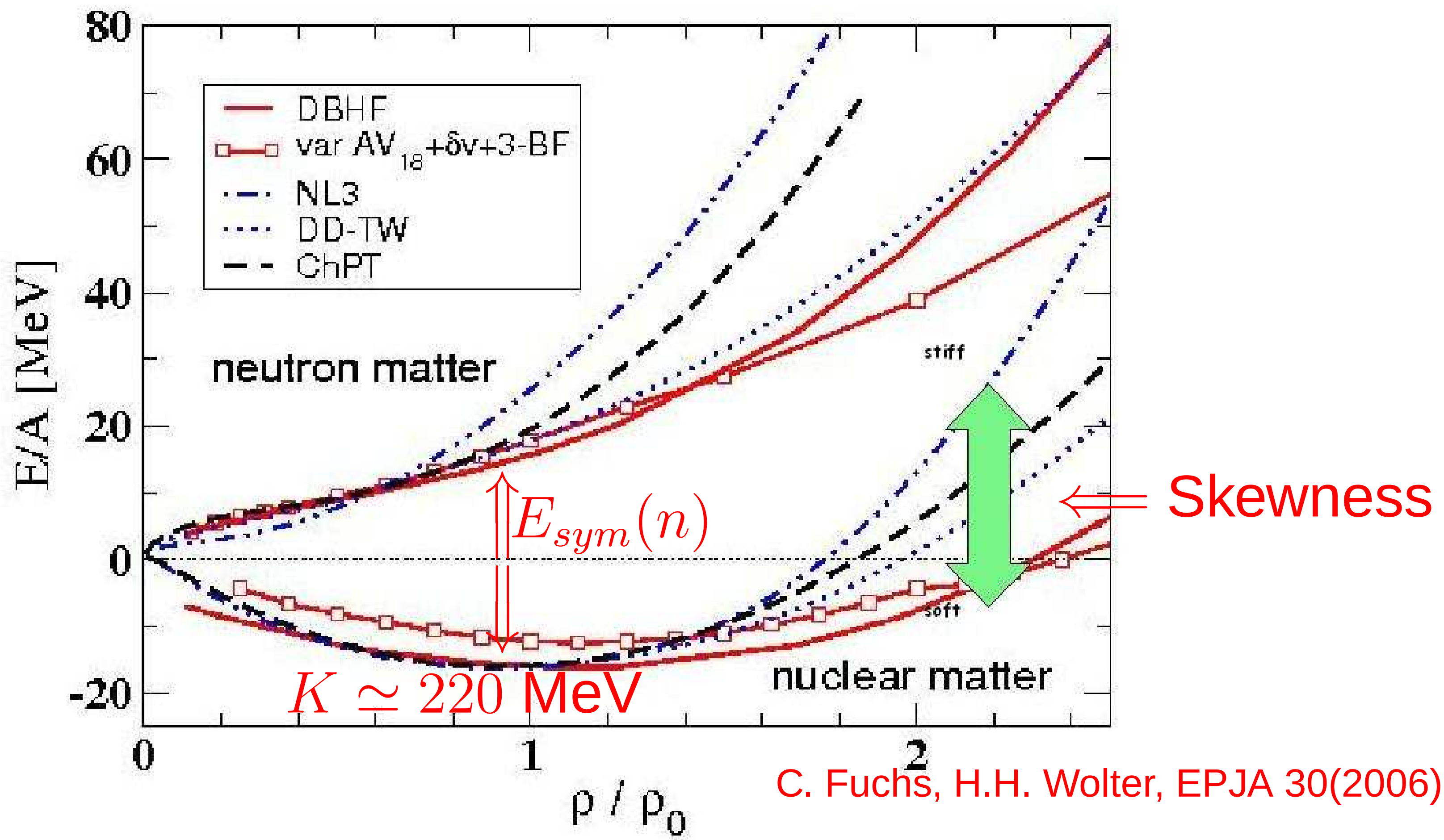
Wide variation:

$$1.2 < \frac{p(n_s)}{\text{MeV fm}^{-3}} < 7$$



The Uncertain Nuclear Force

The density dependence of $E_{sym}(n)$ is crucial but poorly constrained. Although the second density derivative, the incompressibility K, for symmetric matter is known well, the third density derivative, the skewness, is not.



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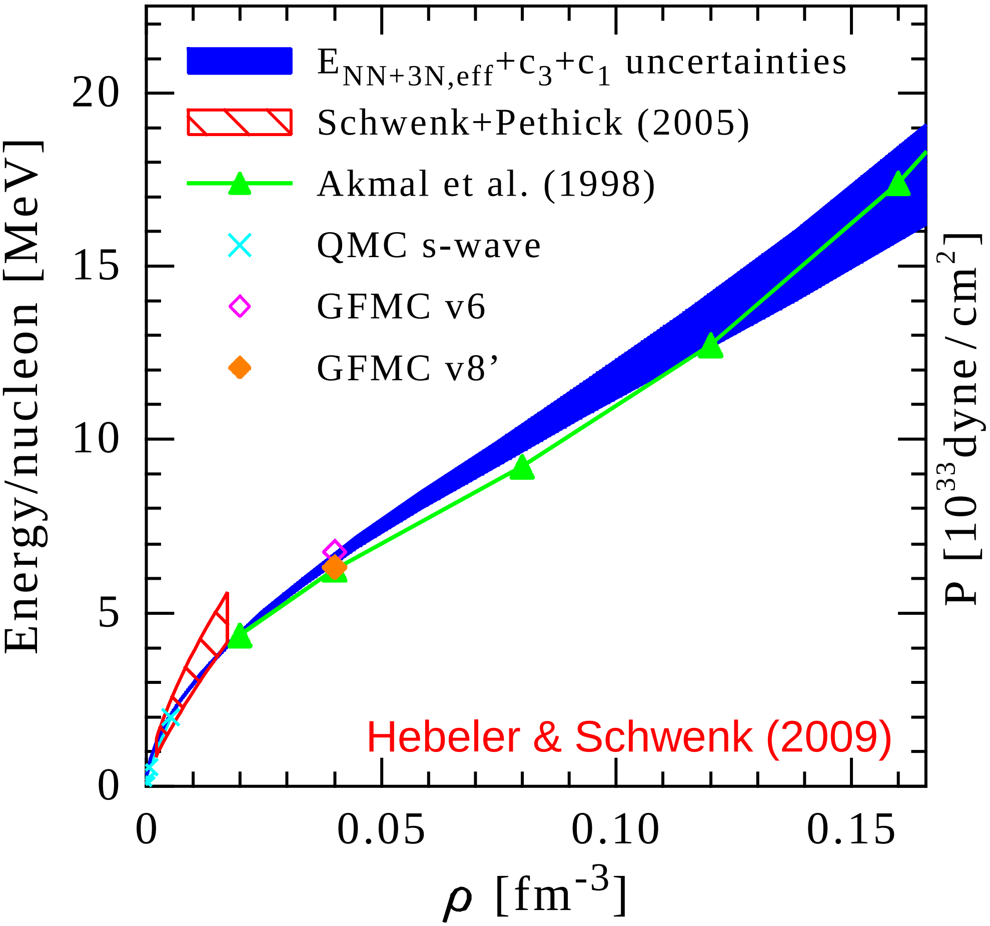
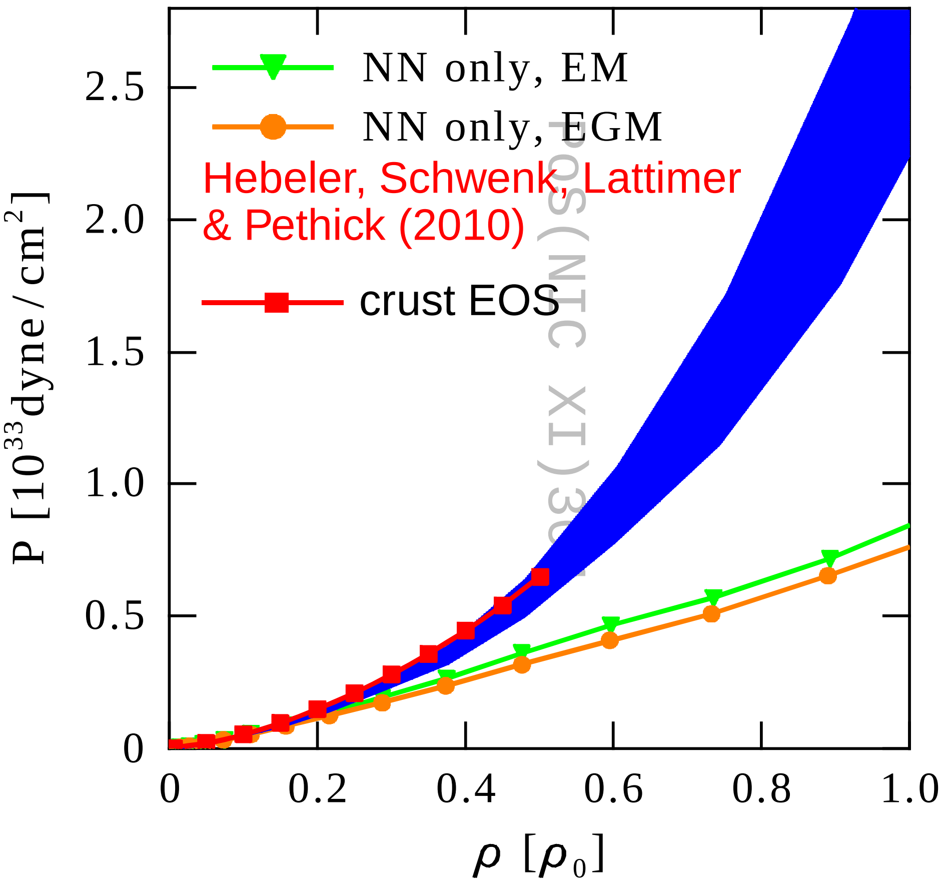
Pure Neutron Matter





Estimating Symmetry Parameters From Neutron Matter

$$E_n = E(n_s, 0) \simeq 16.3 \pm 2.1 \text{ MeV}, \qquad P_n \simeq 2.5 \pm 0.7 \text{ MeV fm}^{-3}$$

$$S'_v \equiv P_n/n_s = 15.6 \pm 4.4 \text{ MeV}$$

- Simple Model

$$E_{sym}(n) = S_v(n/n_s)^p$$

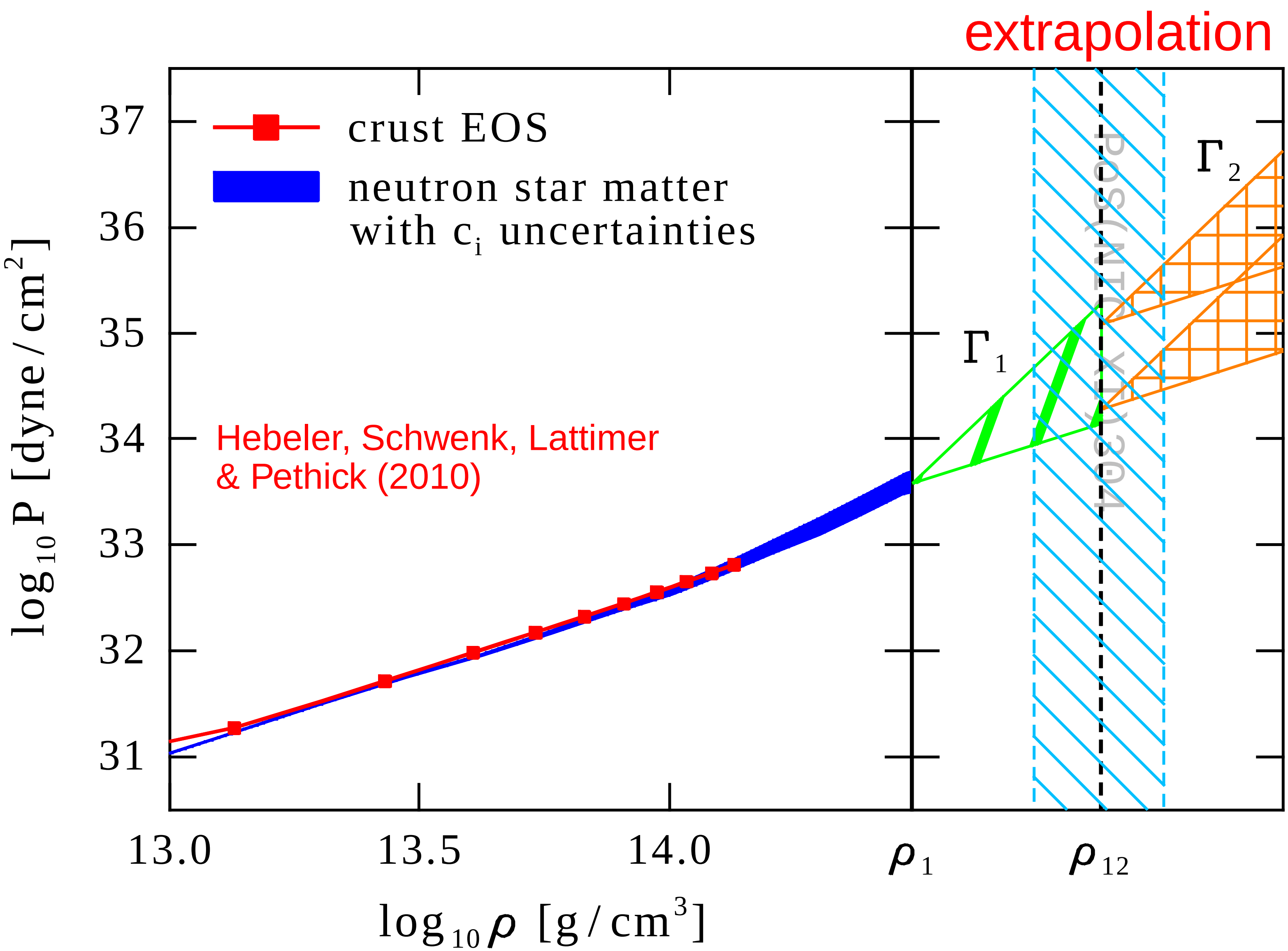
$$S_v = E_n + B \simeq 32.3 \pm 2.1 \text{ MeV}, \qquad p = S'_v/S_v \simeq 0.48 \pm 0.14$$

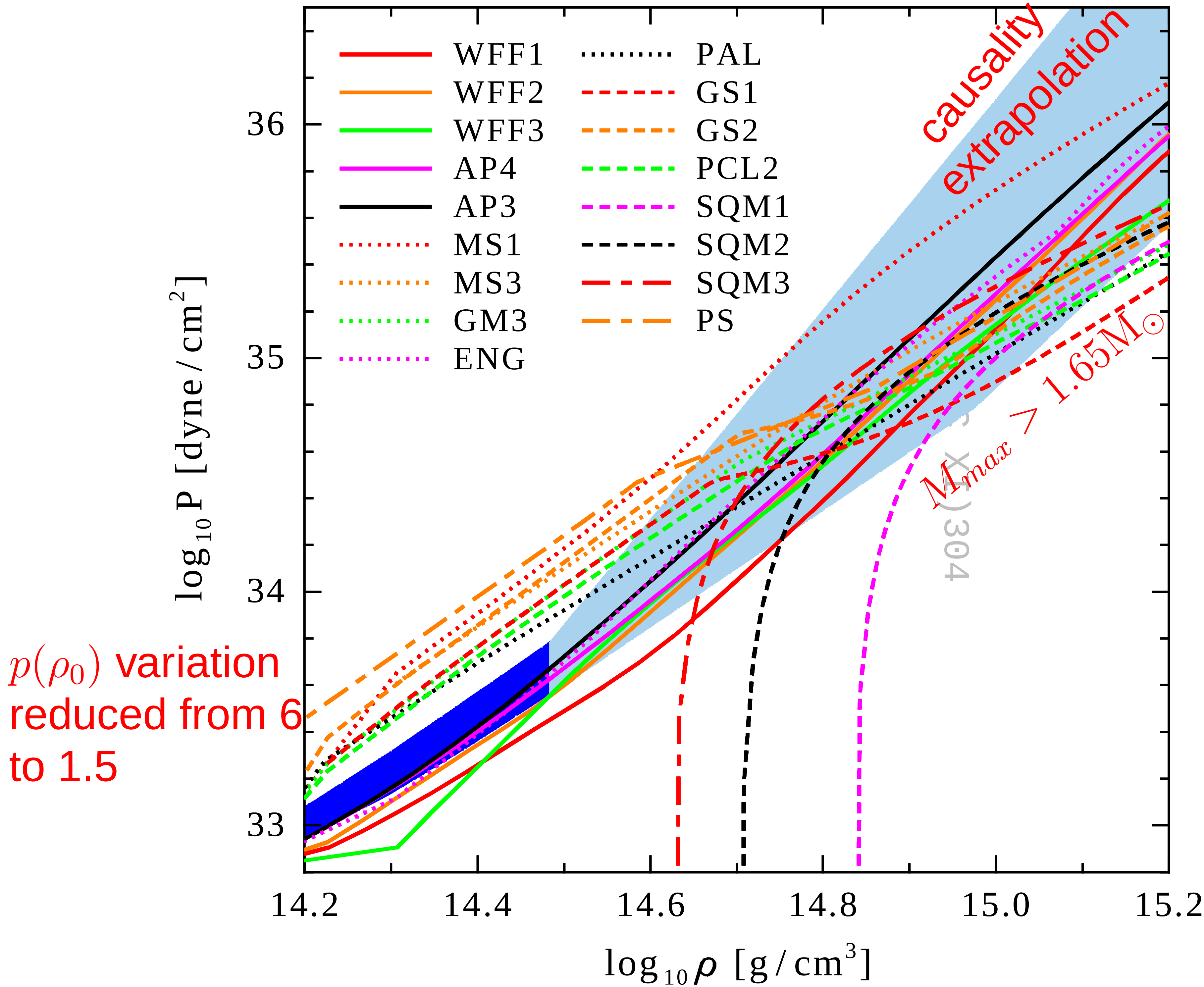
- More Accurate Model

$$E_{sym}(n) = S_k(n/n_s)^{2/3} + (S_v - S_k)(n/n_s)^\gamma$$

$$S_k \simeq 17 \text{ MeV}, \qquad \gamma = \frac{S'_v - 2S_v/3}{S_v - S_k} \simeq 0.28 \pm 0.29$$

Pressure of Neutron Star Matter

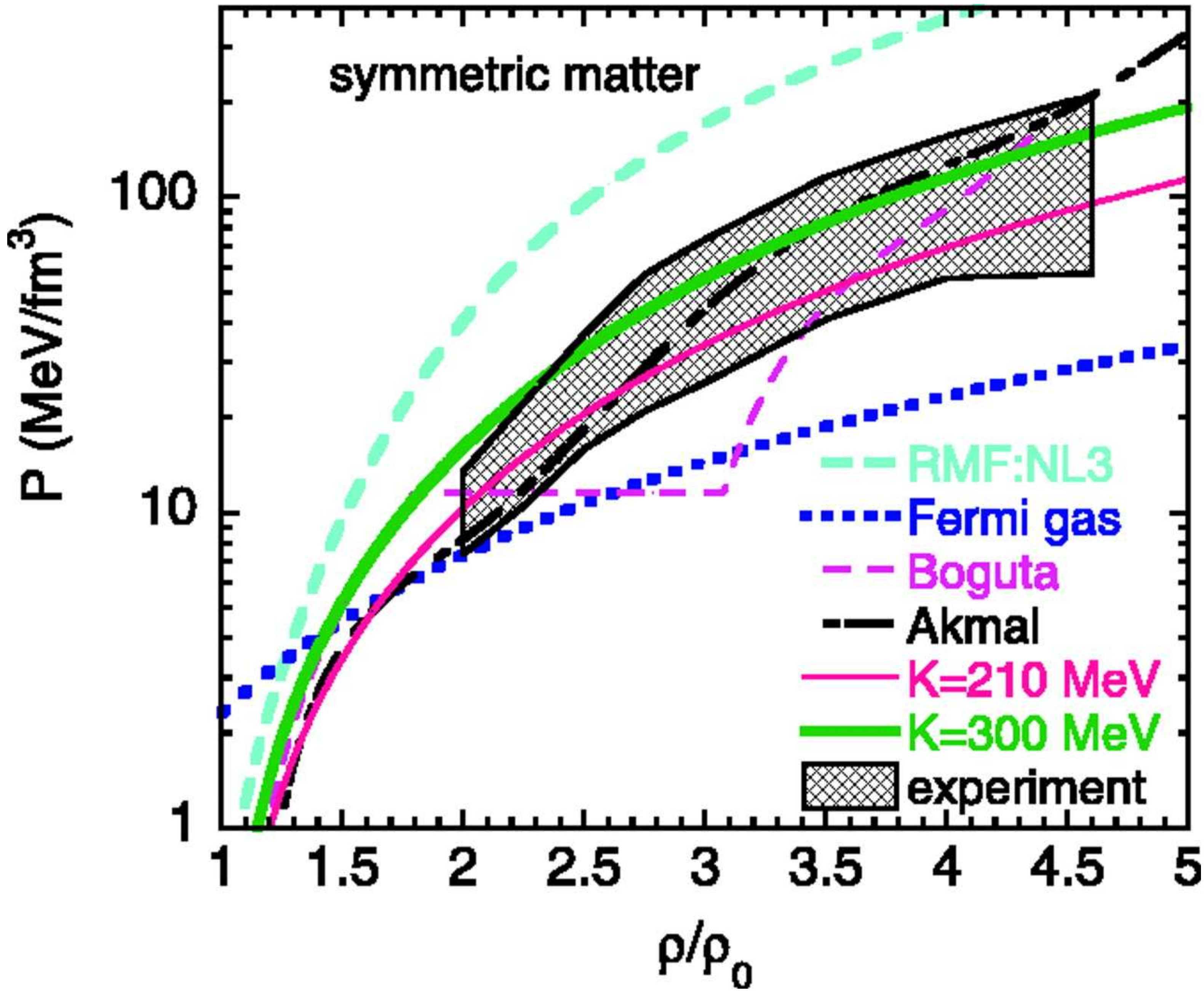




WFF1
WFF2
WFF3
AP4
AP3
MS1
MS3
GM3
ENG
PAL
GS1
GS2
PCL2
SQM1
SQM2
SQM3
PS
causality
extrapolation
$M_{max} > 1.65 M_{\odot}$
$p(\rho_0)$ variation reduced from 6 to 1.5
$\log_{10} P$ [dyne / cm^2]
$\log_{10} \rho$ [g / cm^3]
33
34
35
36
14.2
14.4
14.6
14.8
15.0
15.2

Heavy Ion Flow Data

Danielewicz, Lacey & Lynch 2002



Nuclear Mass Formula

Bethe-Weizsäcker (neglecting pairing and shell effects)

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2.$$

Myers & Swiatecki introduced the surface asymmetry term:

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2 - S_s (N-Z)^2/A^{4/3}.$$

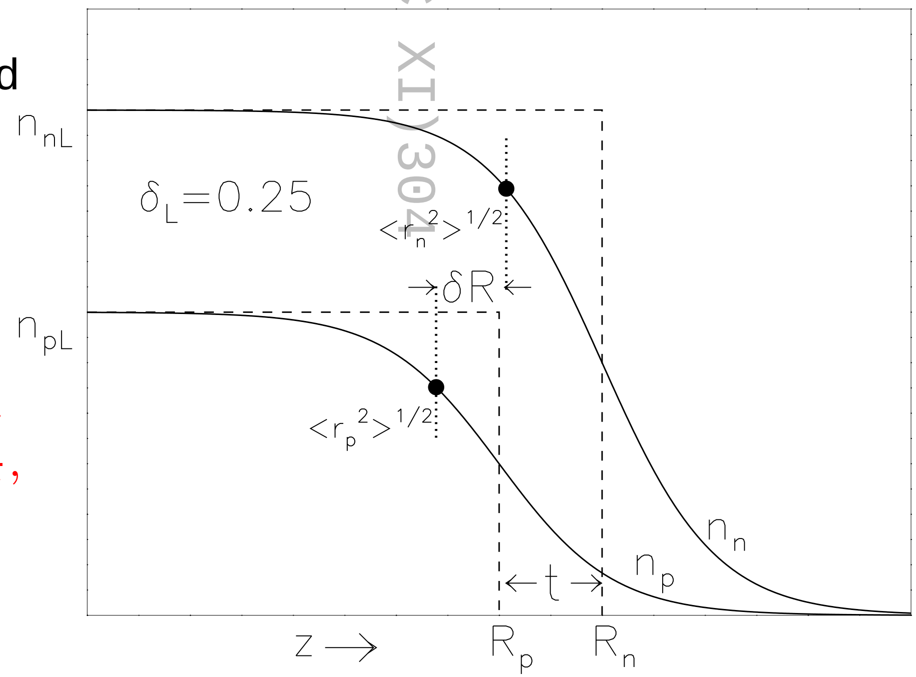
Droplet extension: consider the neutron/proton asymmetry of the nuclear surface.

$$E(A,Z) = (-a_v + S_v \delta^2)(A - N_s) + a_s A^{2/3} + a_C Z^2/A^{1/3} + \mu_n N_s.$$

N_s is the number of excess neutrons associated with the surface, $I = (N-Z)/(N+Z)$, $\delta = 1 - 2x = (A - N_s - 2Z)/(A - N_s)$ is the asymmetry of the nuclear bulk fluid, and μ_n is the neutron chemical potential. From thermodynamics,

$$N_s = -\frac{\partial a_s A^{2/3}}{\partial \mu_n} = \frac{S_s}{S_v}\frac{\delta}{1-\delta} = A\frac{I-\delta}{1-\delta},$$

$$\delta = I\left(1 + \frac{S_s}{S_v A^{1/3}}\right)^{-1},$$



$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2 \left(1 + \frac{S_s}{S_v A^{1/3}}\right)^{-1}.$$

Nuclear Structure Considerations

Information about E_{sym} can be extracted from nuclear binding energies and models for nuclei. For example, consider the schematic liquid droplet model (Myers & Swiatecki):

$$E(A,Z) \simeq -a_v A + a_s A^{2/3} + \frac{S_v}{1+(S_s/S_v)A^{-1/3}} A + a_c Z^2 A^{-1/3}$$

Fitting binding energies results in a strong correlation between S_v and S_s, but not definite values.

Blue: $\Delta E < 0.01$ MeV/b

Green: $\Delta E < 0.02$ MeV/b

Gray: $\Delta E < 0.03$ MeV/b

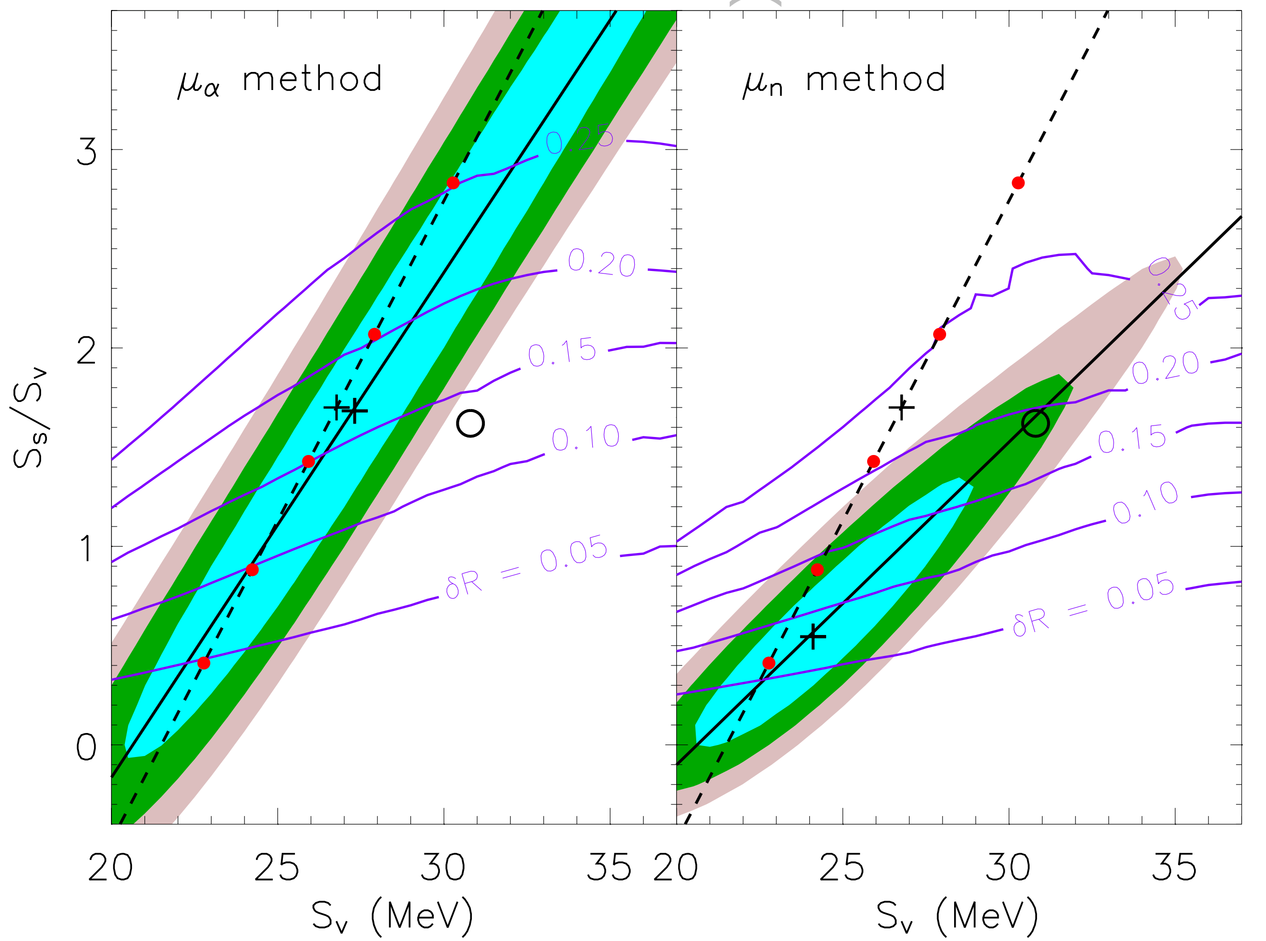
Circle: Moeller et al. (1995)

Crosses: Best fits

Dashed: Danielewicz (2004)

Solid: Steiner et al. (2005)

$\delta R \simeq 0.59 \frac{S_s}{S_v} \frac{\delta}{1-\delta^2}$ fm



Schematic Models

Nuclear Hamiltonian:

$$H = H_B + \frac{Q}{2} n'^2 \,, \quad H_B \simeq n\left[-B + \frac{K}{18}\left(1 - \frac{n}{n_s}\right)^2\right] + E_{sym}(1-2x)^2$$

Lagrangian minimization of energy with respect to n (symmetric matter):

$$H_B - \mu_0 n = \frac{Q}{2} n'^2 = \frac{K}{18} n \left(1 - \frac{n}{n_s}\right)^2 \,, \qquad \mu_0 = -a_v$$

Liquid Droplet surface parameters: $a_s = 4\pi r_0^2 \sigma_0 \,, \qquad S_s = 4\pi r_0^2 \sigma_\delta$

$$\sigma_0 = \int_{-\infty}^{+\infty} [H - \mu_0 n] dz = \int_0^{n_s} (H_B - \mu_0 n) \frac{dn}{n'} = \frac{4}{45}\sqrt{QKn_s^3}$$

$$t_{90-10} = \int_{0.1n_s}^{0.9n_s} \frac{dn}{n'} = 3\sqrt{\frac{Qn_s}{K}} \int_{0.1}^{0.9} \frac{du}{\sqrt{u}(1-u)} \simeq 9\sqrt{\frac{Qn_s}{K}}$$

$$\sigma_\delta = S_v \sqrt{\frac{Q}{2}} \int_0^{n_s} n \left(\frac{S_v}{E_{sym}} - 1\right) (H_B - \mu_0 n)^{-1/2} dn$$

$$= \frac{S_v t_{90-10} n_s}{3} \int_0^1 \frac{\sqrt{u}}{1-u} \left(\frac{S_v}{E_{sym}} - 1\right) du$$

$$E_{sym} \simeq S_v \left(\frac{n}{n_s}\right)^p \Longrightarrow \int \to 0.61 \,, \; 0.93 \,, \; 2.0 \;\; \left(p = \frac{1}{2} \,, \; \frac{2}{3} \,, \; 1\right)$$

$$E_{sym} \simeq S_k \left(\frac{n}{n_s}\right)^{2/3} + (S_v - S_k)\left(\frac{n}{n_s}\right)^\gamma \Longrightarrow \int \to 0.30 \,, \; 0.57 \,, \; 0.86 \;\; (\gamma = .0 \,, \; .3 \,, \; .6)$$

Schematic Models

$$\frac{S_s}{S_v} \simeq \frac{t_{90-10}}{r_0} \int \simeq 2.05 \int \quad \Longrightarrow \quad 1.26\,,\ 1.90\,,\ 4.1\ \ (p = 1/2\,,\ 2/3\,,\ 1)$$

$$\Longrightarrow \quad 0.62\,,\ 1.17\,,\ 1.76\ \ (\gamma = 0.0\,,\ 0.3\,,\ 0.6)\,,$$

For Pb^{208}:

$$\delta R = \sqrt{\frac{3}{5}}\frac{2}{3} r_0 \frac{S_s}{S_v} \frac{\delta}{1-\delta^2} \quad \Longrightarrow \quad 0.13\,,\ 0.18\,,\ 0.31\ \mathrm{fm}\ \ (p = 1/2\,,\ 2/3\,,\ 1)$$

$$\Longrightarrow \quad 0.07\,,\ 0.13\,,\ 0.17\ \mathrm{fm}\ \ (\gamma = 0.0\,,\ 0.3\,,\ 0.6)\,,$$

PREX experiment (E06002) at Jefferson Lab to measure the neutron radius of lead to about 1% accuracy (current accuracy is about 5%) using the parity violating asymmetry in elastic scattering due to the weak neutral interaction. Requires corrections for Coulomb distortions (Horowitz).

Nuclei in Dense Matter

Liquid Droplet Model, Simplified

$$F = u(F_I + f_{LD}/V_N) + (1-u)F_{II}\,, \qquad f_{LD} = f_S + f_C + f_T$$

$$\begin{aligned}
f_C &= \frac{3}{5}\frac{Z^2e^2}{R_N}\left(1 - \frac{3}{2}u^{1/3} + \frac{u}{2}\right) = \frac{3}{5}\frac{Z^2e^2}{R_N}D(u) \\
f_T &= T\ln\left(\frac{u}{n_Q V_N A^{3/2}}\right) - T = \mu_T - T\,, \qquad n_Q = \left(\frac{mT}{2\pi\hbar^2}\right)^{3/2} \\
f_S &= 4\pi R_N^2 \sigma(\mu_s)
\end{aligned}$$

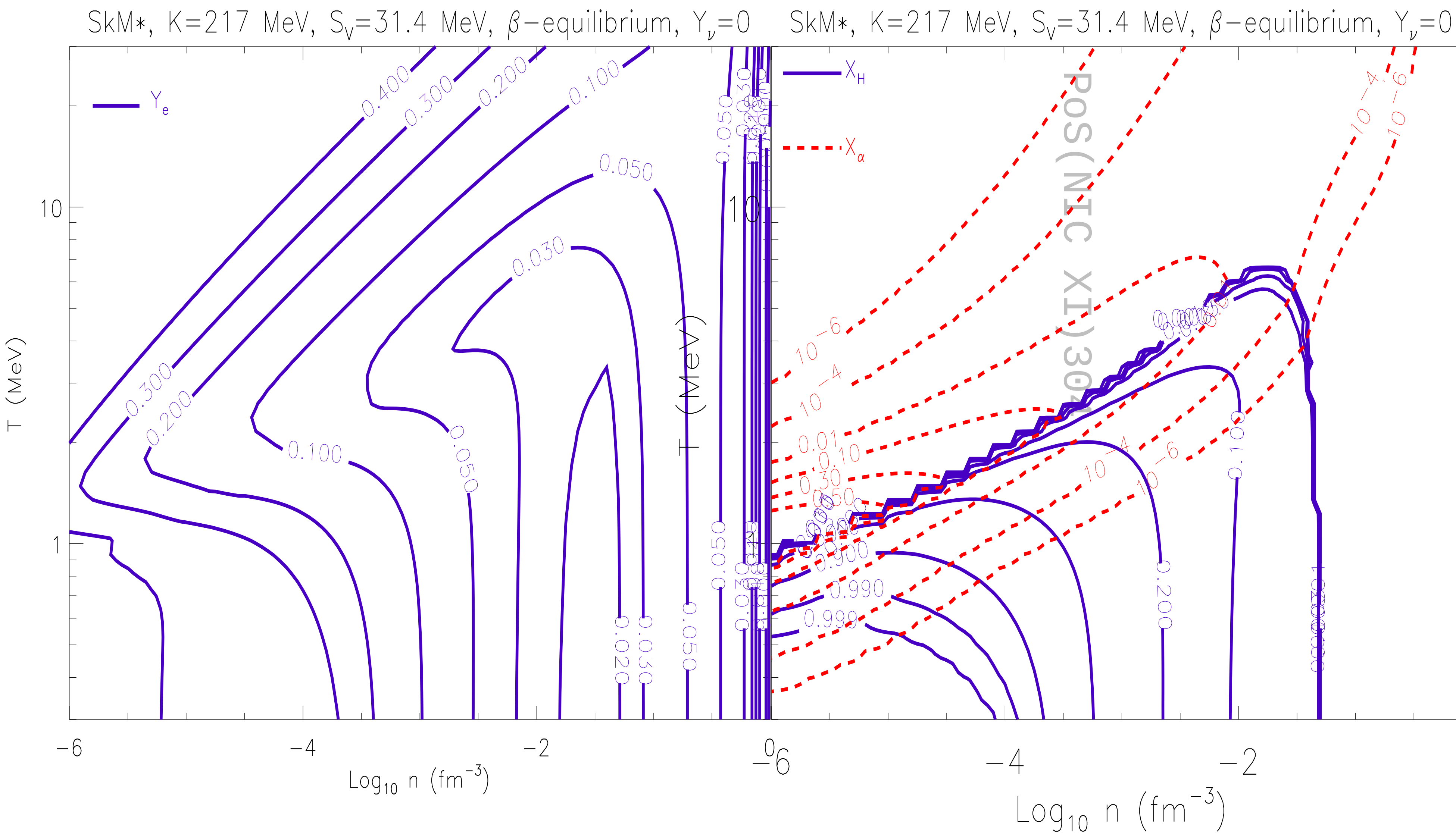
$$n = un_I + (1-u)n_{II}\,, \qquad nY_e = un_I x_I + (1-u)n_{II}x_{II} + u\frac{N_s}{V_N}$$

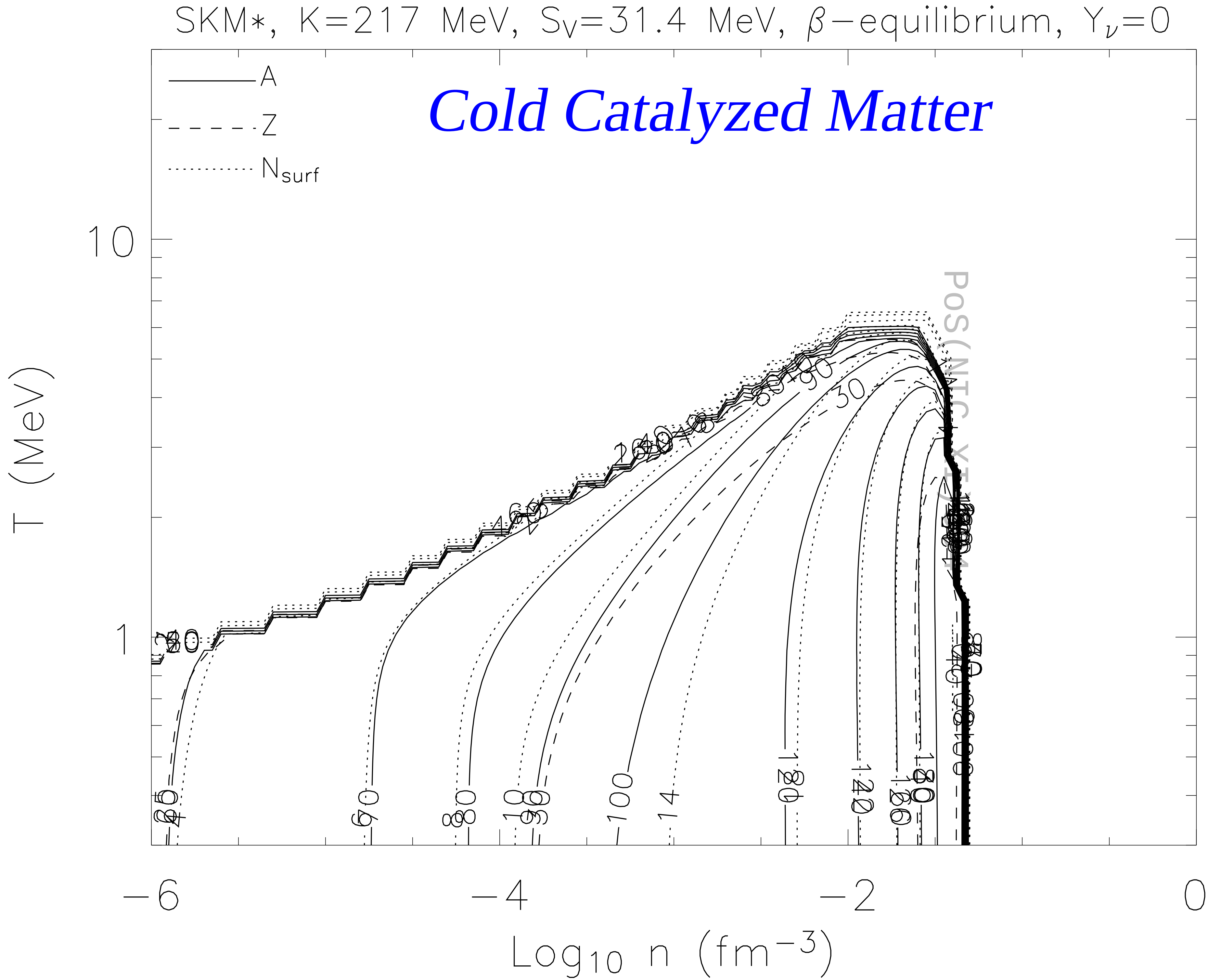
Free Energy Minimization

$$\frac{\partial F}{\partial z_i} = 0\,, \qquad z_i = (n_I, x_I, R_N, u, \nu_s, \mu_s)$$

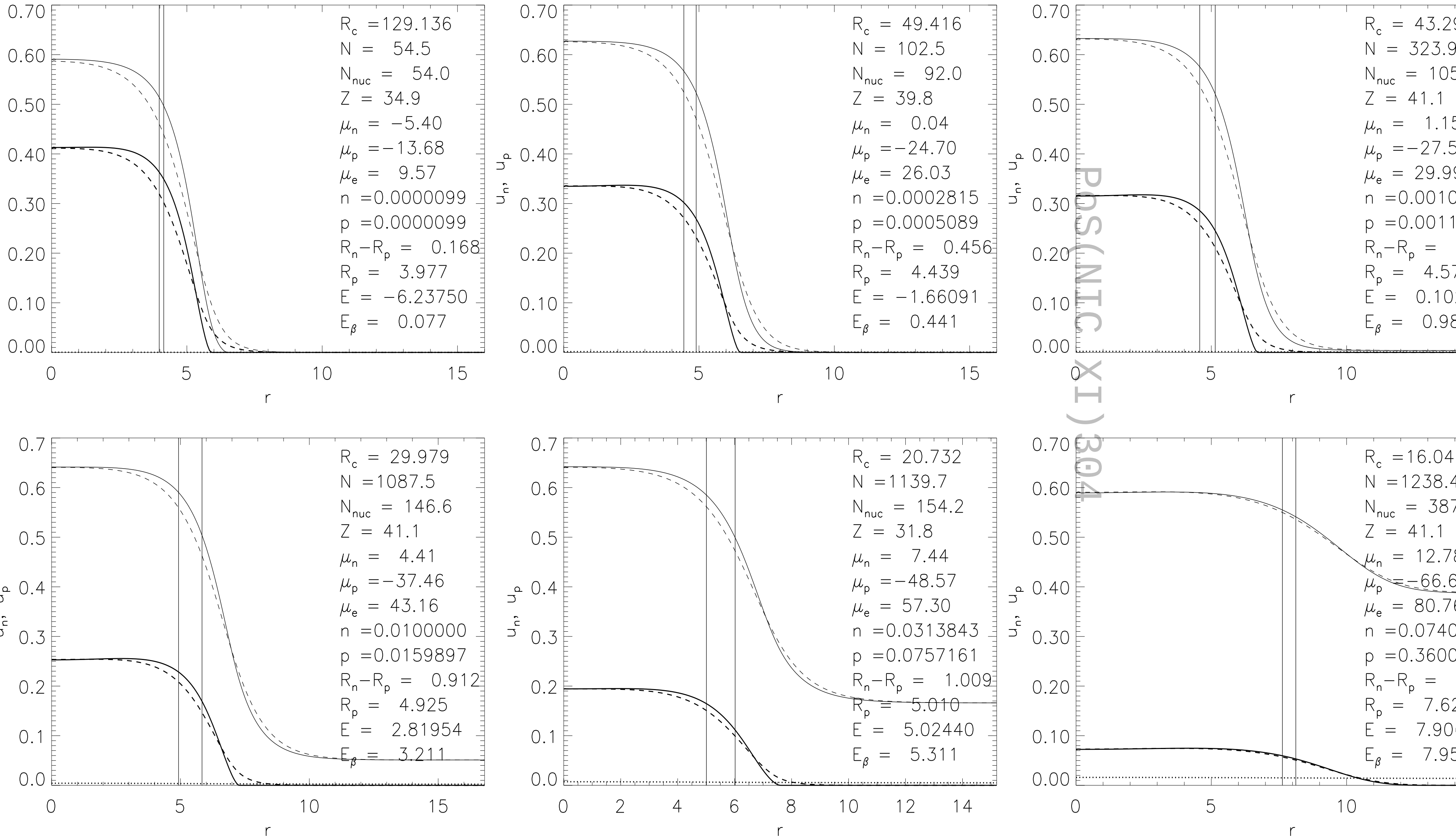
$$\begin{aligned}
\mu_{n,II} &= \mu_{n,I} + \frac{\mu_T}{A}\,, \qquad \hat{\mu}_{II} = \hat{\mu}_I - \frac{3\sigma}{R_N n_I x_i} = -\mu_s\,, \qquad N_s = -4\pi R_N^2 \frac{\partial \sigma}{\partial \mu_s} \\
P_{II} &= P_I + \frac{3\sigma}{2R_N}\left(1 + \frac{uD'}{D}\right)\,, \qquad R_N = \left(\frac{15\sigma}{8\pi n_I^2 x_I^2 e^2 D}\right)^{1/3}
\end{aligned}$$

Cold Catalyzed Matter

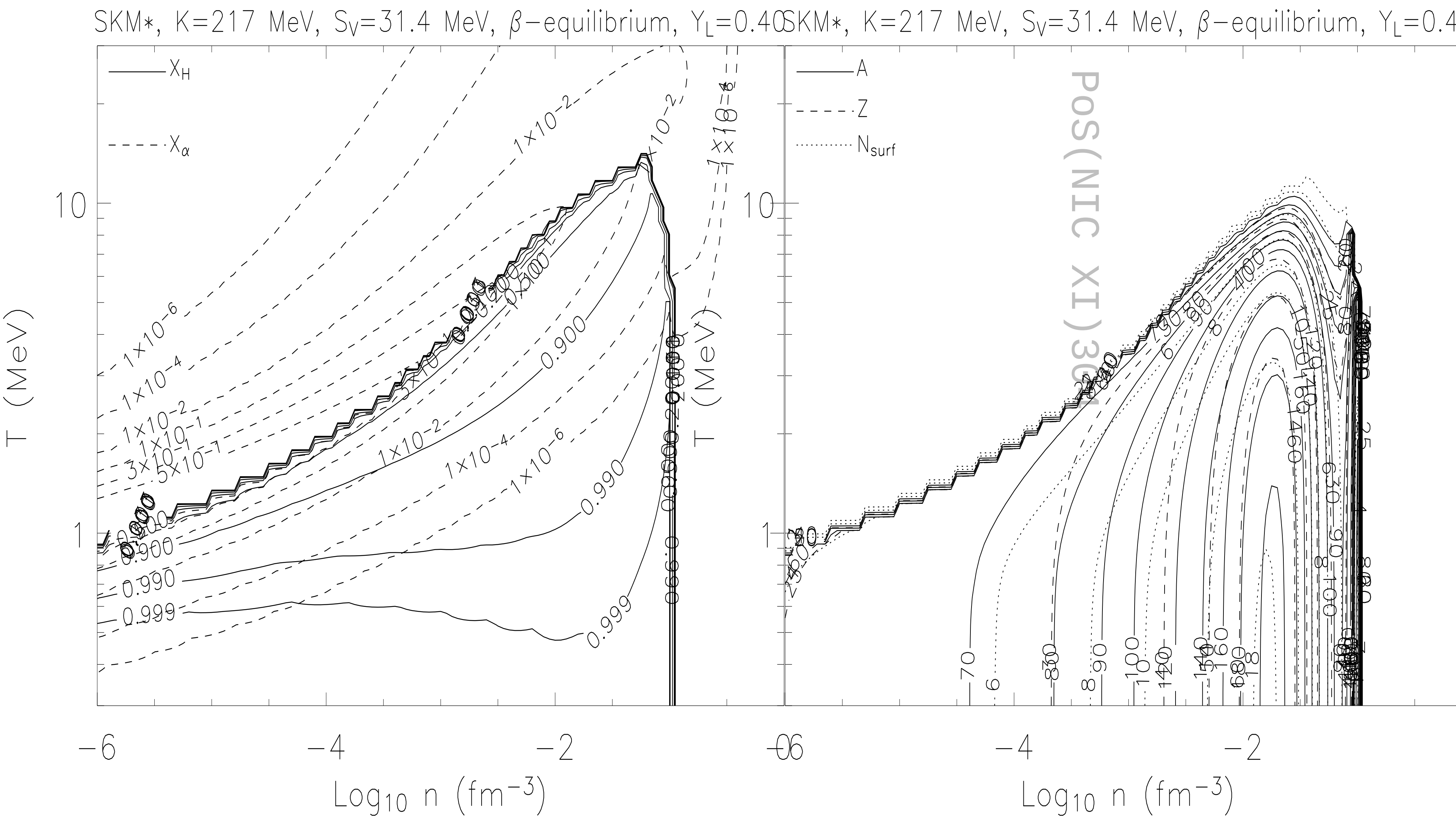


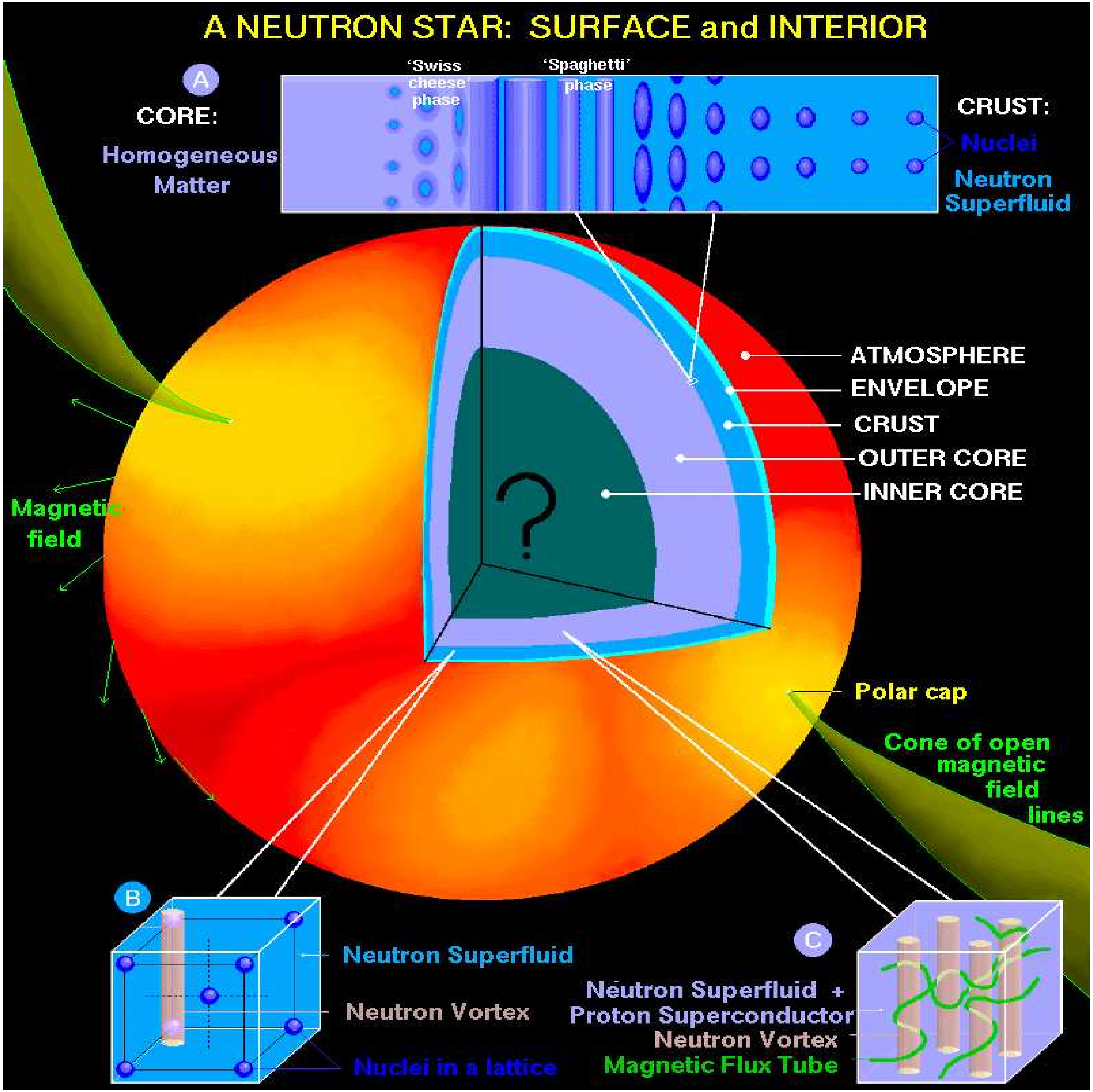

SKM*, K=217 MeV, S_V=31.4 MeV, β−equilibrium, Y_ν=0
A
Z
N_{surf}
Cold Catalyzed Matter
10
1
T (MeV)
−6
−4
−2
0
Log_{10} n (fm^{-3})

Nuclei in Dense Matter



Supernova Matter





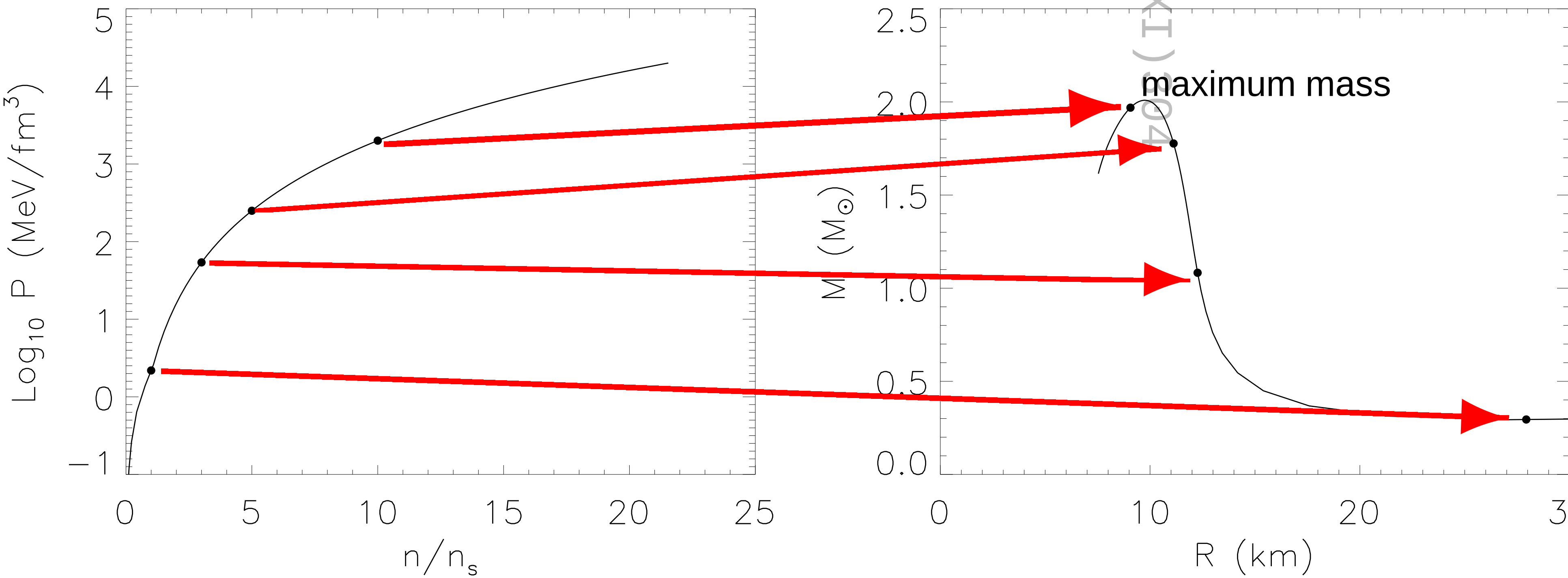
Credit: Dany Page, UNAM

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi p r^3)(\epsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\epsilon}{c^2}r^2$$



Mass-Radius Diagram and Theoretical Constraints

GR:

$R > 2GM/c^2$

$P < \infty$:

$R > (9/4)GM/c^2$

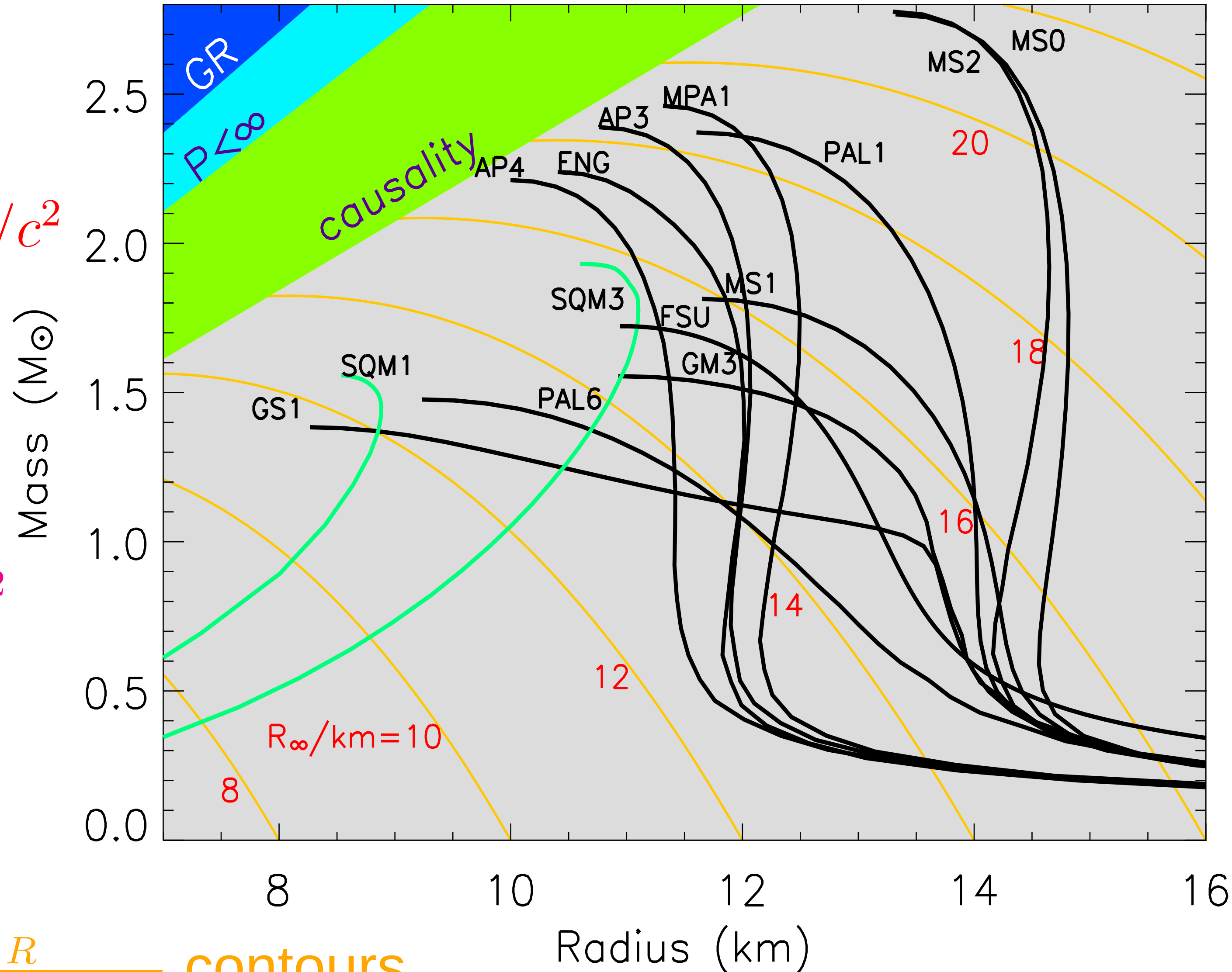
causality:

$R \gtrsim 2.9GM/c^2$

— normal NS

— SQS

— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$ contours



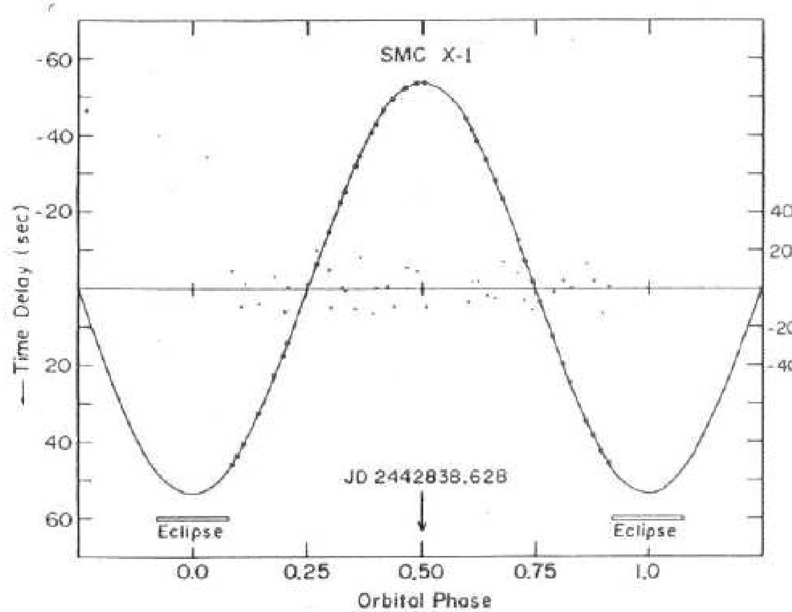
Mass Measurements In X-Ray Binaries

Mass function

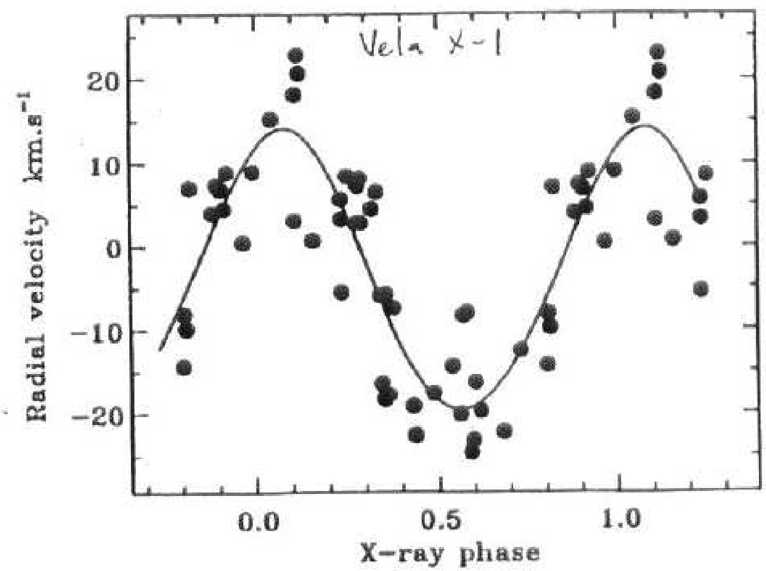
$$f(M_1) = \frac{P(v_2 \sin i)^3}{2\pi G} = \frac{(M_1 \sin i)^3}{(M_1+M_2)^2} > M_1$$

$$f(M_2) = \frac{P(v_1 \sin i)^3}{2\pi G} = \frac{(M_2 \sin i)^3}{(M_1+M_2)^2} > M_2$$

X-ray timing



Optical spectroscopy



In an X-ray binary, $v_{optical}$ has the largest uncertainties. In some cases $\sin i \sim 1$ if eclipses are observed. If eclipses are not observed, limits to i can be made based on the estimated radius of the optical star.

Pulsar Mass Measurements

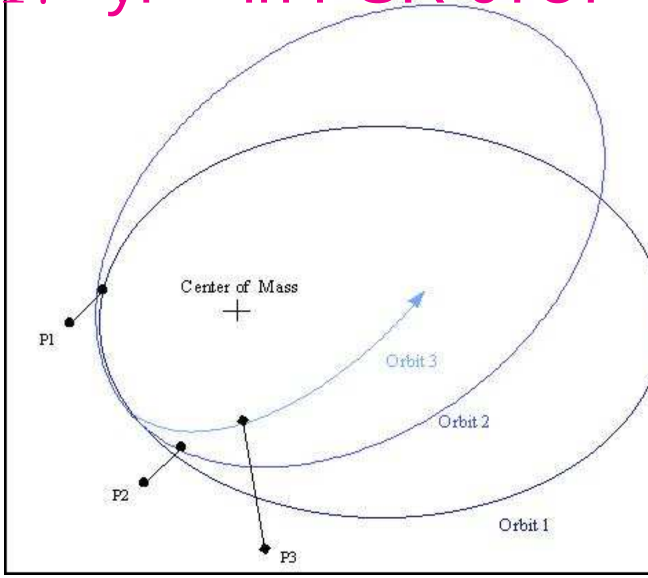
Mass function for pulsar precisely obtained.

It is also possible in some cases to obtain the rate of periastron advance and the Einstein gravitational redshift + time dilation term:

$\dot\omega = 3(2\pi/P)^{5/3}(GM/c^2)^{2/3}/(1-e^2)$

$\gamma = (P/2\pi)^{1/3} e M_2 (2M_2 + M_1)(G/M^2c^2)^{2/3}$

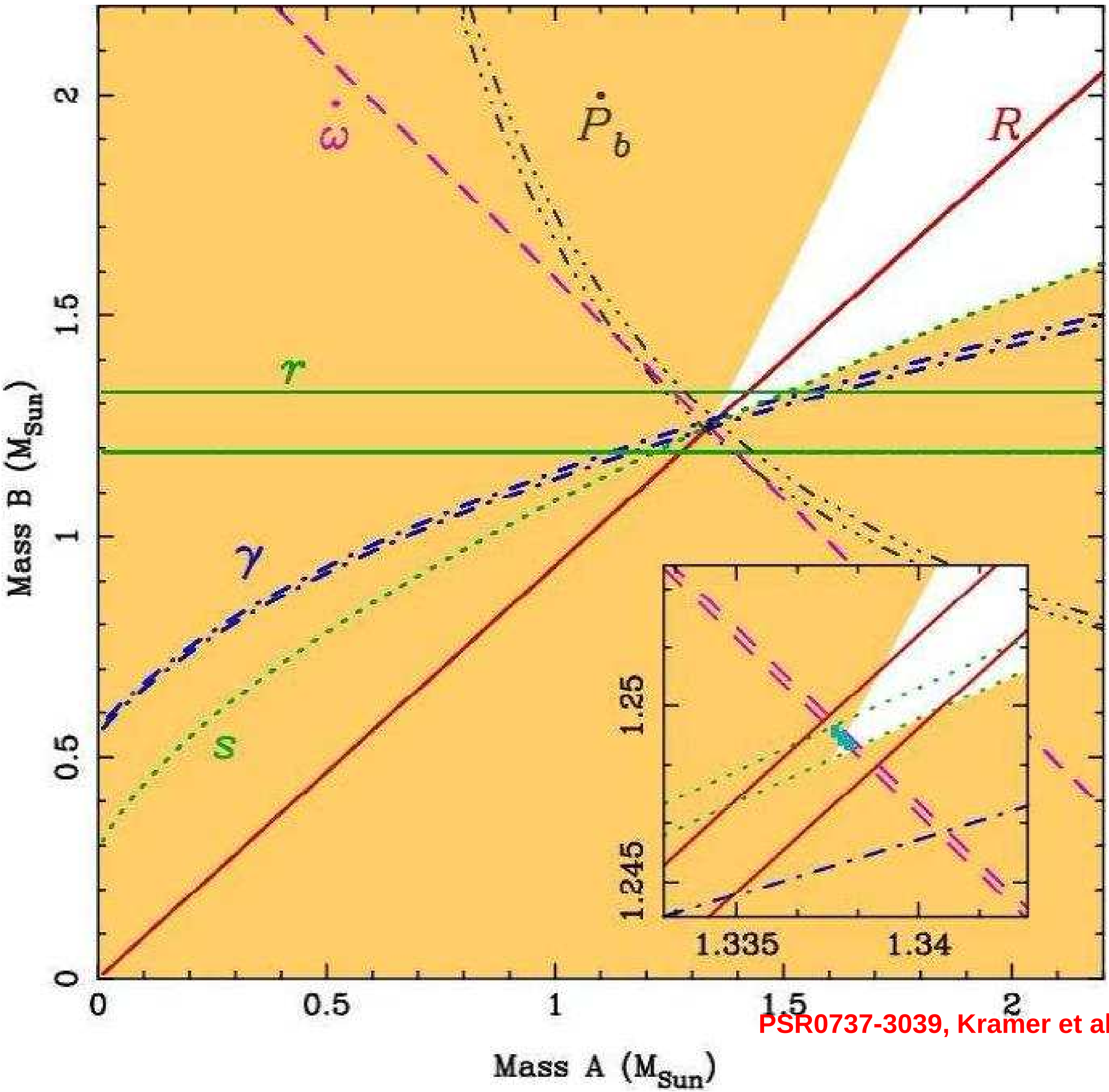
17° yr^{-1} in PSR 0737



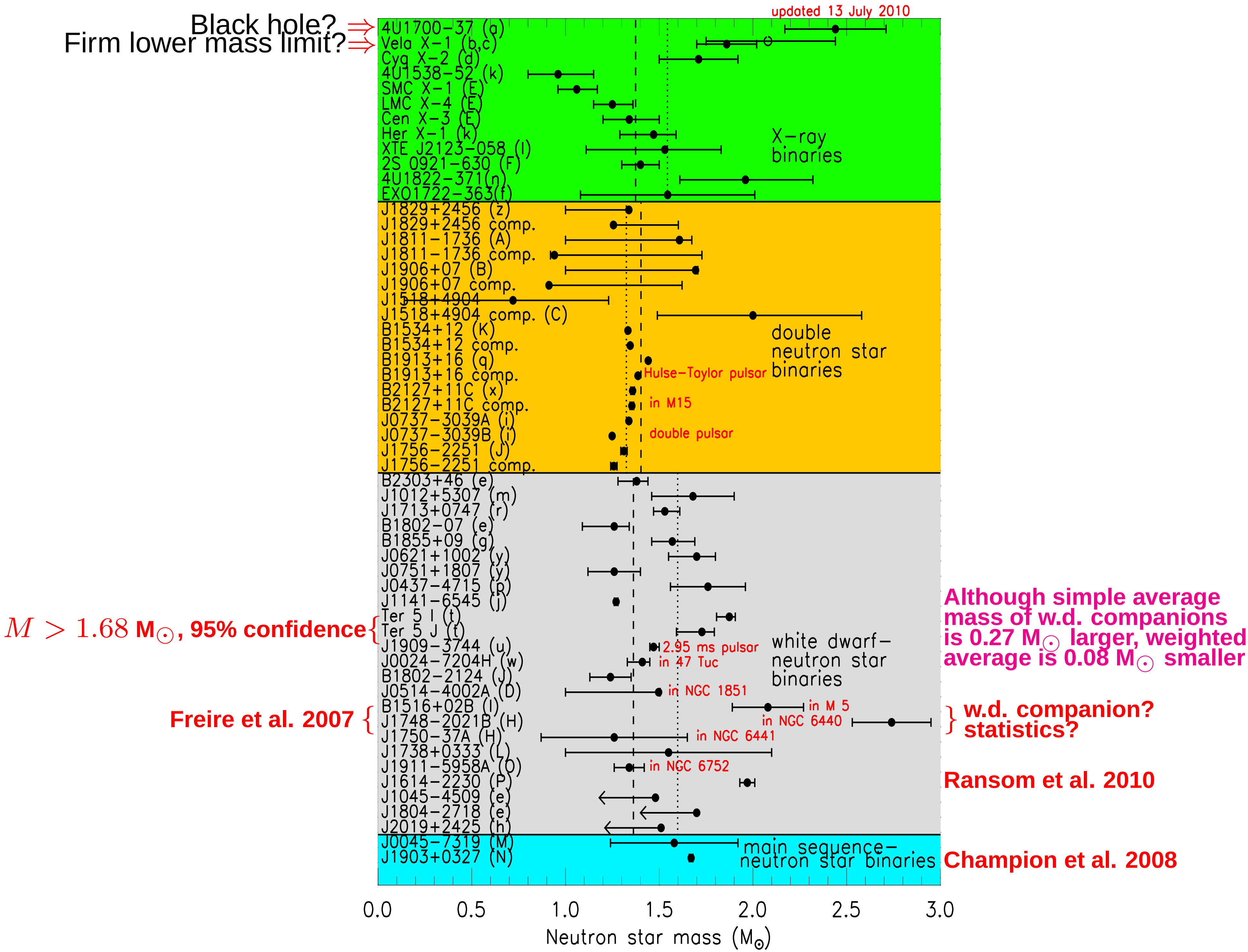
Gravitational radiation leads to orbit decay:

$\dot P = -\frac{192\pi}{5c^5}\left(\frac{2\pi G}{P}\right)^{5/3}(1-e^2)^{-7/2}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)\frac{M_1M_2}{M^{1/2}}$

In some cases, can constrain Shapiro time delay, r is magnitude and $s = \sin i$ is shape parameter.



PSR0737-3039, Kramer et al. (2007)



updated 13 July 2010
Black hole?
Firm lower mass limit?
4U1700−37 (a)
Vela X−1 (b,c)
Cyg X−2 (d)
4U1538−52 (k)
SMC X−1 (E)
LMC X−4 (E)
Cen X−3 (E)
Her X−1 (k)
XTE J2123−058 (l)
2S 0921−630 (F)
4U1822−371(n)
EXO1722−363(f)
X−ray binaries
J1829+2456 (z)
J1829+2456 comp.
J1811−1736 (A)
J1811−1736 comp.
J1906+07 (B)
J1906+07 comp.
J1518+4904
J1518+4904 comp. (C)
B1534+12 (K)
B1534+12 comp.
B1913+16 (q)
B1913+16 comp.
Hulse−Taylor pulsar
B2127+11C (x)
B2127+11C comp.
in M15
J0737−3039A (i)
J0737−3039B (i)
double pulsar
J1756−2251 (J)
J1756−2251 comp.
double neutron star binaries
B2303+46 (e)
J1012+5307 (m)
J1713+0747 (r)
B1802−07 (e)
B1855+09 (g)
J0621+1002 (y)
J0751+1807 (y)
J0437−4715 (p)
J1141−6545 (j)
Ter 5 I (t)
Ter 5 J (t)
J1909−3744 (u)
2.95 ms pulsar
J0024−7204H (w)
in 47 Tuc
B1802−2124 (J)
J0514−4002A (D)
in NGC 1851
B1516+02B (l)
in M 5
J1748−2021B (H)
in NGC 6440
J1750−37A (H)
in NGC 6441
J1738+0333 (L)
J1911−5958A (O)
in NGC 6752
J1614−2230 (P)
J1045−4509 (e)
J1804−2718 (e)
J2019+2425 (h)
white dwarf−neutron star binaries
J0045−7319 (M)
J1903+0327 (N)
main sequence−neutron star binaries
0.0 0.5 1.0 1.5 2.0 2.5 3.0
Neutron star mass (M⊙)
M > 1.68 M⊙, 95% confidence
Freire et al. 2007
Although simple average mass of w.d. companions is 0.27 M⊙ larger, weighted average is 0.08 M⊙ smaller
w.d. companion? statistics?
Ransom et al. 2010
Champion et al. 2008

Roche Model for Maximal Rotation

(*c.f.*, Shapiro & Teukolsky 1983)

$\rho^{-1}\nabla P = \nabla h = -\nabla(\Phi_G + \Phi_c), \qquad \Phi_G \simeq -GM/r, \qquad \Phi_c = -\frac{1}{2}\Omega^2 r^2 \sin^2\theta$

Bernoulli integral: $H = h + \Phi_G + \Phi_c = -GM/R_p$

Enthalpy $h = \int_0^p \rho^{-1} dp = \mu_n(\rho) - \mu_n(0)$ in beta equilibrium

Evaluate at equator: $\frac{\Omega^2 R_{eq}^3}{2GM} = \frac{R_{eq}}{R_p} - 1$

Mass-shedding limit $\Omega^2_{shed} = \frac{GM}{R_{eq}^3} : \frac{R_{eq}}{R_p} = \frac{3}{2}$

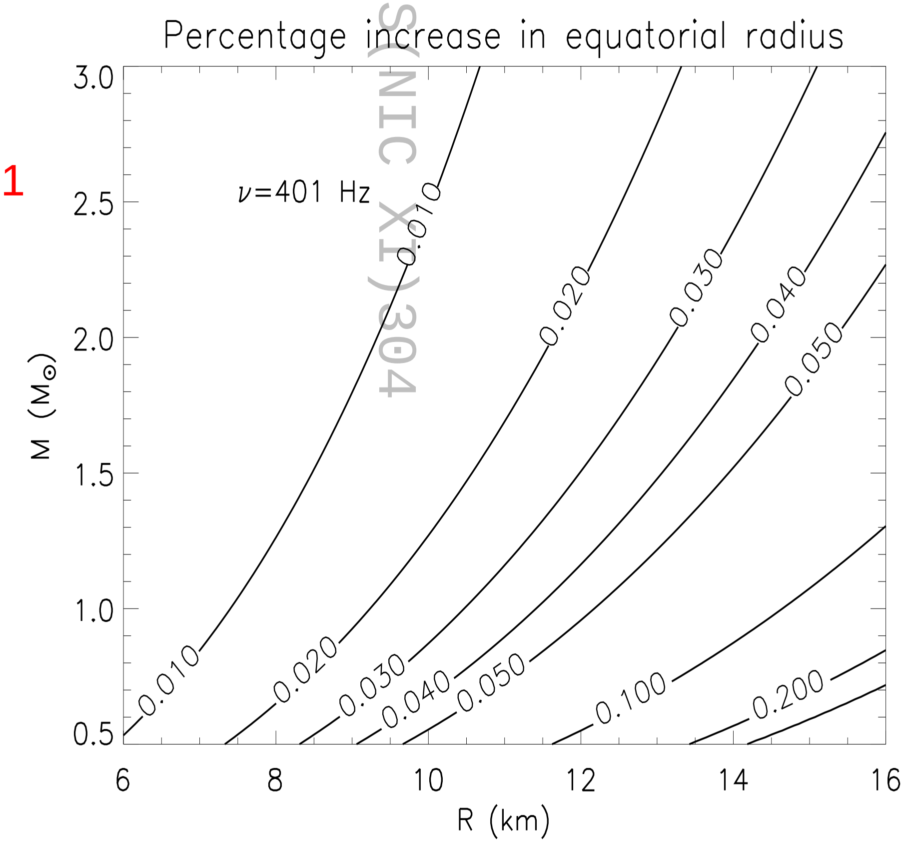
GR: Cook, Shapiro & Teukolsky (1994): 1.43–1.51

Numerical calculations show R_p is nearly constant for arbitrary rotation

$\Omega_{shed} = \left(\frac{2}{3}\right)^{3/2}\sqrt{\frac{GM}{R^3}}$

$P_{shed} = 1.0\ (R/10\ \mathrm{km})^{3/2}(\mathrm{M}_\odot/M)^{1/2}$ ms

GR: Lattimer & Prakash (2005): $0.96 \pm 3\%$



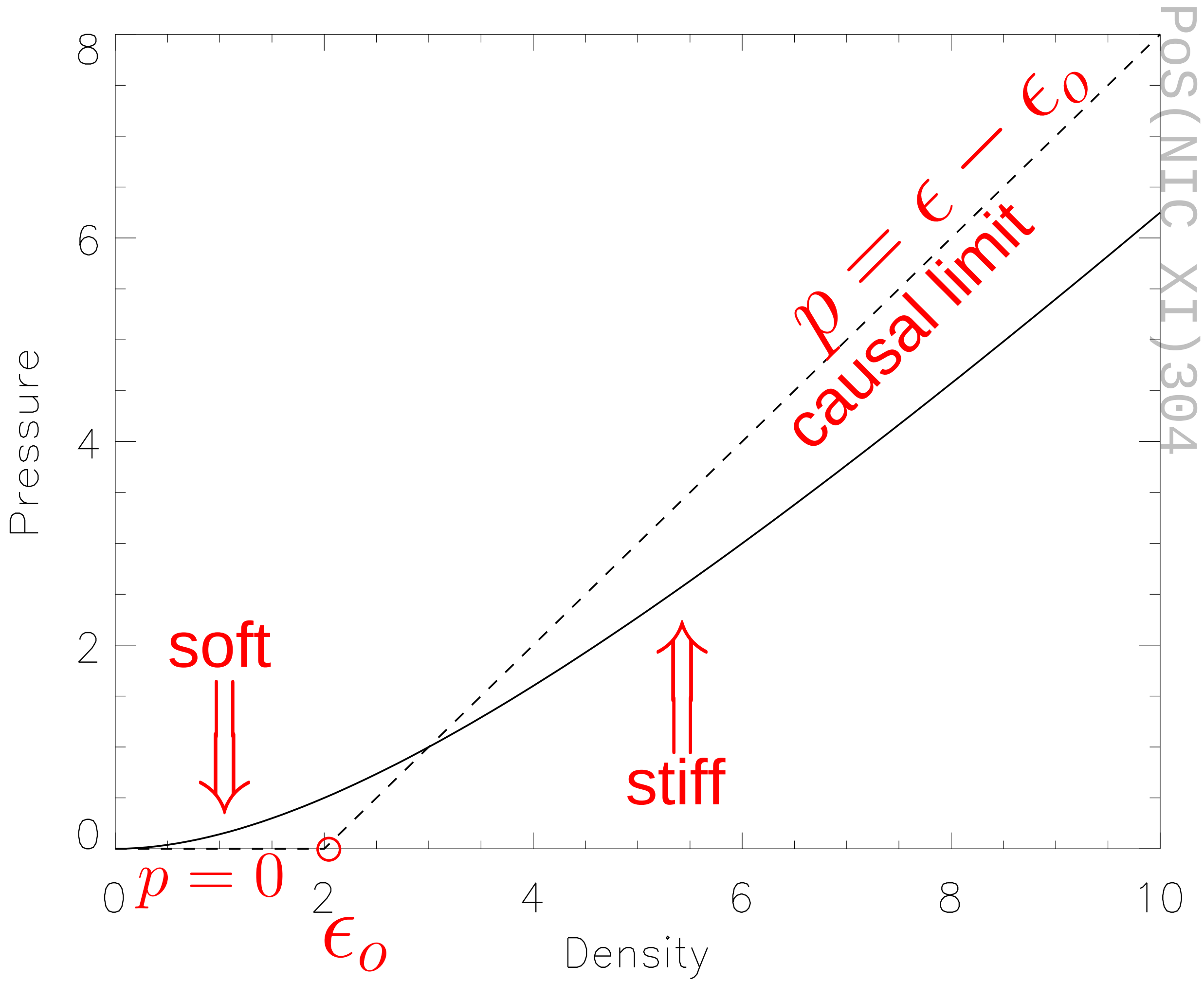
Shape: $\frac{\Omega^2 R(\theta)^3 \sin^2\theta}{2GM} = \frac{R(\theta)}{R_p} - 1$

$\frac{R_p}{R(\theta)} = \frac{1}{3} + \frac{2}{3}\cos[\frac{1}{3}\cos^{-1}(1 - 2(\frac{\Omega\sin\theta}{\Omega_{shed}})^2)]$

Limit: $\frac{R_p}{R(\theta)} = \frac{1}{3} + \frac{2}{3}\cos[\frac{1}{3}\cos^{-1}(1 - 2\sin^2\theta)] = \frac{\sin(\theta)}{3\sin(\theta/3)}.$

Extreme Properties of Neutron Stars

- The most compact configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff".



ϵ_0 is the only EOS parameter

The TOV solutions scale with ϵ_0

Maximally Compact Equation of State

Koranda, Stergioulas & Friedman (1997)

$$\begin{aligned} p(\epsilon) = 0, \qquad \epsilon &\leq \epsilon_o \\ p(\epsilon) = \epsilon - \epsilon_o, \qquad \epsilon &\geq \epsilon_o \end{aligned}$$

This EOS has a parameter ϵ_o, which corresponds to the surface energy density. The structure equations then contain only this one parameter, and can be rendered into dimensionless form using

$$y = m\epsilon_o^{1/2}, \qquad x = r\epsilon_o^{1/2}, \qquad q = p\epsilon_o^{-1}.$$

$$\begin{aligned} \frac{dy}{dx} &= 4\pi x^2(1+q) \\ \frac{dq}{dx} &= -\frac{(y+4\pi q x^3)(1+2q)}{x(x-2y)} \end{aligned}$$

The solution with the maximum central pressure and mass and the minimum radius:

$$\begin{aligned} q_{max} &= 2.026, \qquad y_{max} = 0.0851, \qquad x_{min}/y_{max} = 2.825 \\ p_{max} &= 307\left(\frac{\epsilon_o}{\epsilon_s}\right) \text{ MeV fm}^{-3}, \quad M_{max} = 4.1\left(\frac{\epsilon_s}{\epsilon_o}\right)^{1/2} \text{ M}_\odot, \quad R_{min} = 2.825\frac{GM_{max}}{c^2}. \end{aligned}$$

Moreover, the scaling extends to the axially-symmetric case, yielding

$$P_{min} \propto \left(\frac{M_{max}}{R_{min}^3}\right)^{1/2} \propto \epsilon_o^{-1/2}, \quad P_{min} = 0.82\left(\frac{\epsilon_s}{\epsilon_o}\right)^{1/2} \text{ms} = 0.76\left(\frac{R}{10\text{ km}}\right)^{3/2}\left(\frac{\text{M}_\odot}{M}\right)^{1/2} \text{ms}$$

Maximum Possible Density in Stars

The scaling from the maximally compact EOS yields

$$\epsilon_{c,max} = 3.026 \left(\frac{4.1\ \mathrm{M}_\odot}{M_{max}}\right)^2 \epsilon_s \simeq 13.7 \times 10^{15} \left(\frac{\mathrm{M}_\odot}{M_{max}}\right)^2 \mathrm{\ g\ cm}^{-3}.$$

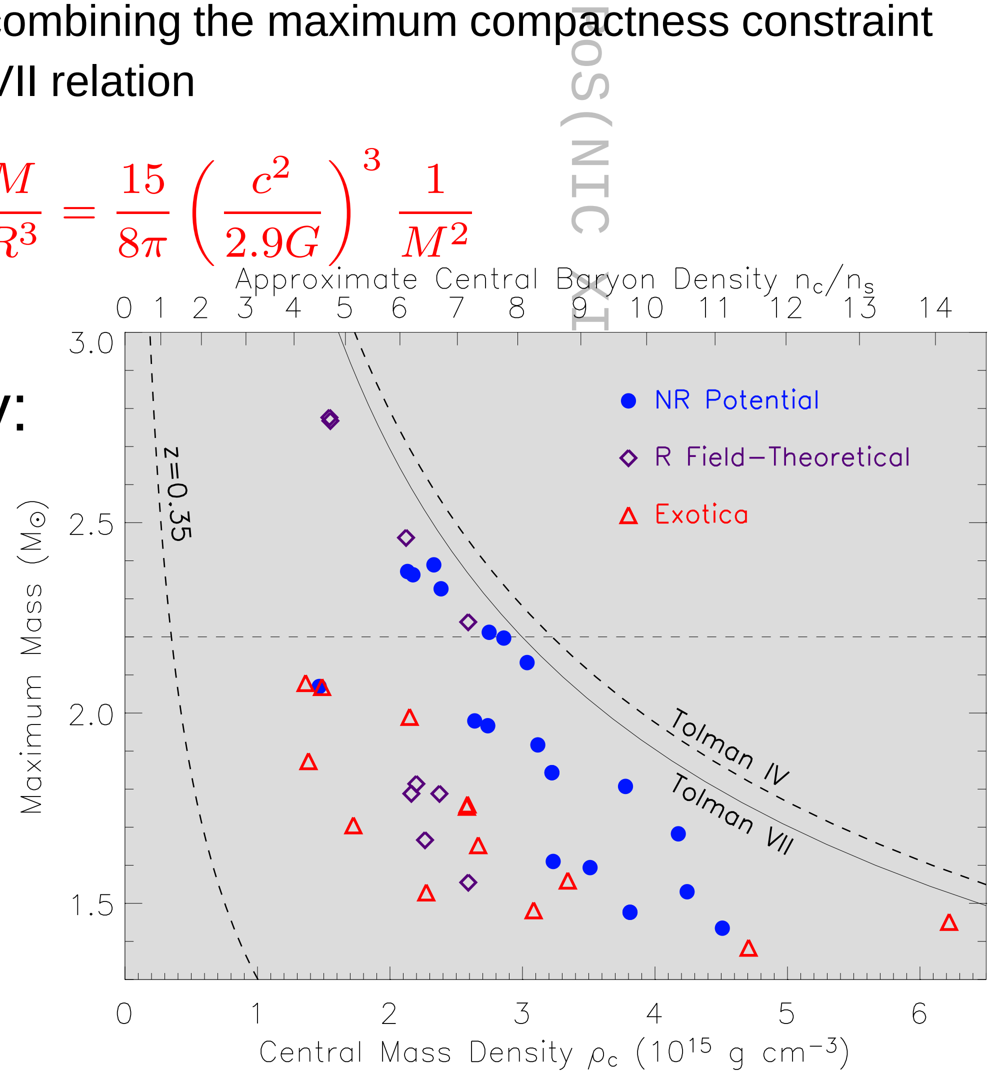
A virtually identical result arises from combining the maximum compactness constraint ($R_{min} \simeq 2.9GM/c^2$) with the Tolman VII relation

$$\epsilon_{c,VII} = \frac{15}{8\pi}\frac{M}{R^3} = \frac{15}{8\pi}\left(\frac{c^2}{2.9G}\right)^3 \frac{1}{M^2}$$

Maximum possible density:

$$2.2\ \mathrm{M}_\odot \Rightarrow$$

$$\epsilon_{max} < 2.8 \times 10^{15} \mathrm{g\ cm}^{-3}$$



Maximum Mass, Minimum Period

Theoretical limits from GR and causality

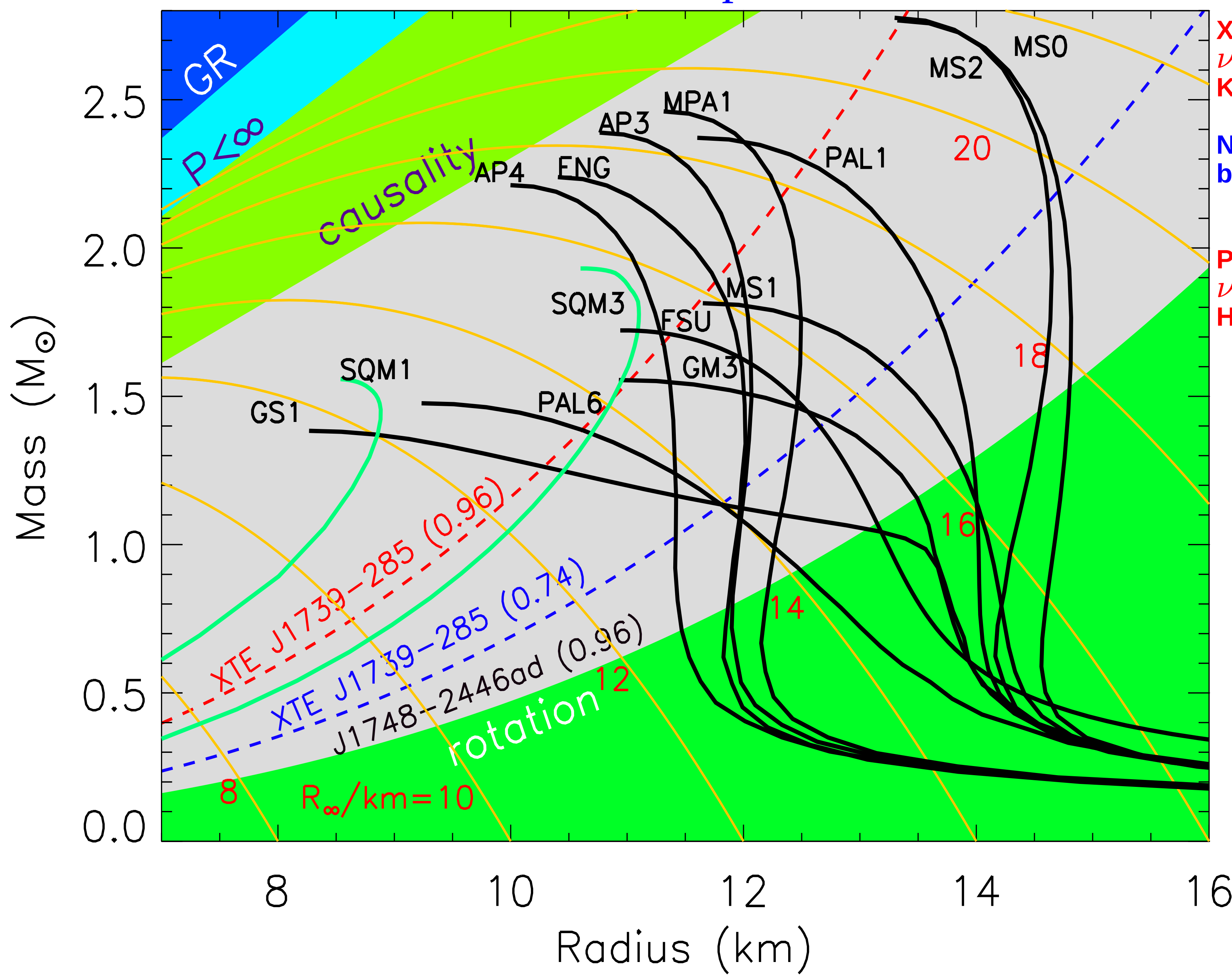
- $M_{max} = 4.2(\epsilon_s/\epsilon_f)^{1/2}\ \mathrm{M}_\odot$ **Rhoades & Ruffini (1974), Hartle (1978)**
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)$ km

  Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)
- $\epsilon_c < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2$ g cm^{-3} **Lattimer & Prakash (2005)**
- $P_{min} \simeq 0.74(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms

  Koranda, Stergioulas & Friedman (1997)
- $P_{min} \simeq 0.96 \pm 0.03(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms

  (empirical) **Lattimer & Prakash (2004)**
- $\epsilon_c > 0.91 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2$ g cm^{-3} (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

Constraints from Pulsar Spins



XTE J1739-285
$\nu = 1122$ Hz
Kaaret et al. 2006

Not confirmed to be a rotation rate

PSR J1748-2446ad
$\nu = 716$ Hz
Hessels et al. 2006

General Relativity

Static spherically symmetric metric ($c = G = 1$):

$$ds^2 = e^{\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) - e^{\nu(r)}dt^2$$

Einstein's equations:

$$\begin{aligned} 8\pi\epsilon(r) &= \frac{1}{r^2}\left(1 - e^{-\lambda(r)}\right) + e^{-\lambda(r)}\frac{\lambda'(r)}{r}, \\ 8\pi p(r) &= -\frac{1}{r^2}\left(1 - e^{-\lambda(r)}\right) + e^{-\lambda(r)}\frac{\nu'(r)}{r}, \\ p'(r) &= -\frac{p(r) + \epsilon(r)}{2}\nu'(r). \end{aligned}$$

Mass:

$$m(r) = 4\pi\int_0^r \epsilon(r')r'^2 dr', \qquad e^{-\lambda(r)} = 1 - 2m(r)/r$$

Boundary conditions:

$$\begin{aligned} r = 0 \qquad & m(0) = p'(0) = \epsilon'(0) = 0, \\ r = R \qquad & m(R) = M, \quad p(R) = 0, e^{\nu(R)} = e^{-\lambda(R)} = 1 - 2M/R \end{aligned}$$

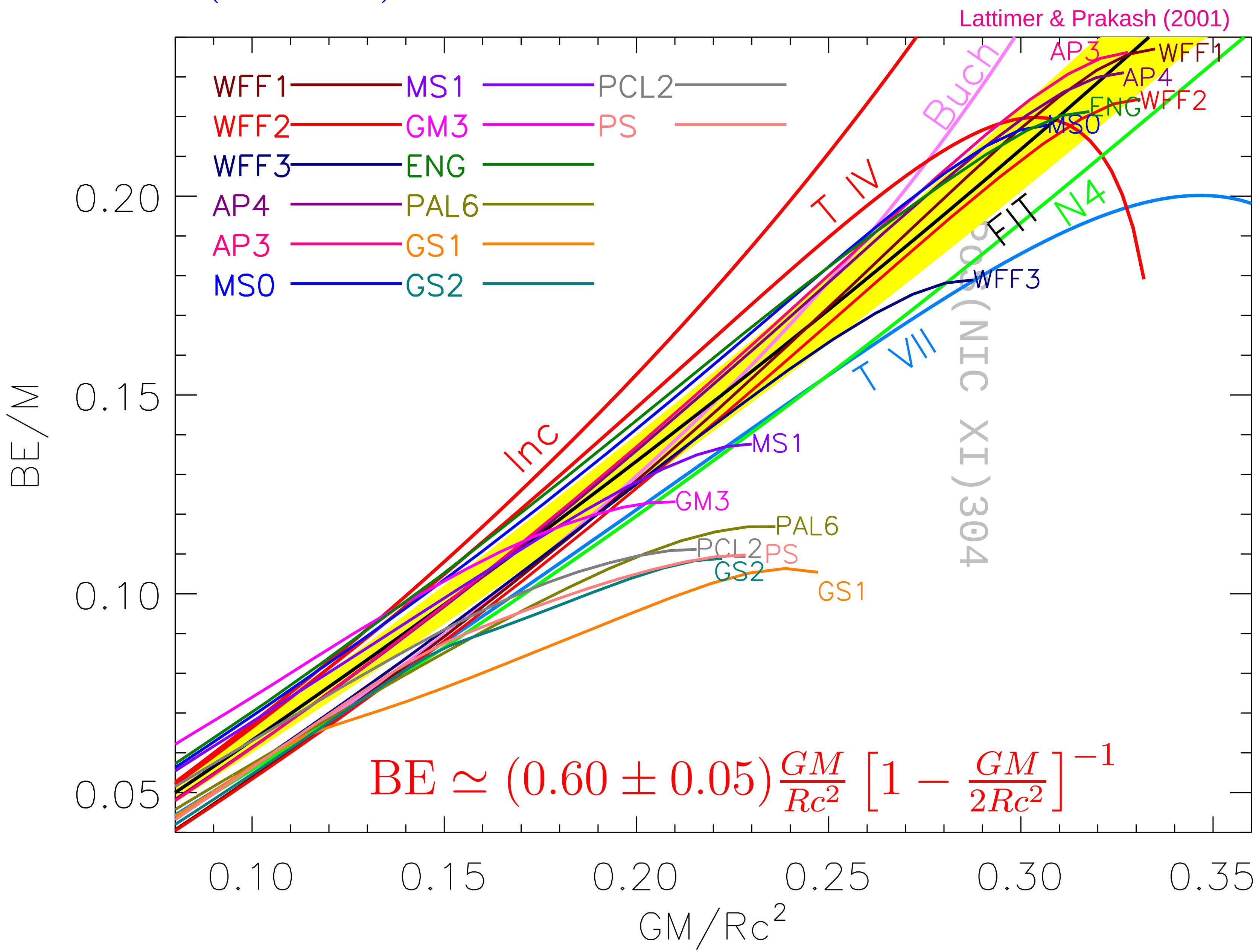
Total baryon number:

$$N = \int_0^R 4\pi r^2 e^{\lambda(r)/2} n(r) dr;$$

Binding energy:

$$\mathrm{BE} = N m_b - M$$

$\mathrm{BE}(M, R)$



Lattimer & Prakash (2001)
WFF1
WFF2
WFF3
AP4
AP3
MS0
MS1
GM3
ENG
PAL6
GS1
GS2
PCL2
PS
Inc
T IV
Buch
FIT
N4
T VII
MS1
GM3
PAL6
PCL2
PS
GS2
GS1
WFF3
MS0
ENG
WFF2
AP4
AP3
WFF1
(NIC XI)304
BE/M
0.05
0.10
0.15
0.20
0.10
0.15
0.20
0.25
0.30
0.35
GM/Rc^2
$\mathrm{BE} \simeq (0.60 \pm 0.05) \frac{GM}{Rc^2} \left[1 - \frac{GM}{2Rc^2}\right]^{-1}$

Moment of Inertia

$$
\begin{aligned}
I &= \frac{8\pi}{3c^4}\int_0^R r^4\left[\epsilon(r)+p(r)\right]e^{(\lambda(r)-\nu(r))/2}\omega(r)dr \\
&= -\frac{2c^2}{3G}\int_0^R r^3\omega(r)\frac{dj(r)}{dr}dr,
\end{aligned}
$$

where

$$
\begin{aligned}
j(r) &= e^{-(\lambda(r)+\nu(r))/2}; \\
\frac{d}{dr}\left[r^4 j(r)\frac{d\omega(r)}{dr}\right] &= -4r^3\omega(r)\frac{dj(r)}{dr}; \\
j(R) &= 1, \qquad \omega(R) = 1-\frac{2GI}{R^3c^2}, \qquad \frac{d\omega(0)}{dr} = 0.
\end{aligned}
$$

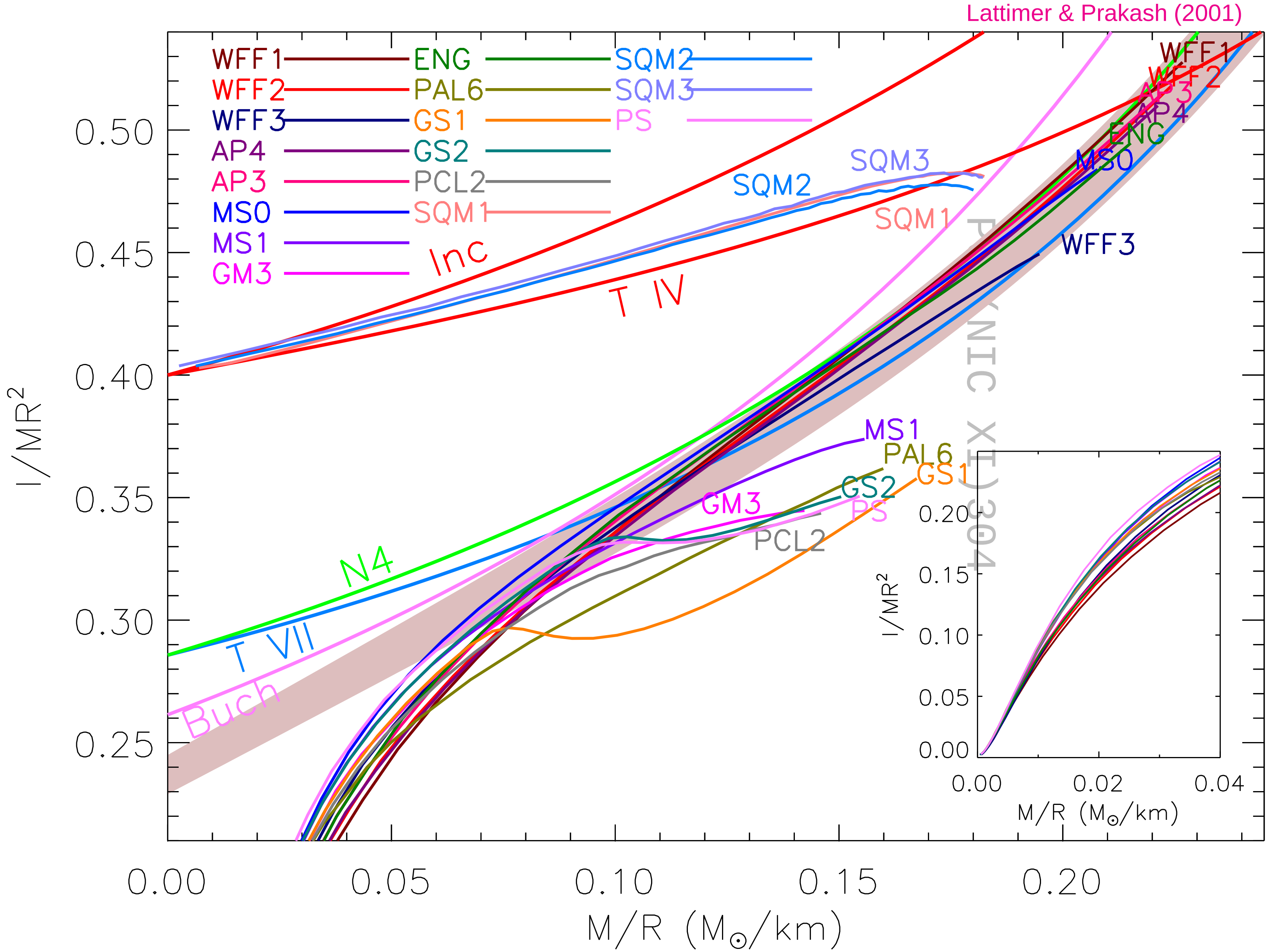
Combining these:

$$
I = \frac{c^2}{6G}R^4\frac{d\omega(R)}{dr}
$$

With $\phi = d\ln\omega/d\ln r$, $\phi(0)=0$,

$$
\begin{aligned}
\frac{d\phi}{dr} &= -\frac{\phi}{r}(3+\phi)-(4+\phi)\frac{d\ln j}{dr}, \\
I &= \frac{\phi_R c^2}{6G}R^3\omega_R = \frac{\phi_R}{6}\left(\frac{R^3c^2}{G}-2I\right) = \frac{R^3\phi_R c^2}{G(6+2\phi_R)}.
\end{aligned}
$$

$I(M,R)$



$$I \simeq (0.237 \pm 0.008) M R^2 \left[1 + 4.2 \frac{M \text{ km}}{R \text{ M}_\odot} + 90 \left(\frac{M \text{ km}}{R \text{ M}_\odot} \right)^4 \right]$$

Polytropes

Polytropic Equation of State: $p = Kn^\gamma$
n is number density, γ is polytropic exponent.
Hydrostatic Equilibrium in Newtonian Gravity:

$$\frac{dp(r)}{dr} = -\frac{Gm(r)n(r)m_b}{r^2}, \qquad \frac{dm(r)}{dr} = 4\pi n m_b r^2$$

Dimensional analysis:

$$M \propto n_c R^3, \quad p \propto \frac{M^2}{R^4}, \quad R \propto K^{1/(3\gamma-4)} M^{(\gamma-2)/(3\gamma-4)}$$

When $\gamma \sim 2$: $$R \propto K^{1/2} M^0 \propto p_f^{1/2} n_f^{-1} M^0$$

General Relativistic analysis using Buchdahl's solution $\epsilon = \sqrt{pp_*} - 5p$:

$$R = (1-\beta)\sqrt{\frac{\pi}{2p_*(1-2\beta)}}, \quad \left.\frac{d\ln R}{d\ln p}\right|_{n,M} = \frac{1}{2}\frac{(1-\beta)(2-\beta)}{(1-3\beta+3\beta^2)}\frac{1-10\sqrt{p/p_*}}{1+2\sqrt{p/p_*}}.$$

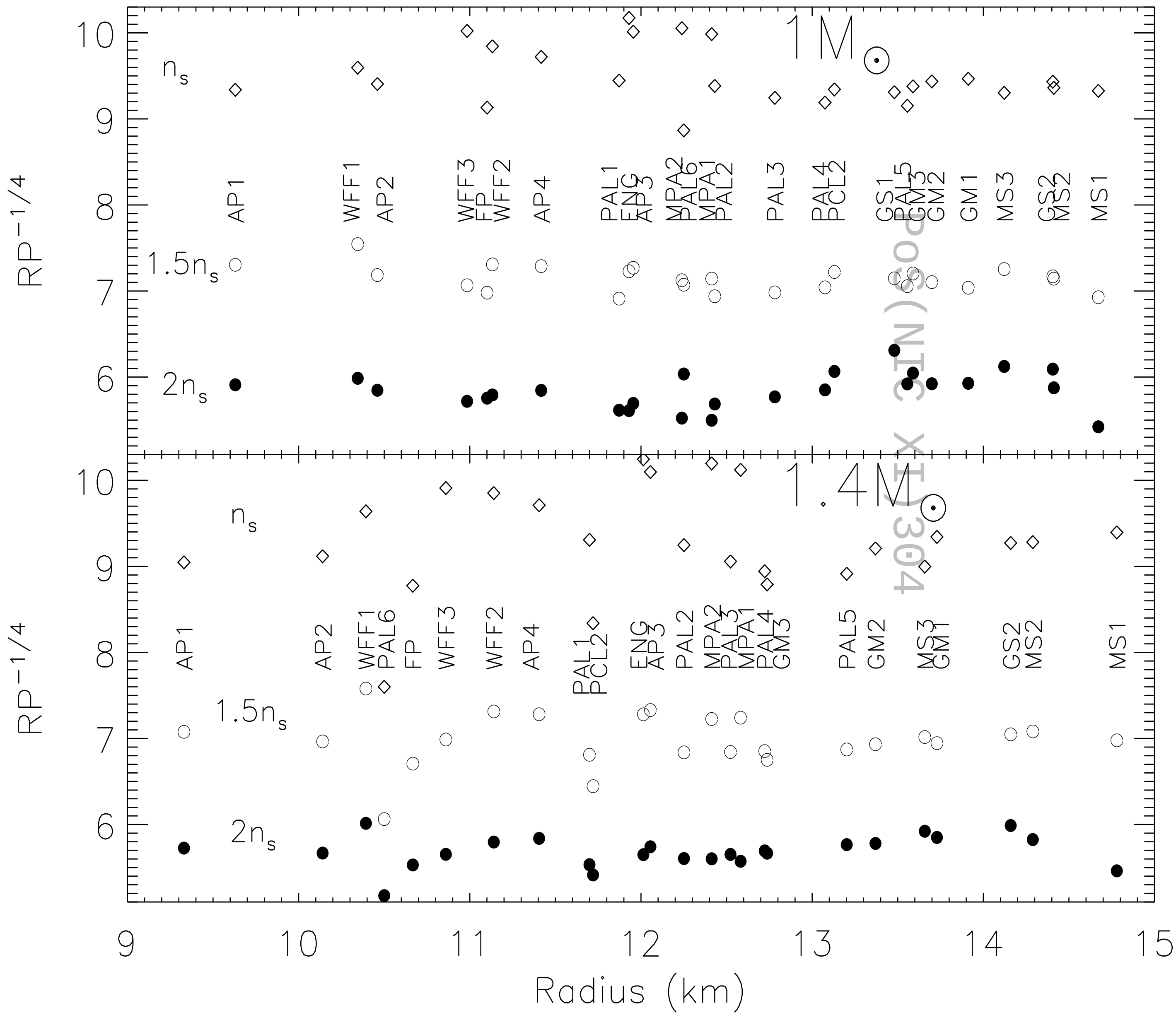
For $M = 1.4\mathrm{M}_\odot$, $R = 14$ km, $n = 1.5n_s$, $\epsilon = 1.5m_b n_s \simeq 3 \times 10^{-4}$ km^{-2}:

$$\beta = 0.148, \quad p_* = 0.00826, \quad p/p_* = 0.00221.$$

$$\left.\frac{d\ln R}{d\ln p}\right|_{n,M} \simeq 0.234$$

The Radius – Pressure Correlation

$R \propto p^{1/4}$



Lattimer & Prakash (2001)

Possible Kinds of Observations

- Maximum and Minimum Mass (binary pulsars)
- Minimum Rotational Period*
- Radiation Radii or Redshifts from X-ray Thermal Emission*
- Crustal Cooling Timescale from X-ray Transients*
- X-ray Bursts from Accreting Neutron Stars*
- Seismology from Giant Flares in SGR's*
- Neutron Star Thermal Evolution (URCA or not)*
- Moments of Inertia from Spin-Orbit Coupling*
- Neutrinos from Proto-Neutron Stars (Binding Energies, Neutrino Opacities, Radii)*
- Pulse Shape Modulations*
- Gravitational Radiation from Neutron Star Mergers* (Masses, Radii from tidal Love numbers)

* Significant dependence on symmetry energy

Potentially Observable Quantities

- Apparent angular diameter from flux and temperature measurements
  $\beta \equiv GM/Rc^2$

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2\beta}} = \sqrt{\frac{F_\infty}{\sigma}}\frac{1}{f_\infty^2 T_\infty^2}$$

- Redshift

$$z = (1-2\beta)^{-1/2} - 1$$

- Eddington flux

$$F_{EDD} = \frac{GMc}{\kappa c^2 D^2}(1-2\beta)^{1/2}$$

- Crust thickness

$$\frac{m_b c^2}{2}\ln\mathcal{H} \equiv h_t = \int_0^{p_t}\frac{dp}{n} = \mu_{n,t} - \mu_{n,t}(p=0)$$

$$\frac{\Delta}{R} \equiv \frac{R-R_t}{R} \simeq \frac{(\mathcal{H}-1)(1-2\beta)}{\mathcal{H}+2\beta-1} \simeq (\mathcal{H}-1)\left(\frac{1}{2\beta}-1\right).$$

- Moment of Inertia

$$I \simeq (0.237 \pm 0.008)MR^2(1+2.84\beta+18.9\beta^4)\ \mathrm{M}_\odot\ \mathrm{km}^2$$

- Crustal Moment of Inertia

$$\frac{\Delta I}{I} \simeq \frac{8\pi}{3}\frac{R^6 p_t}{IMc^2}$$

- Binding Energy

$$\mathrm{B.E.} \simeq (0.60 \pm 0.05)\frac{\beta}{1-\beta/2}$$

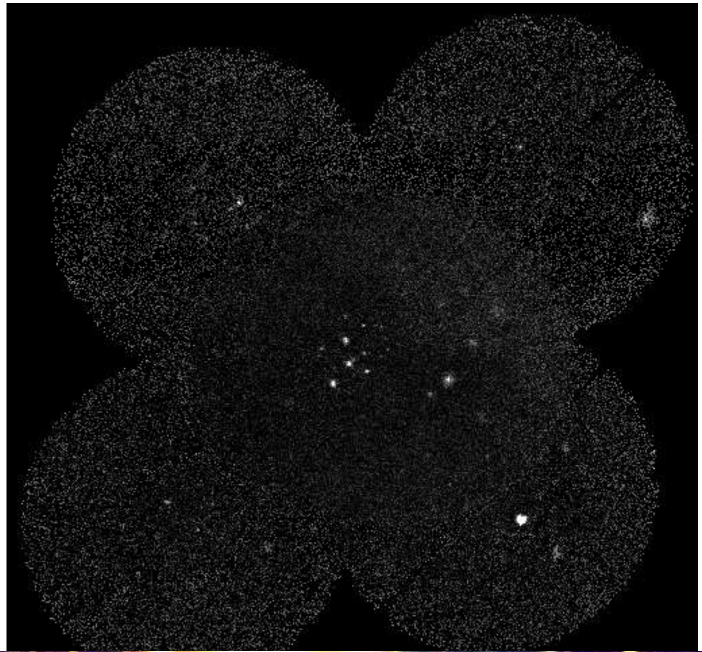
Radiation Radius

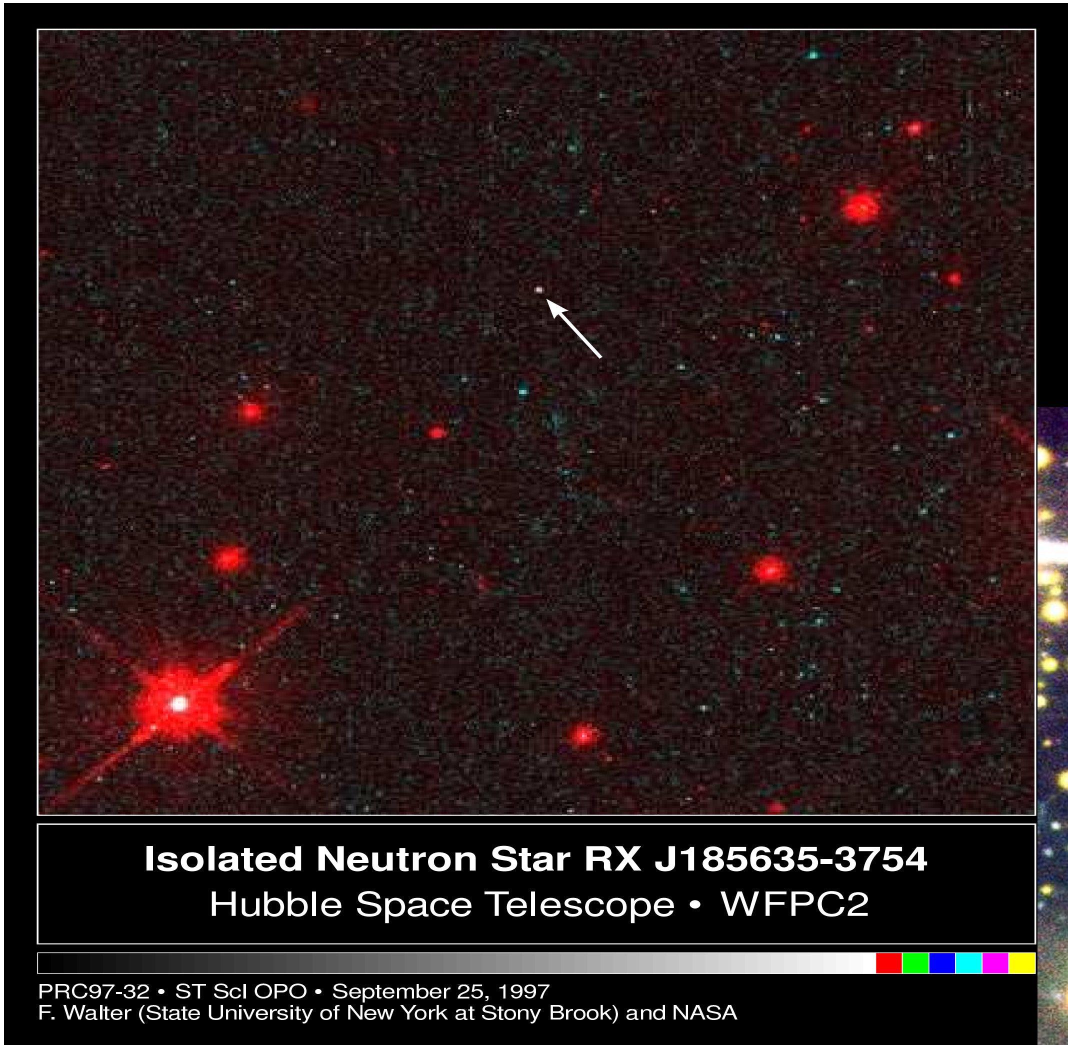
- Combination of flux and temperature measurements yields apparent angular diameter (pseudo-BB):

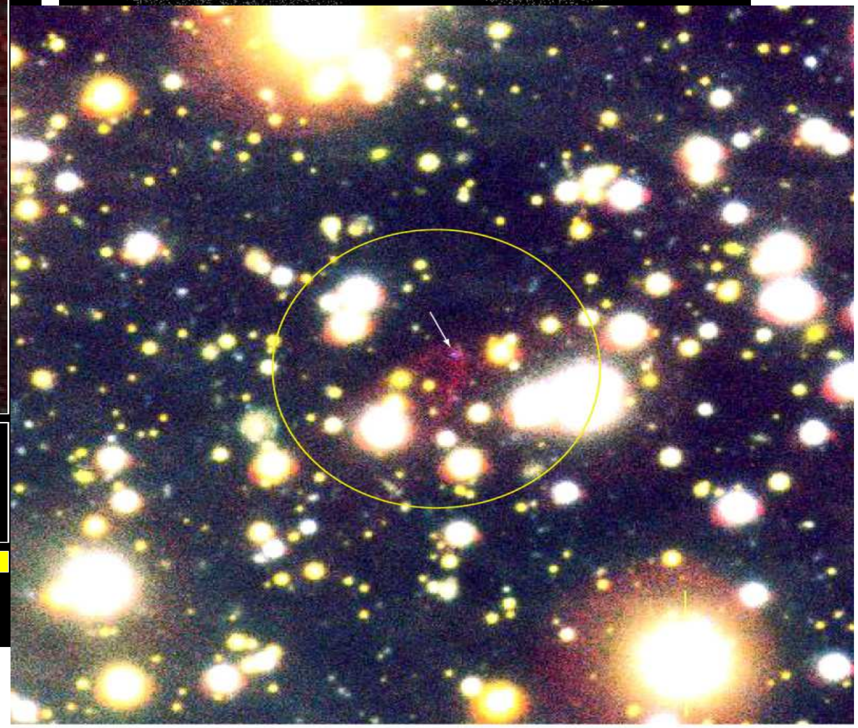
$$\frac{R_\infty}{d} = \frac{R}{d}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

- Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- Best chances for accurate radii are from
  - Nearby isolated neutron stars (parallax measurable)
  - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmospheres)
  - X-ray pulsars in systems of known distance
    - CXOU J010043.1-721134 in the SMC: $R_\infty \geq 10.8$ km (Esposito & Mereghetti 2008)

RX J1856-3754





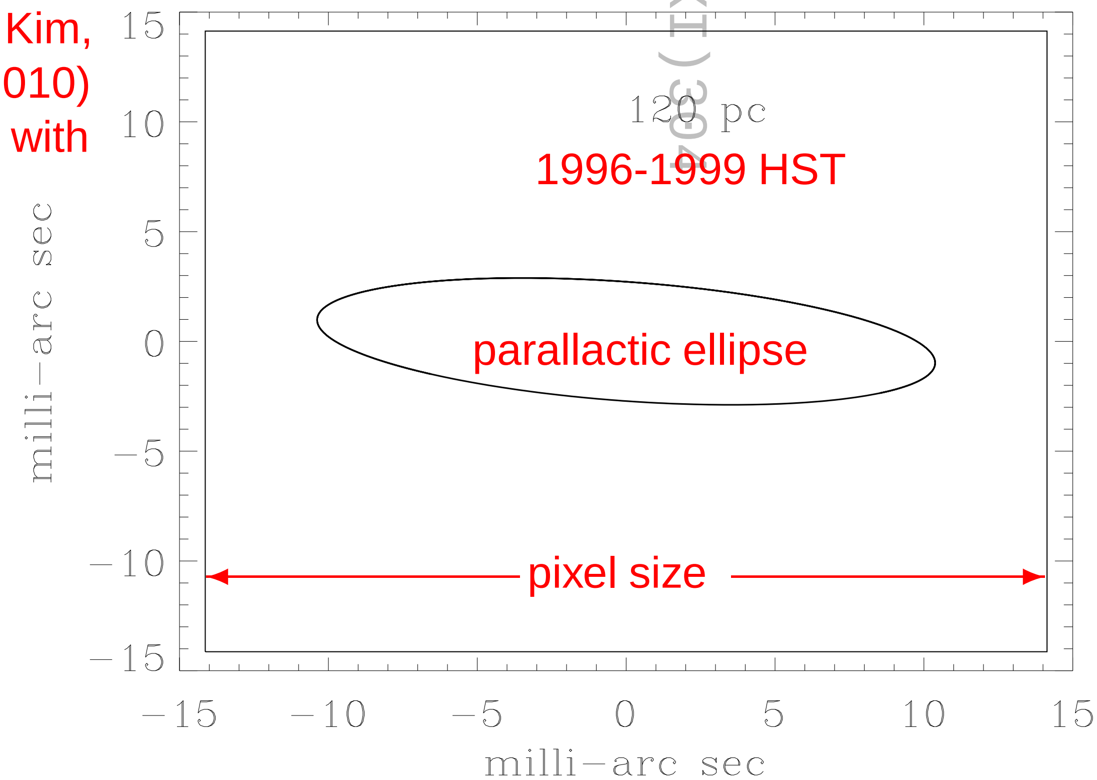


A Bowshock Nebula Near the Neutron Star RX J1856.5-3754 (Detail)
(VLT KUEYEN + FORS2)

ESO PR Photo 23b/00 (11 September 2000)

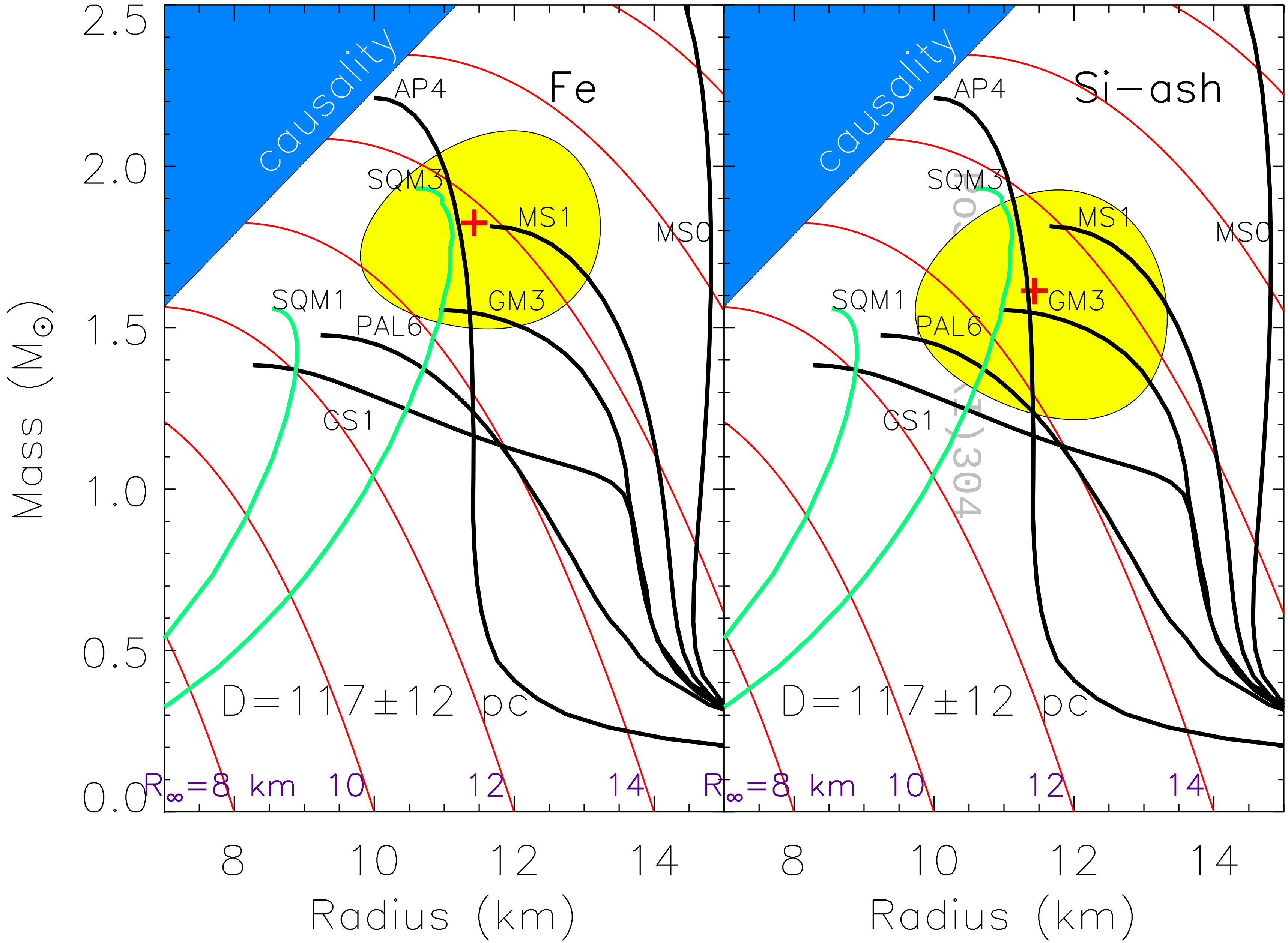
Astrometry of RXJ 1856-3754

- Walter & Lattimer (2002) determined $D = 117 \pm 12$ pc and $v \simeq 190$ km/s from 1996-1999 HST Planetary Camera observations
- Star’s age is probably 0.5 million years
- Kaplan, van Kerkwijk & Anderson (2002): $D = 140 \pm 40$ pc using same data
- van Kerkwijk & Kaplan (2007, conference proceeding) revised this to $D = 161 \pm 16$ pc based on 2002-2004 High-Resolution Camera of the Advanced Camera for Surveys HST observations (double the resolution)
- Walter, Eisenbeiβ, Lattimer, Kim, Hambaryan & Neuhäuser (2010) determined $D \simeq 115 \pm 8$ pc with 2002-2004 HST data

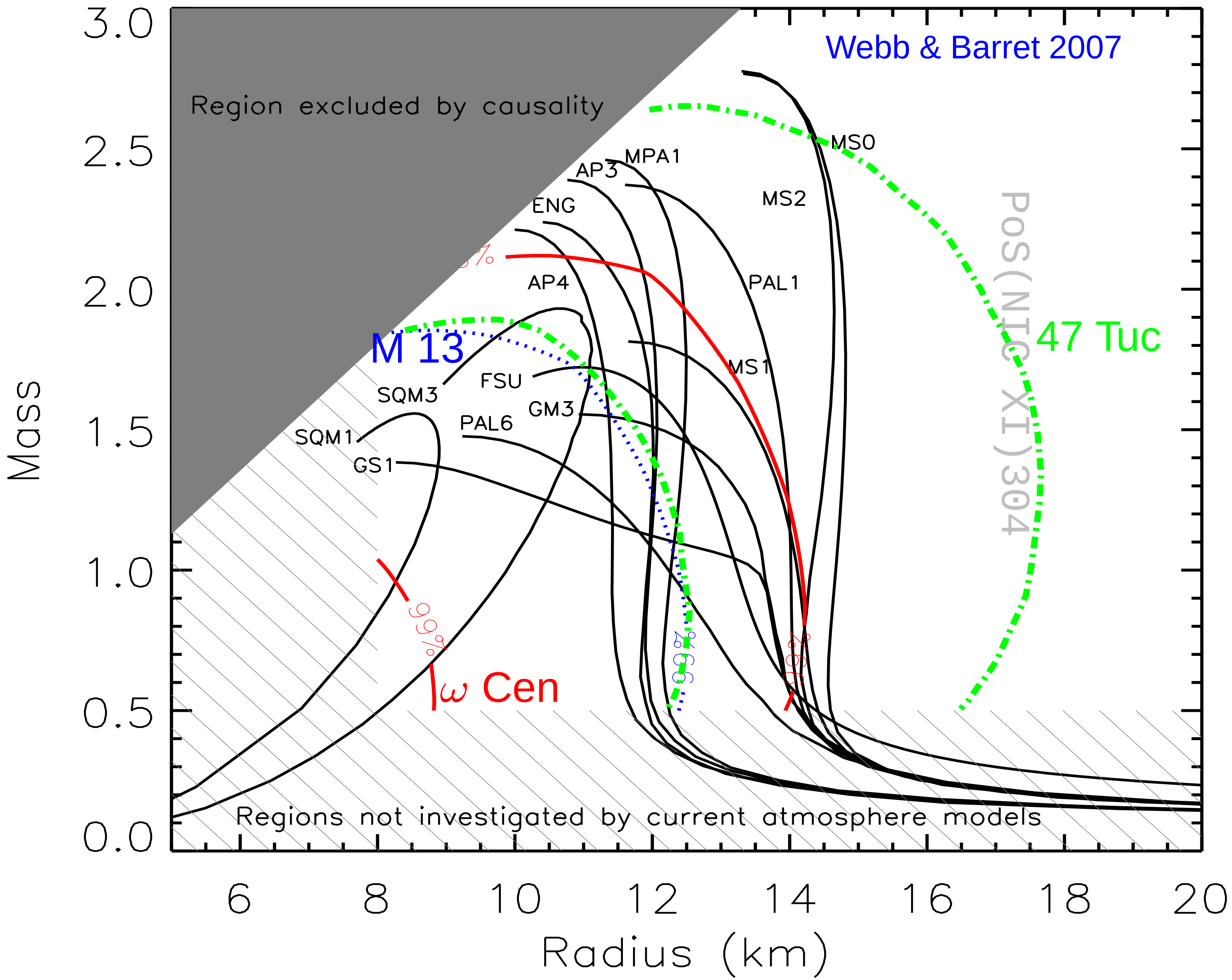


RX J1856-3754

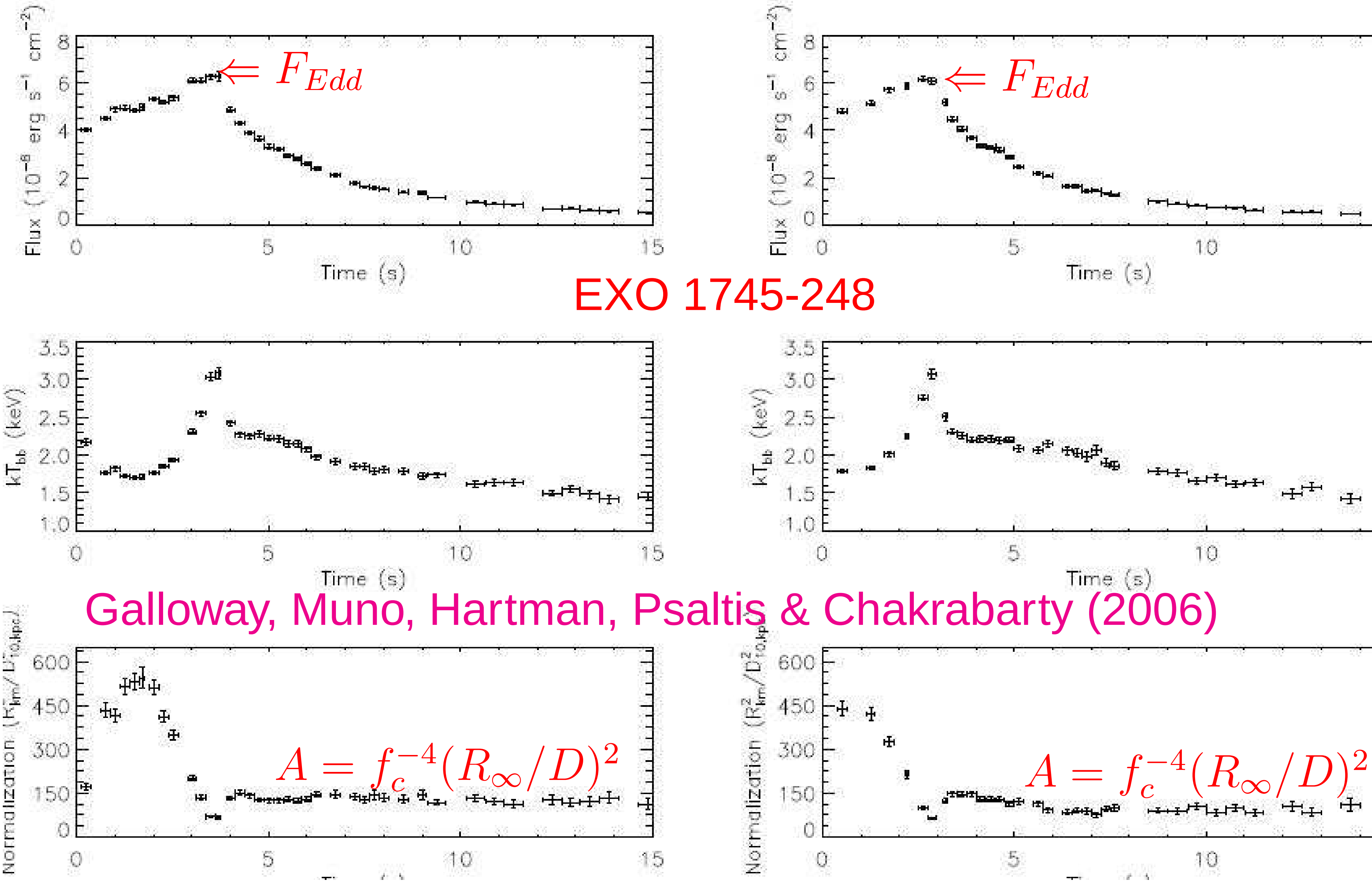
Walter & Lattimer (2002)



Radiation Radius: Globular Cluster Sources



Photospheric Radius Expansion (Type I) X-Ray Bursts



EXO 1745-248

Galloway, Muno, Hartman, Psaltis & Chakrabarty (2006)

Systematics with $R_{ph} = R$

$$F_{Edd} = \frac{GMc}{\kappa D^2}\sqrt{1 - 2\frac{GM}{R_{ph}c^2}} = \frac{GMc}{\kappa D^2}\sqrt{1-2\beta}$$

$$\kappa \simeq 0.2(1+X)\ \mathrm{cm}^2\mathrm{g}^{-1}$$

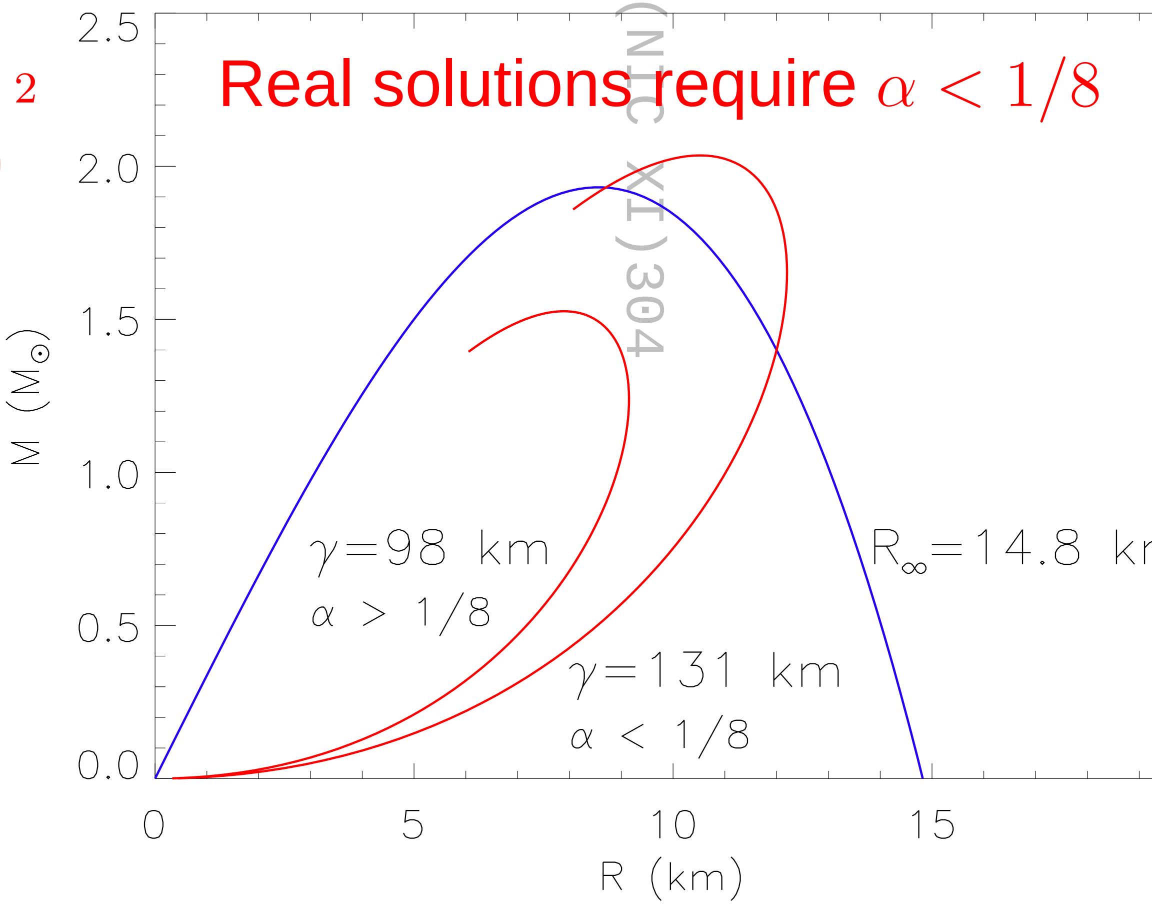
$$A = \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4}\left(\frac{R_\infty}{D}\right)^2$$

$$\alpha = \frac{F_{Edd}}{\sqrt{A}}\frac{\kappa D}{c^3 f_c^2} = \beta(1-2\beta)$$

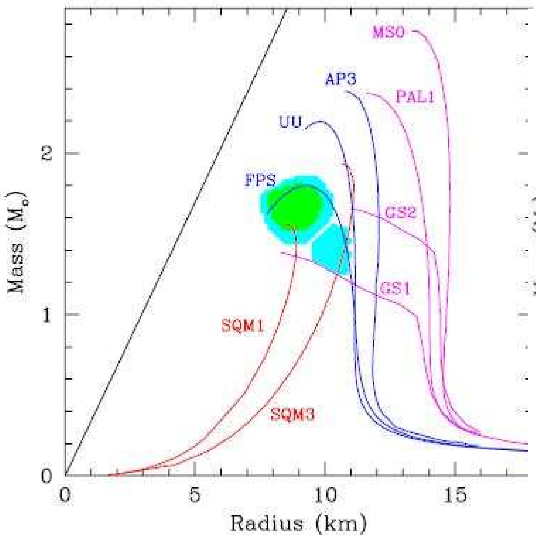
$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd}\kappa} = \frac{R}{\beta(1-2\beta)^{3/2}}$$

$$\beta = \frac{1}{4} \pm \frac{1}{4}\sqrt{1-8\alpha}$$

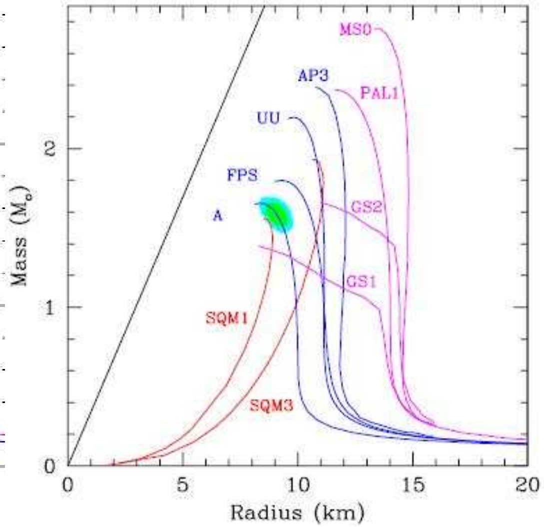
$$R_\infty = \alpha\gamma$$



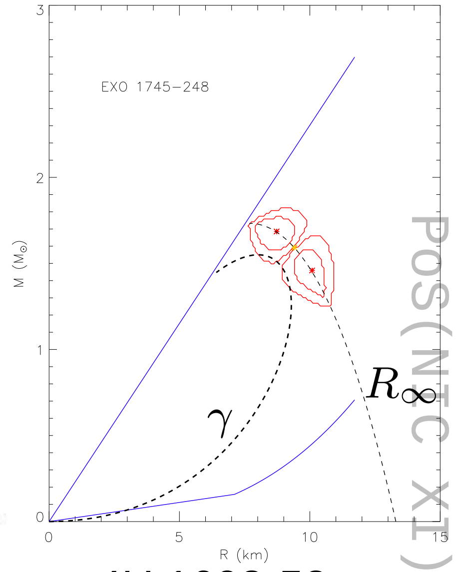
EXO 1745-248
$\alpha = 0.14 \pm 0.02$



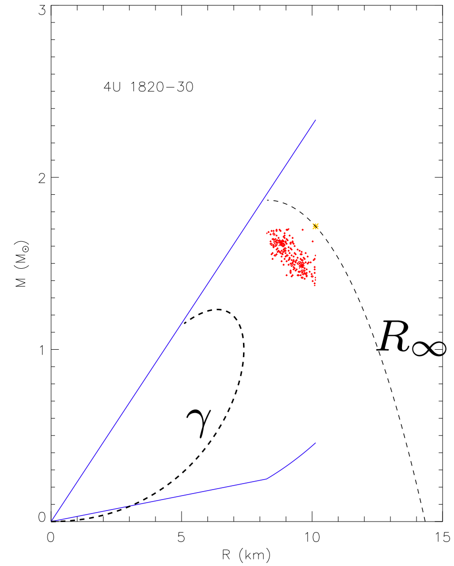
4U 1820-30
$\alpha = 0.18 \pm 0.02$



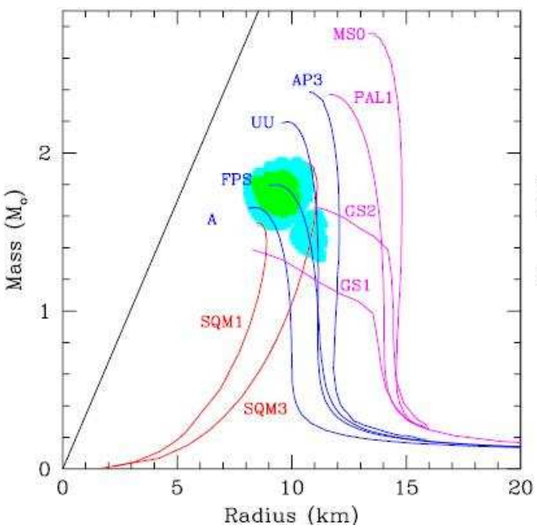
EXO 1745-248
$\alpha = 0.14 \pm 0.02$



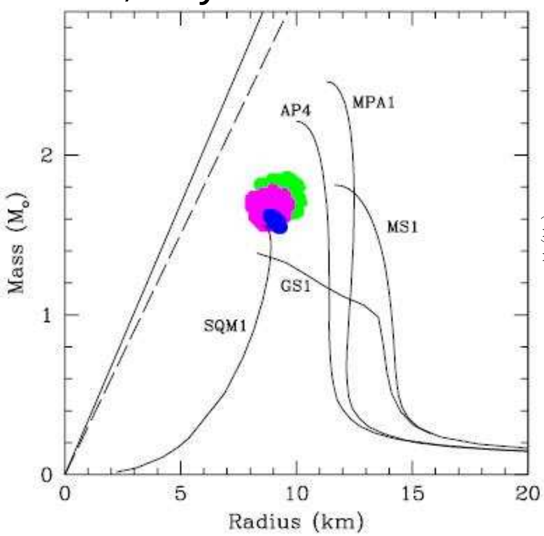
4U 1820-30
$\alpha = 0.18 \pm 0.02$



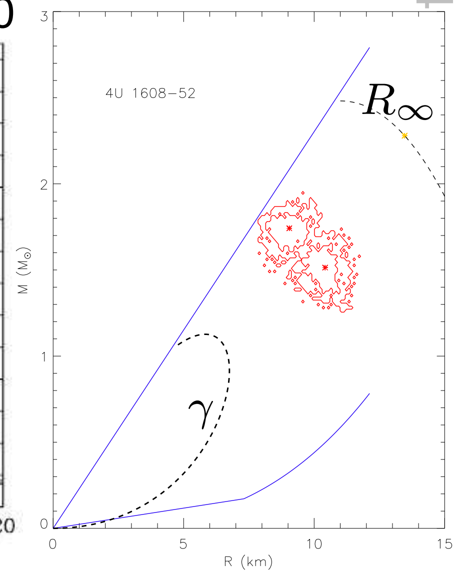
4U 1608-52
$\alpha = 0.26 \pm 0.11$



Ozel, Baym & Güver 2010



4U 1608-52
$\alpha = 0.26 \pm 0.11$



Systematics with $R_{ph} >> R$

$$F_{Edd} = \frac{GMc}{\kappa D^2}\sqrt{1 - 2\frac{GM}{R_{ph}c^2}} \simeq \frac{GMc}{\kappa D^2}$$

$$\kappa \simeq 0.2(1+X)\ \mathrm{cm}^2\mathrm{g}^{-1}$$

$$A = \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4}\left(\frac{R_\infty}{D}\right)^2$$

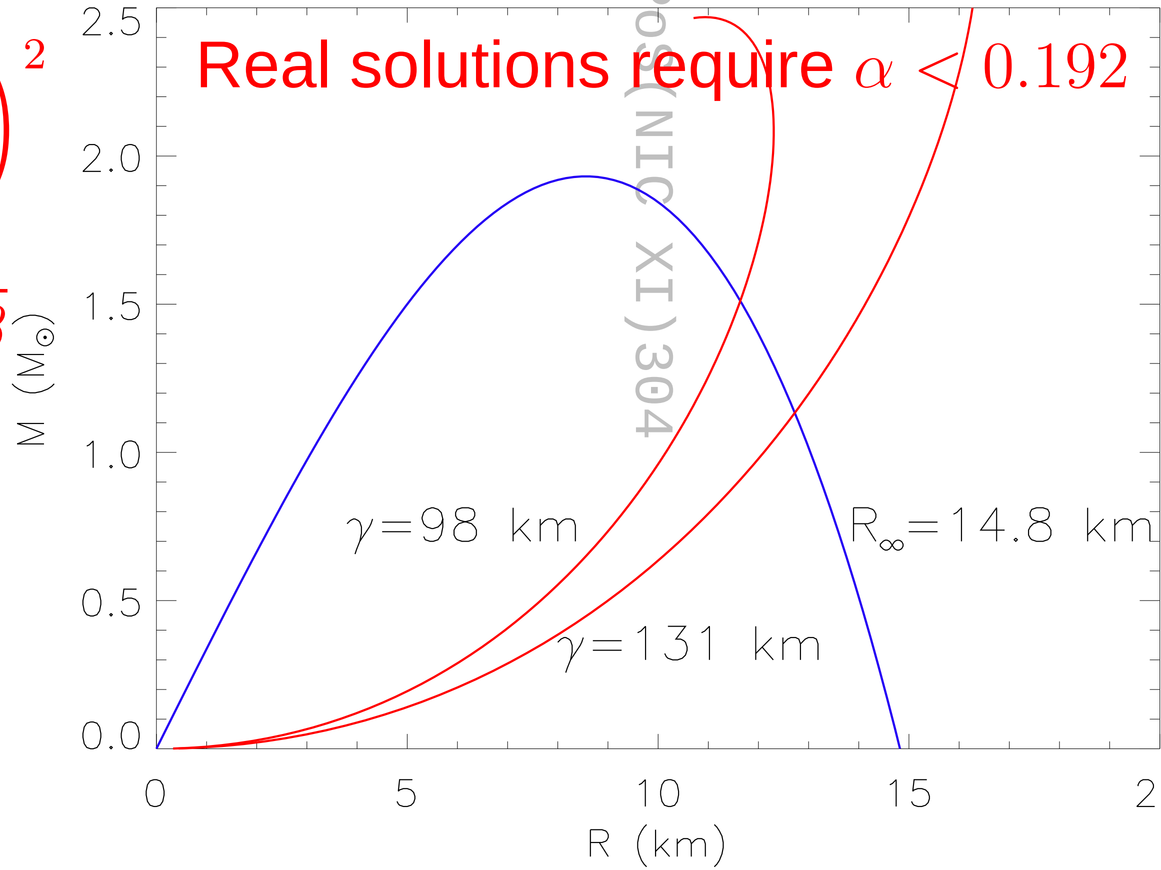
$$\alpha = \frac{F_{Edd}}{\sqrt{A}}\frac{\kappa D}{c^3 f_c^2} = \beta\sqrt{1-2\beta}$$

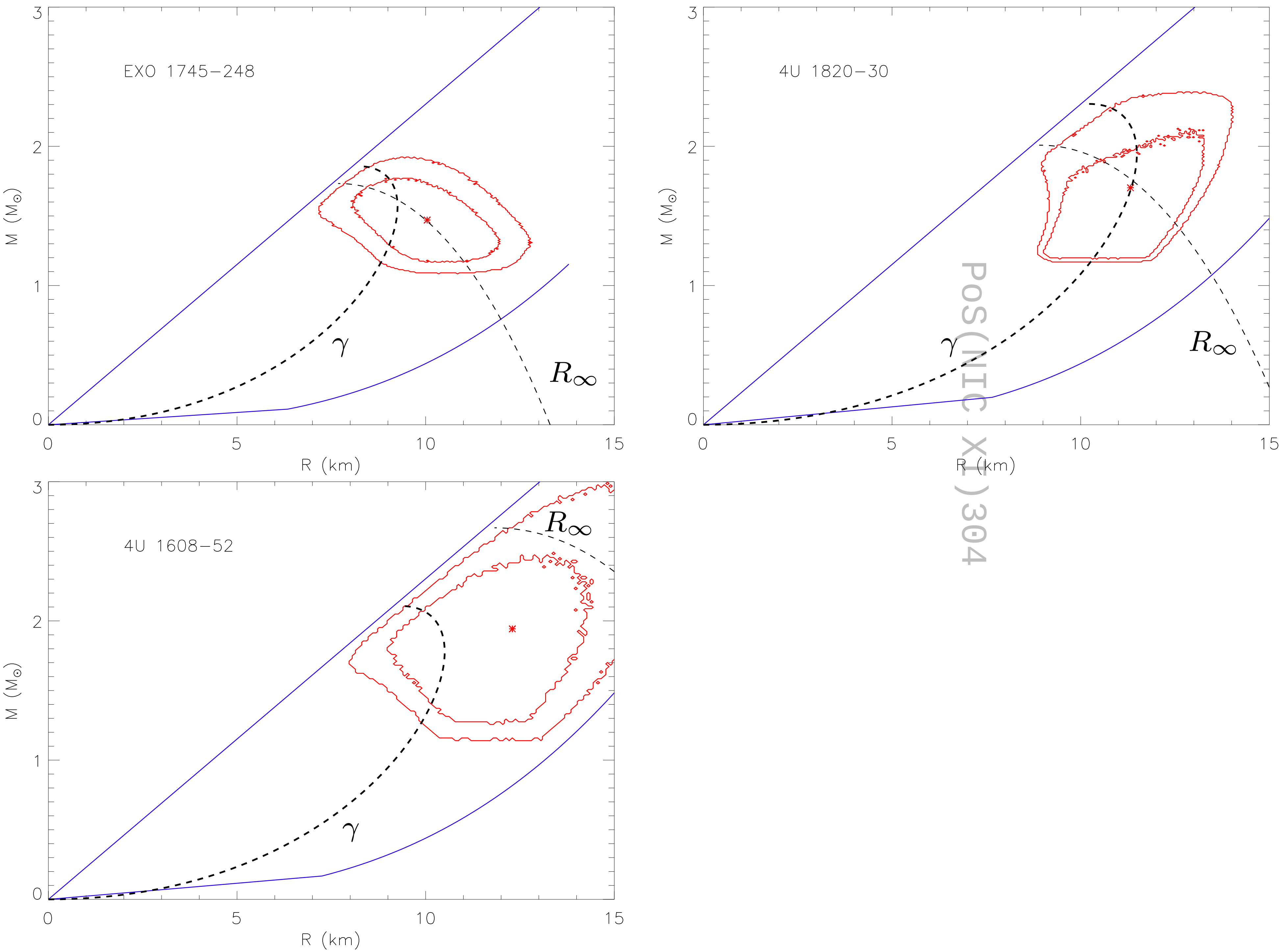
$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd}\kappa} = \frac{R}{\beta(1-2\beta)}$$

$$\beta = \frac{1}{6}\left[1 + \sqrt{3}\sin\theta - \cos\theta\right]$$

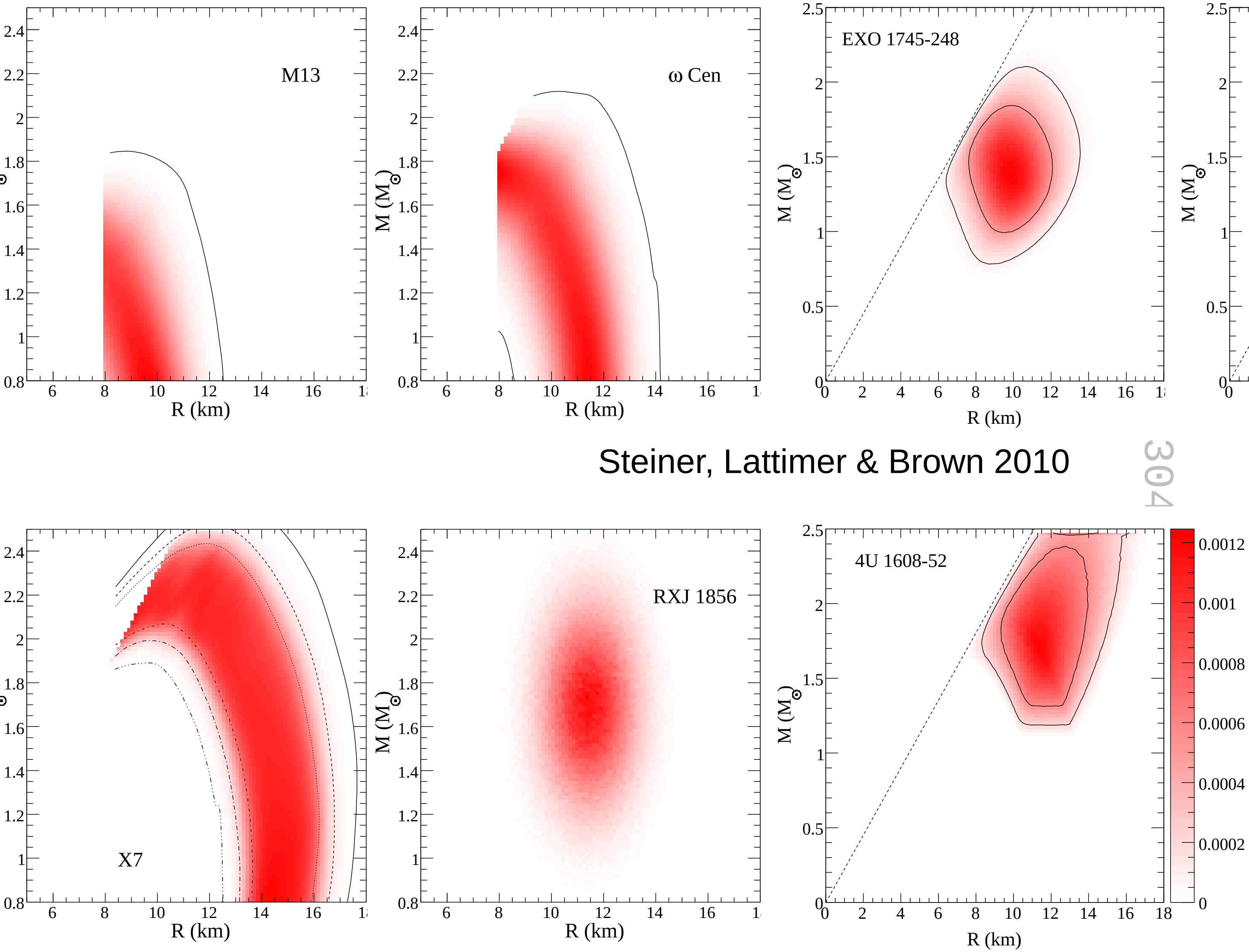
$$\theta = \frac{1}{3}\cos^{-1}(1 - 54\alpha^2)$$

$$R_\infty = \alpha\gamma$$




EXO 1745−248
4U 1820−30
4U 1608−52
M (M$_\odot$)
R (km)
γ
R_∞
0
1
2
3
5
10
15

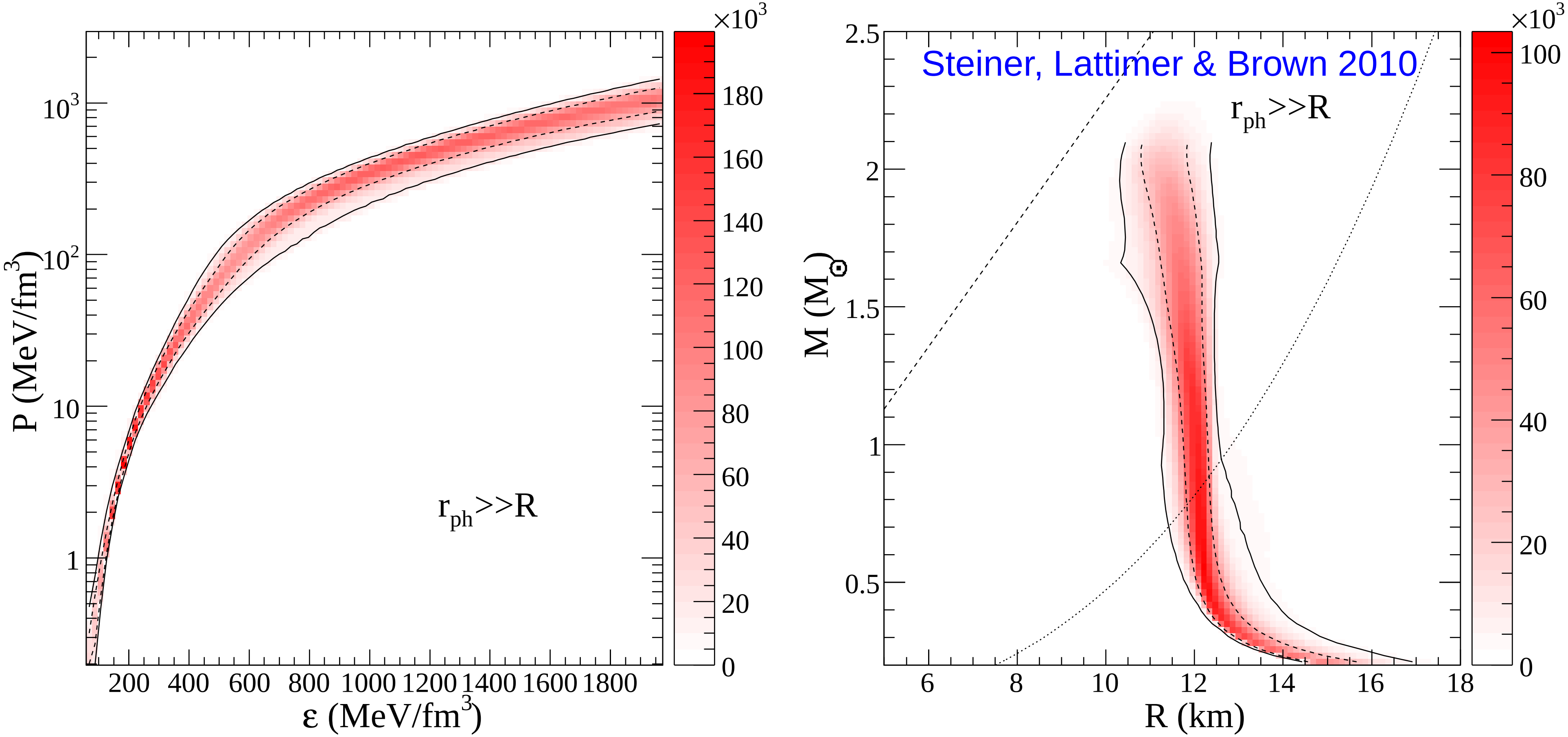
M-R Probability Distributions



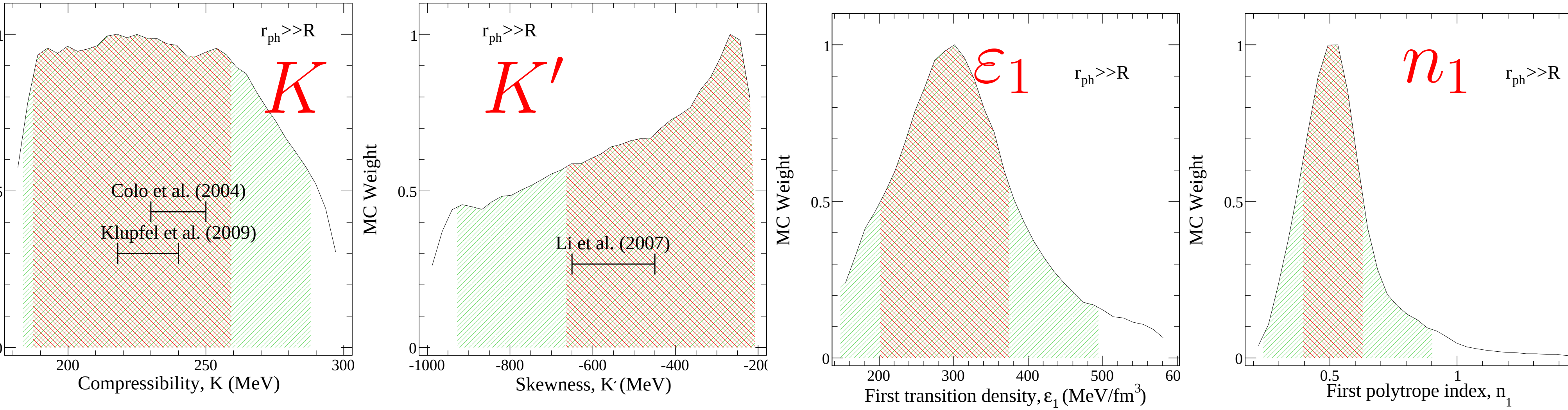
Steiner, Lattimer & Brown 2010

Bayesian TOV Inversion

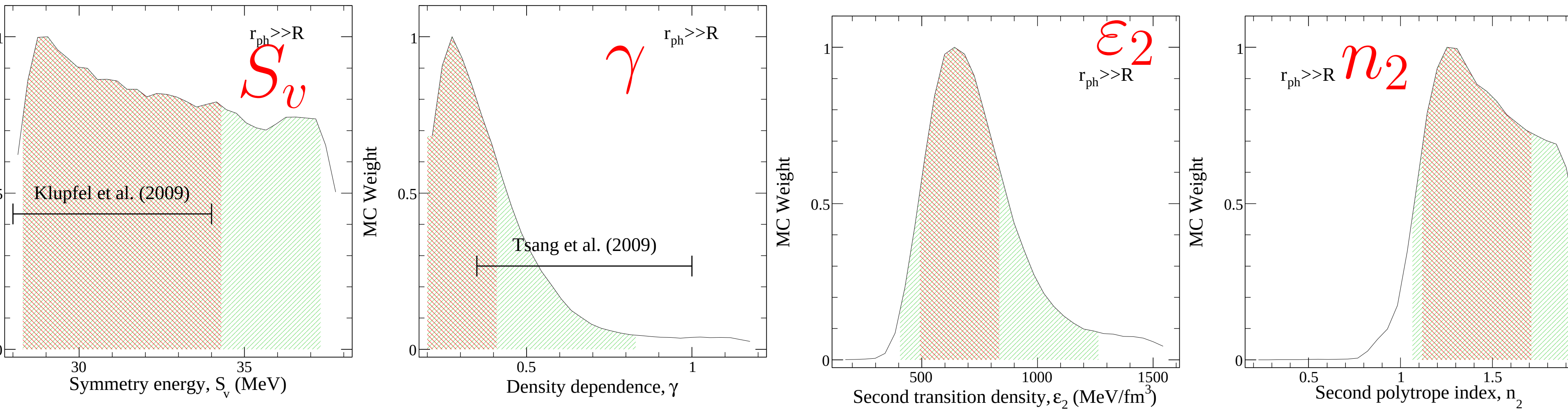
- $\varepsilon < 0.5\varepsilon_0$: EOS from BBP and NV
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- $\varepsilon_1 < \varepsilon < \varepsilon_2$: EOS is polytrope with n_1; $\varepsilon > \varepsilon_2$: EOS is polytrope with n_2
- A-priori EOS parameters ($K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$) uniformly distributed
- M and R probability distributions for 7 neutron stars treated equally ($0.8\ \mathrm{M}_\odot < M < 2.5\ \mathrm{M}_\odot$; $5\ \mathrm{km} < R < 18\ \mathrm{km}$)



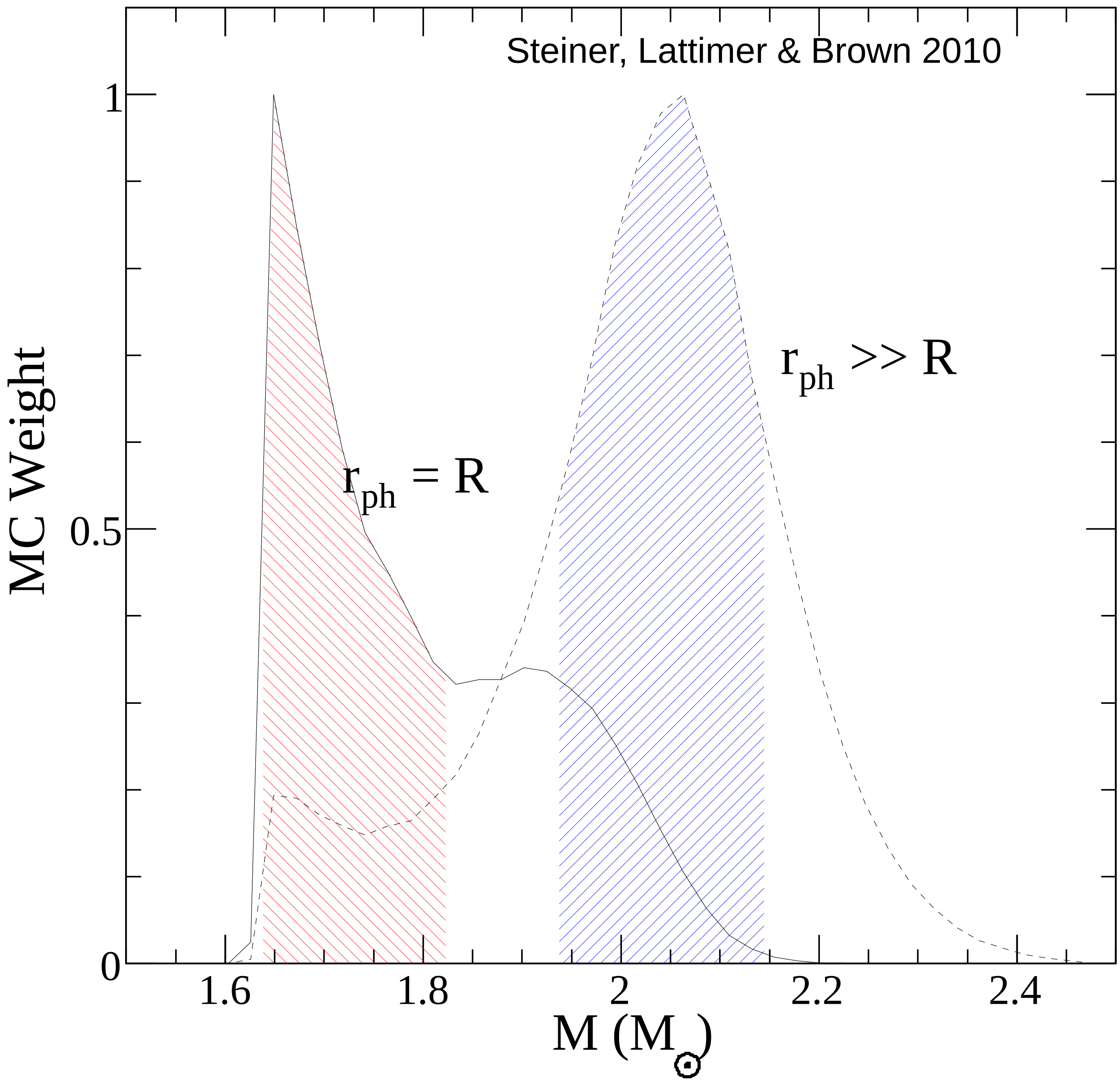
Inferred Model EOS Parameters



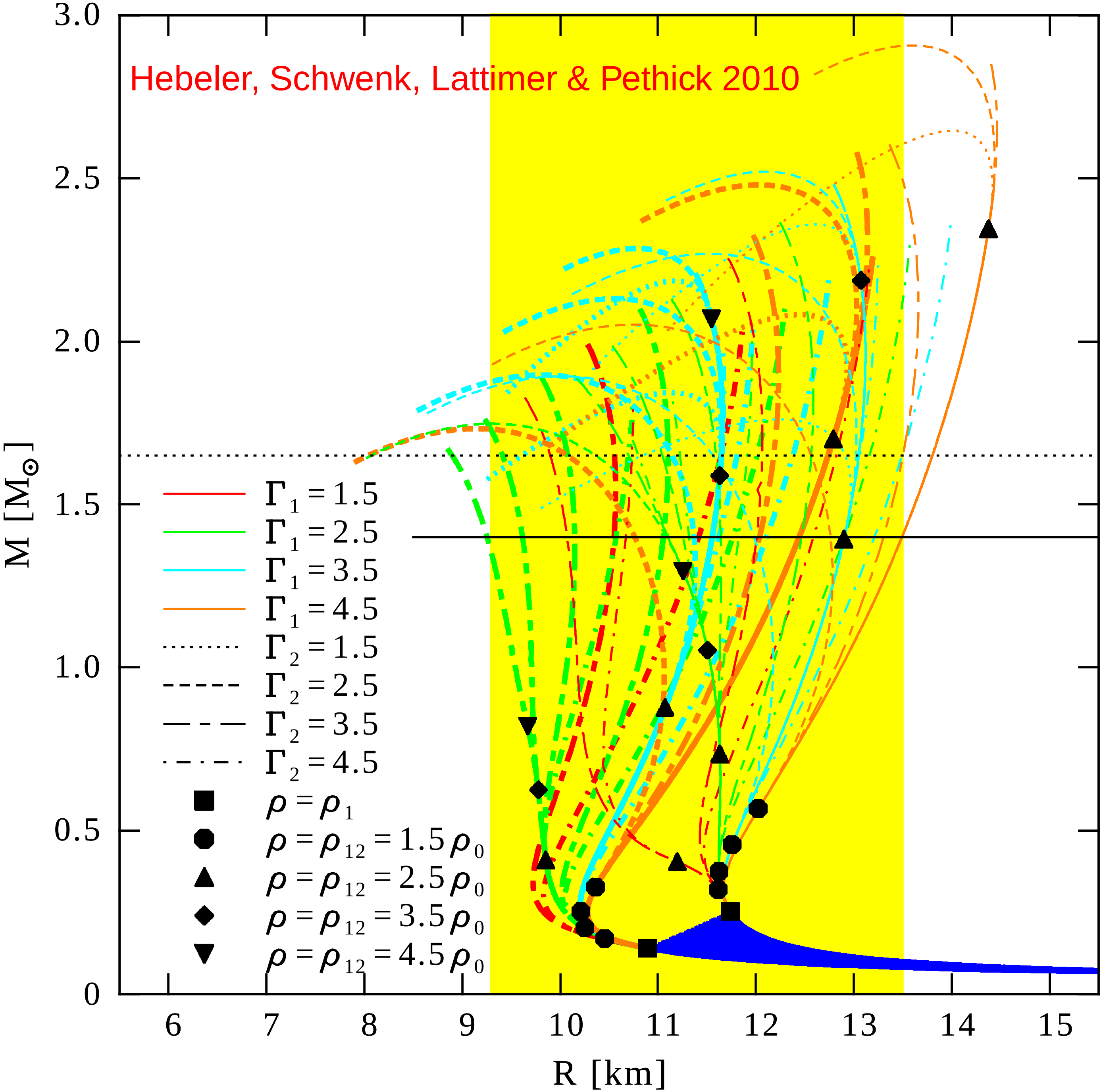
Steiner, Lattimer & Brown 2010



Maximum Mass Probability Distributions



Neutron Matter $M - R$ Implications



$P_\beta \simeq 2.1 \pm 0.5$ MeV fm^{-3}

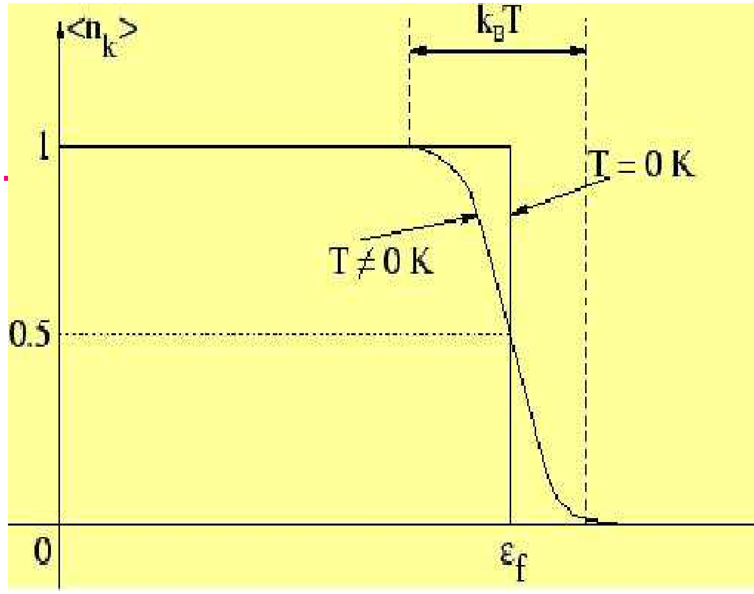
$R_{1.4} \simeq 11.2 \pm 1.5$ km

Neutron Star Cooling

Gamow & Schönberg proposed the direct Urca process: nucleons at the top of the Fermi sea beta decay. c

$$n \rightarrow p + e^- + \nu_e\,, \qquad p \rightarrow n + e^+ + \bar{\nu}_e$$

Energy conservation guaranteed by beta equilibrium
$\mu_n - \mu_p = \mu_e$
Momentum conservation requires $|k_{Fn}| \leq |k_{Fp}| + |k_{Fe}|$.
Charge neutrality requires $k_{Fp} = k_{Fe}$,
therefore $|k_{Fp}| \geq 2|k_{Fn}|$.
Degeneracy implies $n_i \propto k_{Fi}^3$, thus $x \geq x_{DU} = 1/9$.
With muons $(n > 2n_s)$, $x_{DU} = \frac{2}{2+(1+2^{1/3})^3} \simeq 0.148$
If $x < x_{DU}$, bystander nucleons needed:
modified Urca process is then dominant.



$$(n,p) + n \rightarrow (n,p) + p + e^- + \nu_e, \qquad (n,p) + p \rightarrow (n,p) + n + e^+ + \bar{\nu}_e$$

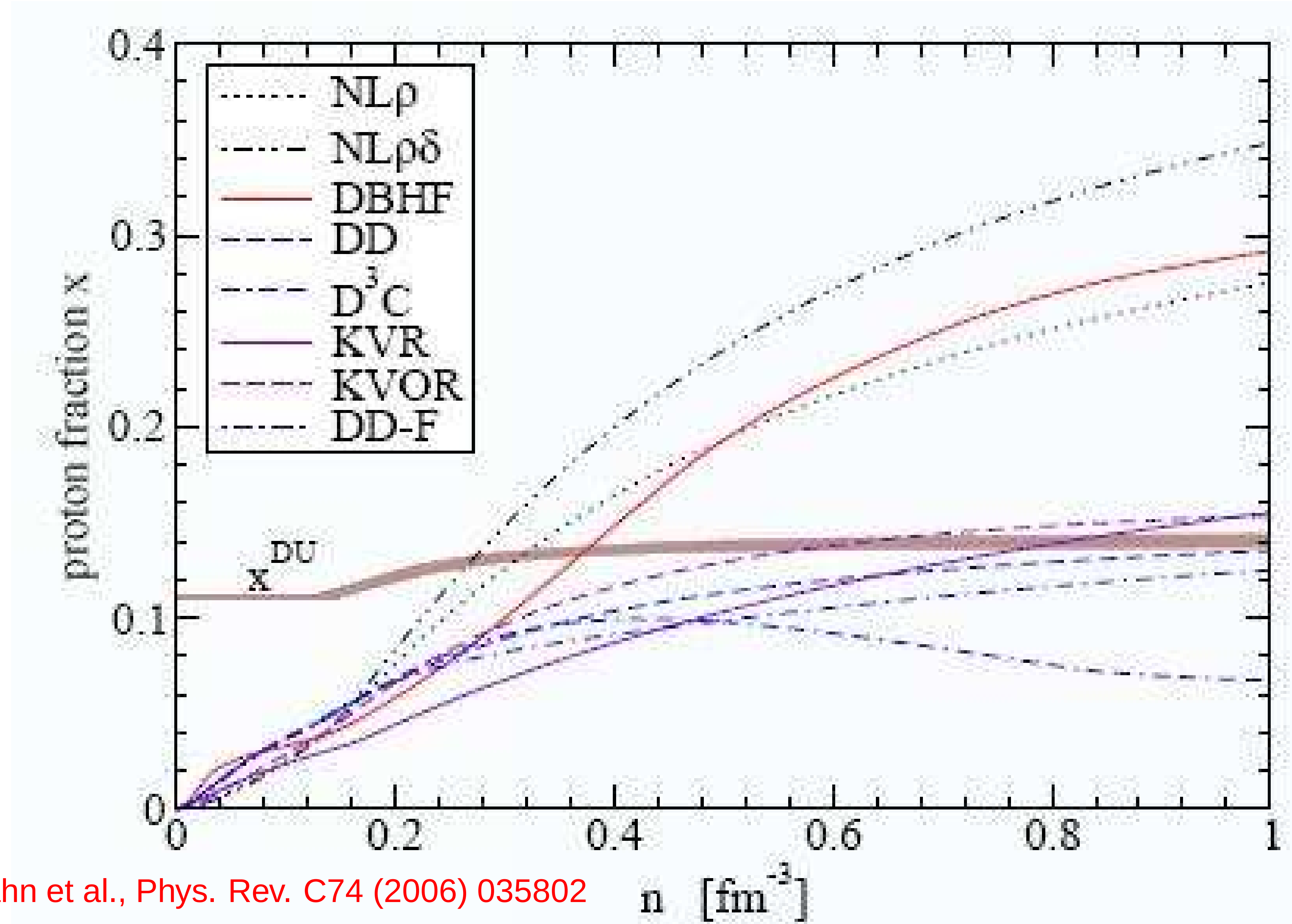
Neutrino emissivities:

$$\dot{\epsilon}_{MURCA} \simeq \left(\frac{T}{\mu_n}\right)^2 \dot{\epsilon}_{DURCA}\,.$$

Beta equilibrium composition:

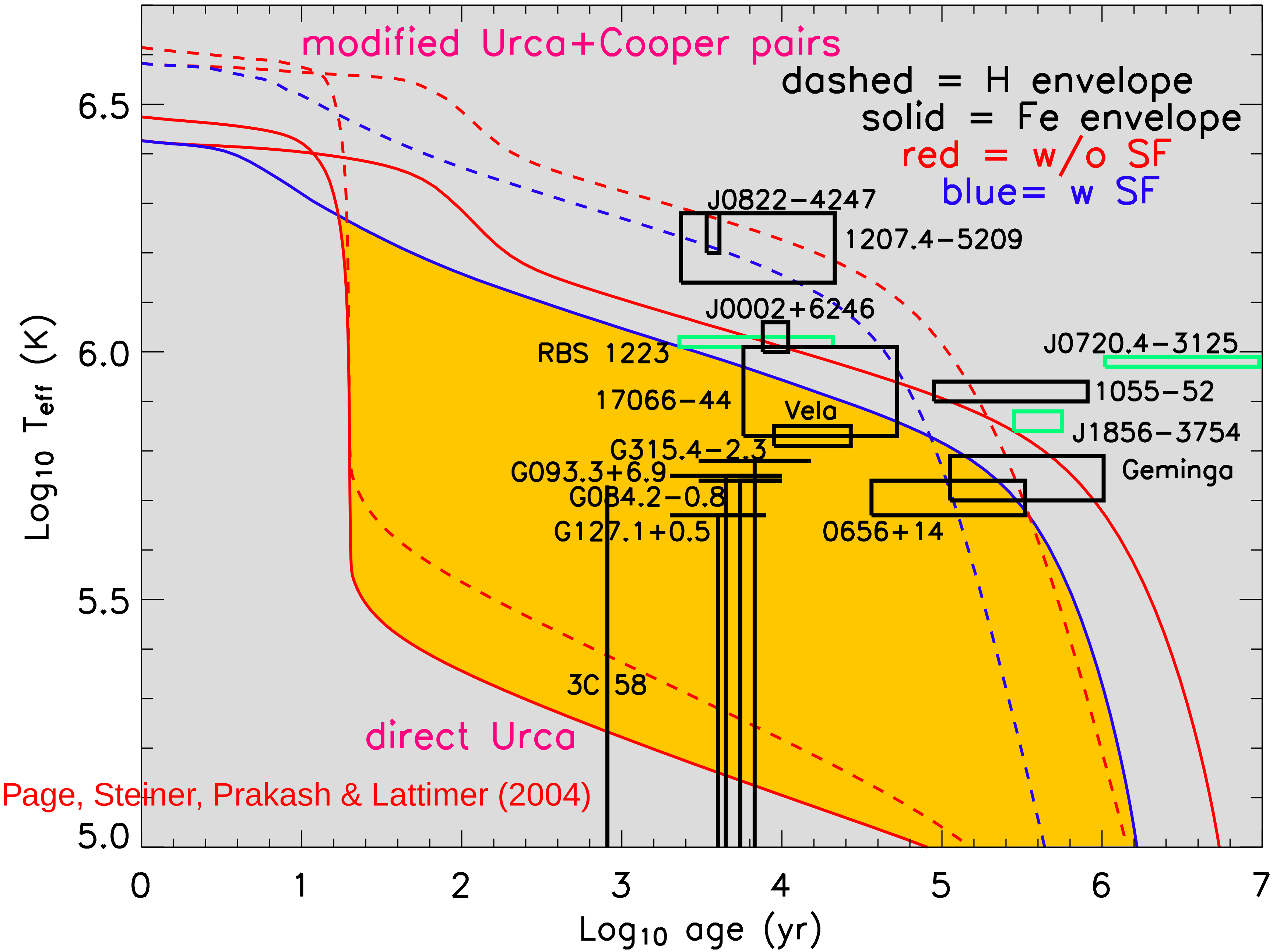
$$x_\beta \simeq (3\pi^2 n)^{-1}\left(\frac{4E_{sym}}{\hbar c}\right)^3 \simeq 0.04\left(\frac{n}{n_s}\right)^{0.5-2}\,.$$

Direct Urca Threshold



Klähn et al., Phys. Rev. C74 (2006) 035802

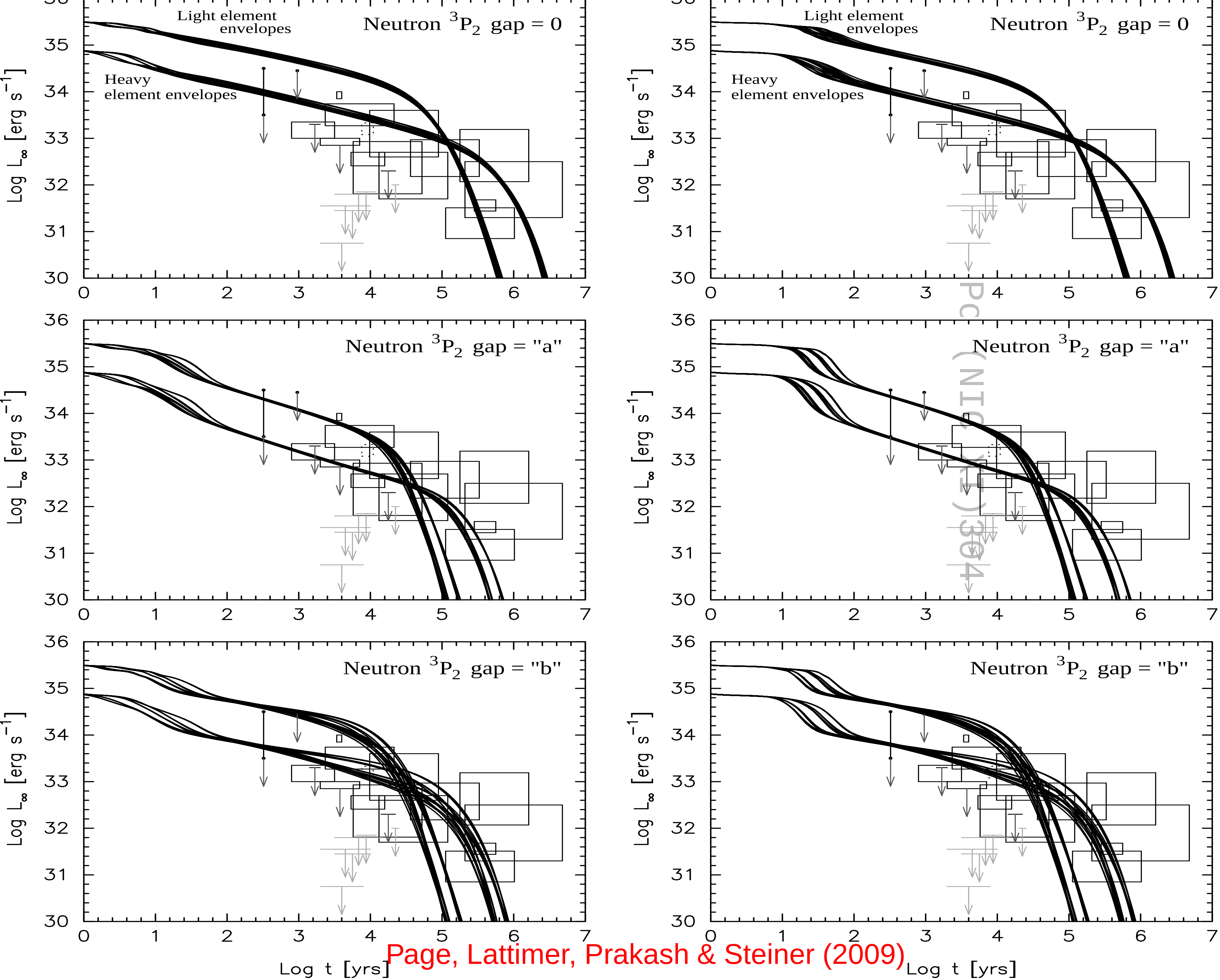
Neutron Star Cooling



Minimal Cooling Paradigm

- Minimal Cooling Paradigm: Neutron star cooling including effects of superfluidity, such as Cooper-Pair breaking and formation, but no “rapid” neutrino cooling processes such as direct Urca involving nucleons or exotica. (Page et al. 2004)
- If some observations are inconsistent with the MCP, then according to Sherlock Holmes, rapid cooling must occur for these exceptions.

Minimal Cooling Paradigm



Page, Lattimer, Prakash & Steiner (2009)

Minimal Cooling Paradigm

- If some observations are inconsistent with the MCP, then according to Sherlock Holmes, rapid cooling must occur for these exceptions.

- All sources are consistent with the MCP only IF
  - tight conditions are placed on the magnitude and density dependence of the neutron ^{3}P$_2$ gap, AND
  - some neutron stars have heavy Z envelopes and others have light Z envelopes, AND
  - ALL core-collapse supernova remnants with no observable thermal emission contain black holes.

- Highly suggestive that rapid cooling occurs in some neutron stars (of higher masses?)

- A possible constraint on $E_{sym}(n)$ or $n_{central}$.